Dissipative solitons in the coupled dynamics of light and cold atoms

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Abstract: We investigate the coupled dynamics of light and cold atoms in a unidirectional ring cavity, in the regime of low saturation and linear single-atom response. As the dispersive opto-mechanical coupling between light and the motional degrees of freedom of the atoms makes the dynamics nonlinear, we find that localized, nonlinearity-sustained and bistable structures can be encoded in the atomic density by means of appropriate control beams.

OCIS codes: (020.1335) Atom optics; (190.4420) Nonlinear optics, transverse effects in; (190.6135) Spatial solitons.

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Spatial dissipative solitons are stable, self-localized structures which exist in open systems driven far from thermal equilibrium, and with general features which cut across nonlinear sciences [1]. In the context of optics they can be realized in nonlinear pattern-forming systems, such as nonlinear cavities [2, 3], where a self-localized structure for the light is encoded in the internal excitation of the nonlinear medium. In warm atomic gases, the internal state is constituted by the populations and coherences of the atoms, while the motional degrees of freedom are irrelevant as optical forces are overwhelmed by thermal effects. In recent years, however, spatial self-organization due to the coupling of light and the motional degrees of freedom of cold [4–11] and ultracold [12–14] matter attracted remarkable interest. The spontaneous appearance of off-axis far field sidebands indicating transverse structuring in a low aspect ratio regime was reported in [15]. Spontaneous symmetry breaking leading to large-scale patterns in the plane transverse to the propagation of a single beam has been predicted in [16] for a cavity configuration, and was observed in a single-mirror setup with Rb atoms [17, 18] .
We shall demonstrate in the following that the model discussed in [16] for pattern formation can also support dissipative solitons. This model is developed assuming strong viscous damping for the atomic velocities (see also [10, 11]) and can be extended to soft matter systems, where solitonic self-trapping of beams has already been predicted [19–21] and observed [22]. Recently coupled energy-matter dissipative solitons have also been proposed [23]. The system analyzed here also shares some formal similarities with the optomechanical cavity system analyzed in [24], where dissipative structures are found for an empty cavity with a deformable mirror. However, we remark that in [24] the relevant optical forces are given by radiation pressure on the mirror, while we deal here with dipole forces on the atoms. An important consequence of this is that the system analyzed here does not support plane-wave bistability.
A fundamental property of dissipative solitons is the fact that they are bistable, and thus can be switched on and off 'at will' by means of appropriate control beams [2]. In the cold-atoms situation studied here, bistable structures are thus encoded in the spatial density distribution of the gas. At difference with atomic lithography [25], however, here one is allowed to optically 'write' and 'erase' a given density structure which remains unaffected after the control beams have been turned off, sustained just by a homogeneous pump (see Fig. 1).

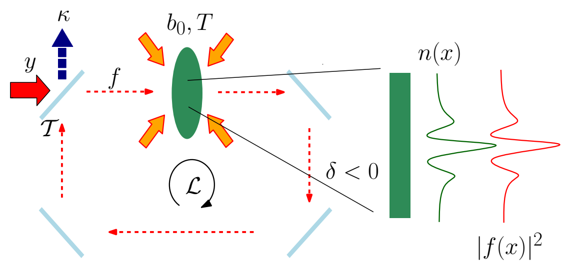

Fig. 1. A plane wave of amplitude A_{in} drives a single-longitudinal-mode cavity characterized by a length $\mathscr{L}$, mirror transmittivity $\mathscr{T}$, and lifetime κ^{-1}. The intracavity field f interacts with a cloud of two-level laser-cooled atoms (optical density b_0, temperature T). Optical molasses are assumed to act on the cloud during such interaction. A self-localized state for the cloud density and the optical field can be sustained by the homogeneous driving (right inset, the red-detuned case is shown).

We consider a sample of N two-level atoms laser-cooled at a temperature $T \sim 100\,\mu$K and placed along the axis of a single-longitudinal-mode ring cavity of length $\mathscr{L}$. In this work we assume low saturation and treat the atoms as linear scatterers, but we allow for density redistributions of the atomic density in the plane transverse to the cavity axis z, so that $N_0 n(\mathbf{x})$ denotes the atomic density at the transverse coordinate $\mathbf{x} = (x, y)$, where N_0 is the homogeneous density. The optical cavity is driven by a plane wave of amplitude A_{in} and frequency ω_0, so that using the slowly varying envelope, rotating wave, paraxial, and mean field approximations the intracavity field f is governed by the wave equation (see also [3]):

$$\frac{\partial f}{\partial t} = -(1 + i\theta) f + A_{\text{in}} - i\gamma n f + i\nabla^2 f \,. \tag{1}$$

All the quantities appearing in Eq. (1) are adimensional. We specifically rescaled time to the cavity lifetime $\kappa^{-1} = \mathscr{L}/(c\mathscr{T})$ (c speed of light in vacuum, $\mathscr{T}$ mirror transmittivity) and space to the diffraction length $\sqrt{a} = [\lambda_0 \mathscr{L}/4\pi\mathscr{T}]^{1/2}$, where λ_0 is the radiation wavelength. The light-cavity detuning is given by $\theta = (\omega_0 - \omega_{\text{cav}})/\kappa$, while γ parametrizes the cloud susceptibility. Assuming the optical field to be detuned far from the atomic resonance ω_{at}, absorption can be neglected and the susceptibility is given by $\gamma = b_0\Delta/[2(1+\Delta^2)]$, with b_0 the optical density in resonance, $\delta = \omega_0 - \omega_{\text{at}}$ the light-atom detuning and $\Delta = 2\delta/\Gamma$ the half-linewidth detuning (Γ is the inverse atomic lifetime). Scaling is such that $I = |f|^2$ gives the off-resonance saturation parameter associated to the intracavity intensity.

In warm gases Eq. (1) has to be modified to account for excitation of the electronic transition, giving at lowest order a Kerr-like nonlinear response. This leads to the paradigmatic Lugiato-Lefever model of nonlinear optics [3], which is well known to display plane-wave bistability, pattern formation, and localized structures. However, even in the limit of a linear single-atom response density redistributions due to optical forces lead to nonlinear effects, so that pattern formation is made possible [16]. Bubble instabilities driven by diffused light were also predicted to occur in such 'linear optical' regime [26]. We also remark that in correspondence with the homogeneous state ($n_{\text{h}} = 1$) there is no nonlinearity, so that no plane-wave bistability is possible from Eq. (1).

The wave equation (1) for the optical field must now be supplemented with a material equation for the atomic density distribution. The density modulation n is coupled to the field dynamics through the action of dipole forces, which in the limit of large detuning and low saturation are given by $\mathbf{F}_{\text{dip}} = -(\hbar\delta/2)\nabla|f|^2$. Assuming optical molasses to damp the atomic velocities during the interaction with the field, a Fokker-Planck equation can be derived for the density

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modulation [10, 11]:

$$\frac{\partial n}{\partial t} = \frac{\hbar\delta}{2k_B T} D\nabla \cdot \left[n\nabla |f|^2 \right] + D\nabla^2 n \,, \tag{2}$$

where D is a diffusion constant depending on the details of the optical molasses [27]. The stationary solution of Eq. (2) is the equilibrium distribution

$$n_{\rm eq}(\mathbf{x}) = \frac{\Omega \exp\left[-\sigma |f(\mathbf{x})|^2\right]}{\int_\Omega d\mathbf{x}' \exp\left[-\sigma |f(\mathbf{x}')|^2\right]} \qquad \sigma = \frac{\hbar\delta}{2k_B T} \,, \tag{3}$$

where Ω is the transverse size of the cloud, so that a stationary solution for the system can be obtained by inserting (3) into (1), and solving the latter until a stationary state is reached. We remark that we are interested here in determining the stationary state which attracts a given basin of initial conditions and not in the temporal dynamics of the system. The method described above has been used in [16] to numerically investigate spontaneous structuring due to optomechanical forces. In fact, it can be shown that the homogeneous solution $f_{\rm h} = A_{\rm in}/[1 + i(\theta + \gamma)]$, $n_{\rm h} = 1$ is unstable to transverse perturbations when the intracavity intensity $I_{\rm h} = |f_{\rm h}|^2$ exceeds the critical intensity $I_{\rm c} = 1/(\sigma\gamma)$. Hexagons for the optical intensity emerge spontaneously in the transverse plane as the result of a symmetry breaking process, with complementary structures encoded in the atomic density distribution. Recently, opto-mechanical self-structuring was observed experimentally in cold Rb gases with a single-mirror feedback [18].

Whenever spatial structures emerge as a result of an instability, it is interesting to ask whether such structures can coexist with a stable homogeneous background. If this happens, in fact, self-localized solutions (e.g. dissipative solitons) are possible for the system as different regions of the transverse domain can display different stationary solutions (structured/homogeneous). Such structures are localized by the nonlinearity and have the fundamental property of being bistable, i.e. they can be switched on and off by appropriate control beams, a property that makes them interesting candidates for optical information processing and storage [1]. A peculiar feature of the system analyzed here is that a stationary, self-localized structure would be encoded in the density distribution of the atoms, whereas the internal excitation of the system is typically involved in hot-atoms pattern formation. This opens up new opportunities for the shaping and control of potentially quite complex and reconfigurable atomic density distributions, which only need homogeneous driving to sustain after they have formed.

We thus envisage a situation where one can 'write' and 'erase' coupled light-matter structures in a controllable way by inducing localized structures in atomic density. Suppose in fact that a region of high density is created in the atomic cloud during the interaction with the pump beam. This could be achieved for instance by using additional control beams, completely incoherent with the pump. The 'slow' timescale for this process is essentially dictated by the time for atomic motion, $\tau_{\rm ext} \sim 10\,\mu$s (to move by $\sim 1\,\mu$m at $T \sim 100\,\mu$K). Light from a red-detuned (and spatially homogeneous) pump beam would then be guided towards high density regions, effectively creating a bright light spot. The 'fast' timescale for this process will be determined by the cavity lifetime, $\kappa^{-1} \ll \tau_{\rm ext}$. If localized states are stable for the system (i.e. if bistability is obtained for the 'homogeneous' and 'patterned' solutions), such localized spot of light can sustain itself via nonlinear dipole forces. Analogously, one could create a 'hole' in the atomic cloud (on the timescale of $\tau_{\rm ext}$), which in turns would attract blue-detuned light from the spatially homogeneous pump (on the timescale of κ^{-1}). Such a bright localized spot of light can then sustain itself by expelling atoms from the high intensity region, again by means of dipole forces. In both cases, self-localized structures are eventually sustained just by a homogeneous driving. Additional beams can then be used to erase any given density structure present in the cloud, thus removing the entire light-matter structure.

To demonstrate that a bistable regime for the homogeneous/patterned solutions is indeed possible, Fig. 2(a) shows the bifurcation diagram for the instability obtained from one-dimensional

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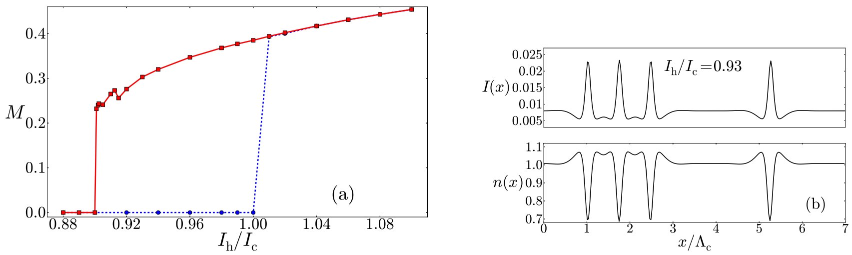

Fig. 2. (a) Crossing the intensity threshold I_c for self-structuring the homogeneous solution becomes unstable and a 'patterned' solution for the density $n(x)$ bifurcates subcritically (blue circles). As the control parameter I_h is decreased, the pattern survives below the critical value I_c and loses stability for $I_h/I_c < 0.9$ (red squares). Parameters are: $\sigma = 25$, $\gamma = 4.5$, and $\theta = -3.7$. M denotes the maximum deviation from the background value $n_h = 1$. The right panel (b) shows a solution obtained at $I_h/I_c = 0.93$, displaying a dissipative soliton and a three-peaks localized pattern.

simulations with $\sigma = 25$, $\gamma = 4.5$, and $\theta = -3.7$. Taking the D_2 line of ^{87}Rb as a reference (transition wavelength $\lambda_0 = 780.27\,$nm), these parameters would correspond to a beam blue-detuned by $\delta = +10\,\Gamma$ from the resonance of a cloud with optical density $b_0 = 180$ and temperature $T \simeq 60\,\mu$K. Eq. (1) is solved with $n = n_{eq}$ using a split-step Fourier method until a final time of 10^3 cavity lifetimes (using 10^5 time steps). Periodic boundary conditions are implemented over a domain of 7 critical wavelengths $\Lambda_c = 2\pi/[1-(\theta+\gamma)]$ [16]. Discretization is performed on a grid of 256 (256×256) points for one (two) dimensional simulations. The homogeneous intensity $I_h = |f_h|^2$ is varied crossing the critical value $I_c = 8.8\times10^{-3}$, and the resulting density pattern is monitored through the maximum deviation from the background value, $M = \max|n(x) - n_h|$. As the 'patterned' branch bifurcates subcritically, a bistable region $0.9 < I_h/I_c < 1$ is obtained where both the spatially modulated and the spatially homogeneous solutions are stable. Coupled light-matter dissipative solitons are therefore stable in this domain. The system also allows for the simultaneous and independent coexistence of cavity solitons and localized patterns, as shown in Fig. 2(b). The small variations in the quantity M along the upper branch are a known effect of the discretization of the reciprocal domain. We remark that, although these results are obtained from one-dimensional simulations, no differences are expected in two dimensions as the stability properties of the system depend only on the wavenumber $|\mathbf{q}|$. However, the fact that the bifurcation is subcritical already in one dimension indicates that soliton formation is a robust feature of our system. Hexagons are in fact always subcritical in nature, and the subcriticality domain is expected to widen moving to two transverse dimensions.

Fig. 3 shows the results from two-dimensional simulations, for the same parameters as Fig. 2(a) and $I_h/I_c = 0.93$. Here we start the simulations with a localized light-matter state (with complementary gaussian profiles), and find that these states are attracted towards the structures displayed in Fig. 3. As discussed earlier, such an initial condition could be prepared by means of appropriate control beams. Self-localized solutions are obtained in the transverse plane (x,y) for the atomic density (right panels), with corresponding structures encoded in the optical intensity (left panels). The spatial size $\sim \Lambda_c\sqrt{a}$ of these structures can be controlled by varying the cavity detuning θ, the susceptibility γ and/or the diffraction length $\sqrt{a}$. For a mirror transmittivity $\mathscr{T} = 0.1$ and a cavity length $\mathscr{L} = 1\,$mm we obtain a size $\sim 0.5\,$mm, which is well within the transverse size of modern setups. As can be seen from Fig. 3, light accumulates in atom-depleted areas in the blue-detuned regime, while it is guided towards atom-rich areas

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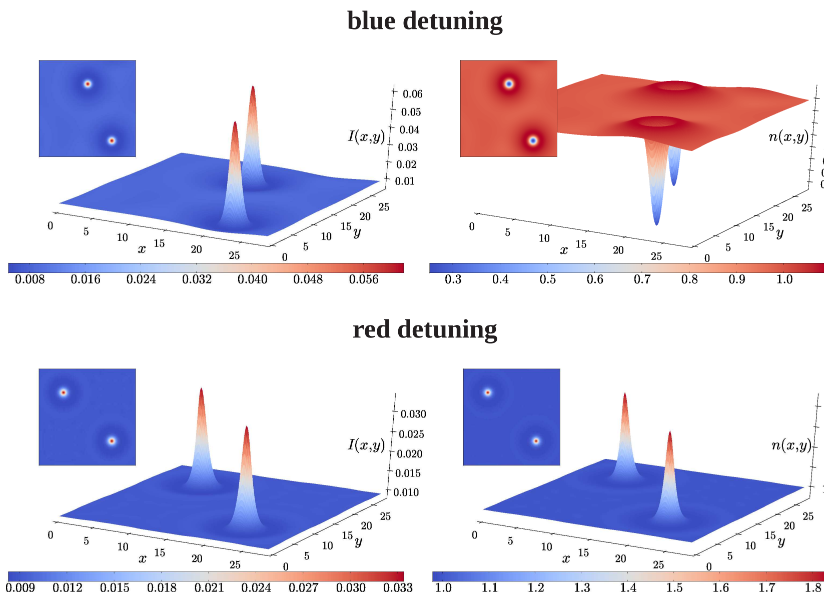

Fig. 3. Two-dimensional intensity (left) and density (right) profiles obtained from numerical simulations for the same parameters as Fig. 2(a) and $I_{\mathrm{h}}/I_{\mathrm{c}} = 0.93$. Two stable, self-localized structures are formed for the atomic density, sustained by expelling atoms from (attracting atoms towards) regions of high optical intensity in the blue (red) detuned regime.

in the red-detuned case. However, as in Ref. [16] we find that the blue-detuned regime displays higher stability, as the expulsion of atoms results in a strong saturation effect. As in the one-dimensional case we find that these structures can coexist independently on the same homogeneous background. Moreover, they can be switched on and off independently or, if the system is prepared with broader initial localized profiles, multi-peaked localized states can be obtained (see also [28]). This paves the way to the development of addressable techniques in which a spatial structure is encoded 'at will' in the spatial density distribution of a cold gas.

In conclusion, we investigated the formation of dissipative solitons in the coupled dynamics of light and cold atoms. In the limit of low saturation no nonlinearities arise from the internal state of the atoms, but nonlinear redistribution effects can lead to spontaneous structuring, see [16]. We numerically demonstrated the existence and stability of self-localized light-matter solitons, encoded at will in the transverse atomic density distribution. The model developed here assumes optical molasses to provide velocity damping, but this was found to be inessential for the self-structuring process, both theoretically and experimentally. The recent observation of hexagonal self-structuring in cold Rb [17, 18] is a promising starting point for the experimental study of coupled light-matter dissipative solitons. Future work could include electronic nonlinearities arising from saturation of the involved optical transitions. Another interesting line of work is to extend the study presented here to the regime of quantum degeneracy where matter wave coherence could yield novel features.

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