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An impact assessment of electricity and emission allowances pricing in optimised expansion planning of power sector portfolios

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Abstract

The present work concerns a systematic investigation of power sector portfolios through discrete scenarios of electricity and CO2 allowance prices. The analysis is performed for different prices, from regulated to completely deregulated markets, thus representing different electricity market policies. The modelling approach is based on a stochastic programming algorithm without recourse, used for the optimisation of power sector economics under multiple uncertainties. A sequential quadratic programming routine is applied for the entire investigation period whilst the time-dependent objective function is subject to various social and production constraints, usually confronted in power sectors. The analysis indicated the optimal capacity additions that should be annually ordered from each competitive technology in order to substantially improve both the economy and the sustainability of the system. It is confirmed that higher electricity prices lead to higher financial yields of power production, irrespective of the CO2 allowance price level. Moreover, by following the proposed licensing planning, a medium-term reduction of CO2 emissions per MWh by 30% might be possible. Interestingly, the combination of electricity prices subsidisation with high CO2 allowance prices may provide favourable conditions for investors willing to engage on renewable energy markets.

Keywords

Electricity Prices; Power Sector Portfolio; Optimisation; Operational Research; Emissions trading
1. Introduction

Optimal expansion planning of electricity production has been an important goal for grid designers, energy policy makers, operational researchers and economic analysts. The incomes from electricity selling to the grid, as well as the power production costs, both determine the economics of the power sector, though to a different extent. The compensation of the electricity producers depends on the System Marginal Prices (SMP), which are announced by the grid operators based on the market dynamics. On the other hand, the costs of power production may depend on the installation of mature but proven workhorse technologies or, on the displacement of obsolete plants by emerging eco-friendly technologies necessitated by the recent environmental directives. The fuel costs and the power loads demanded by the end-users may additionally impact the aggregate power production costs. Moreover, the economics of the power sector depend on the current generation mix thus leading to initiatives related with its expansion planning. Such initiatives may include deregulated pricing policies as well as the determination of capacity shares from each competing technology.

Electricity production planning may be influenced by interventions on licensing policies, fiscal conditions and price volatility. The evaluation of policies used for the assistance of project designers usually requires the discounting of future incomes and expenses. The Discounted Cash Flow (DCF) method has been extensively used in past energy investment studies. These were characterised by incorporating arbitrary values of risk premium in the interest rates, which were assumed to remain constant throughout the projects’ operational life time. In practice, high values of interest rate were used and moreover, immediate investments were considered.

Modern business plans have focused on time-varying methods and therefore, previously irreversible investment decisions are progressively replaced by flexible planning, subject to optimisation processes. Thus, the minimisation of production costs and the determination of the optimal generation mix may be inquired over time. Significant contributions on power sector
portfolio analysis and optimisation have been recently available, focusing on non-probabilistic or probabilistic models [1, 2] for the representation of multiple uncertainties, usually confronted in energy investment analysis, expansion planning and the related CO2 mitigation strategies.

With these advances on hand, the analysis of energy investments may now utilise powerful computational tools applicable for many case studies. The present article addresses the case of the Greek Power Sector. The capacity additions -recently installed and connected to the national grid- have been questioned for their environmental implications and financial performance, especially when considered in the context of the EU emissions trading system (EU-ETS), which has set the rules of the emissions’ trading market. On the other hand, the power demand is marginally served in peak seasons (e.g. in temperature extremum) thus stressing the underlying grid commitment issues. Moreover, the Greek Power Sector has been criticised for not complying with the emission constraints, set for its power generation mix. Therefore, an urgent need has emerged for a thorough analysis of the Greek Power portfolio aiming at the minimisation of the aggregate production cost, while keeping the power supply failures (system black-outs) to a minimum and complying with the European environmental constraints. This type of analysis may also contribute to the identification of investment opportunities based on emerging technologies. Attractive projects might then be realised, given that an admissible modelling of electricity and emission allowance pricing is available.

The objective of the research is the assessment of the impact of electricity and CO2 allowance prices to the future structure of the Greek Power Sector. The ultimate goal is the determination of the optimal generation mix which is inquired on an annual basis in order to set the power licensing policy for the oncoming 25 years. Four different electricity price scenarios are investigated: a) Prices regulated and evolving according to stochastic inflation rates, b) deregulated and simulated through random walk process based on historical data, c) semi-regulated to the extent that the prices for conventional producers are deregulated and stochastically evolving according to historical data, while the prices for renewable energy producers are fixed and evolving according to stochastically evolving inflation rates and d) semi-regulated to the extent that the prices for conventional producers are deregulated and
stochastically evolving according to historical data, while the prices for renewable energy producers are fixed and follow the drift of the conventional prices. Furthermore, two scenarios of CO2 allowance prices evolution are assumed thus dealing with the uncertain behaviour of emissions trading markets: I) The allowance prices regress around their current levels by following a random walk process and II) the allowance prices revert to a long-run mean of 31 Euros/tn CO2. The above 8 scenarios (A,B,C,D) – (I, II) may be interesting for policy makers willing to provide optimal electricity production in terms of system economics and anticipated CO2 emissions, while meeting the power demand targets and complying with the imposed market and environmental constraints.

Apart from the analysed pricing impact in optimised expansion planning, the contribution of the present study also lies on the non-linear stochastic programming approach, the utilised constraints as well as on the statistical analysis of the results. The above will be analysed in detail in the next sections, which are essentially structured as follows: In Section 2, the related recent studies are presented. In Section 3 a description of the case study is given and the mathematical modelling of the problem is formulated. The technical details of the numerical algorithm and the input data are described in Section 4. Section 5 includes the results of the numerical experiments conducted, with critical comments on the relevant graphical representations. A statistical error analysis of the results is performed in Section 6 to validate the model’s reliability. Finally, in Section 7 the concluding remarks of the research are summarised.

2. Background

2.1 Power portfolios, electricity pricing impact and risk analysis

Bar-Lev and Katz (1976) [3] introduced the mean variance portfolio analysis in fossil fueled Power Sector. More recent research [4, 5, 6] extended the analysis to various power expansion mixes. Focus on mean-variance portfolios has been shown in some applications testing different risk measures [7, 8]. Mean-variance frameworks have been proposed to address the energy portfolio planning and the optimal allocation of positions in peak and off-peak forward price contracts [9]. It has been shown that optimal allocations are based on the risk premium
differences per unit of day-ahead risk as a measure of relative costs of hedging risk in the day-ahead markets. The influence of the risk management has been also analysed in further studies concerning either solely electricity production or multi-objective functions comprising of combined heat and power production [10, 11]. Multiple objectives have been also addressed in some case studies of discrete regional power portfolios under demand uncertainty [12, 13]. In [13] the multi-objective function was extended by assigning cost penalties to non-cost attributes to force the optimisation to satisfy non-cost criteria, while still complying with environmental and demand constraints. The impacts of uncertain energy prices on the supply structures and their interaction with the demand sectors have been analysed in the work of Krey et al. (2007) [14].

Decision support tools have been developed [15] seeking for globally optimal solutions, taking into account financial and economical conditions and constraints imposed at an international level.

The analysis of individual power-plant strategies (i.e. [16, 17, 18]) as well as the optimisation of power expansion planning (i.e. [19, 20, 21]) might be separated in two discrete research categories. The present study might be classified in the latter category in which the optimal structure of power generation may be inquired, though considering individual investment opportunities in a broader sense: Suboptimal generation mixes imply that individual plants might sometimes operate under imperfect financial conditions. On the contrary, compliance with optimal investment timing might lead to optimal system NPV thus allowing significant profit chances for individual players. The optimisation of energy portfolios relies on the determination of the objective function representing the aggregate Net Present Value (NPV) of the system (Power Sector). This function has to be optimised while being subject to an appropriate set of constraints. The resulting optimal point determines the power generation mix for which the aggregate system NPV is maximised, thus indicating the optimal investing time as well as the upper bound share of capacities from each technology, allowed to be ordered in a specific time point.

The optimisation of the power generation mix may entail complicated algorithms for handling the non-linear functional relationships of multi-variable sets and constraints. Multiple uncertainties may be introduced as inputs, whilst the Lagrangian functions usually become discontinuous due
to various conditional algorithmic statements, thus inhibiting smooth, global results. Reliable trust-region algorithms have been recently available [22, 23] thus allowing more accurate results within acceptable computational times. They are widely used in operational research applications, but the competing Sequential Quadratic Programming (SQP) solvers may outperform them both in terms of accuracy and reliability. SQP routines are based on the solution of the Karush-Kuhn-Tucker (KKT) equations [24] which are comprised of a combined Lagrangian formulation of the objective function and the constraints set. The SQP solvers are used in order to approximate the KKT equations with a convex quadratic formulation, thus ensuring continuity. Moreover, convexity constitutes both necessary and sufficient condition for the KKT equations to be resolved thus leading to global optimisation of the quadratic approximation. An overview of the KKT equations and details of the computational SQP solvers may be found in Fletcher, (1987) [25] or Biggs and Hernandez (1995) [26], but they are also outlined here in Appendix A.

In the present research, a non-linear SQP routine is embedded in a stochastic programming algorithm without recourse. This method leads to a single strategy for the entire time horizon, operating on the average of all the stochastic input future states. The stochastic programming approach has been effectively used in past and recent researches of power generation planning under uncertainty [13, 21, 27, 28]. The research analysed in [13] in particular, comprises of a stochastic programming algorithm with recourse, thus embedding the uncertainties’ modelling in the optimisation iterations. Nonetheless, linear solvers were mainly used in the above studies, instead of the non-linear SQP solver implemented in the present research. Moreover, different objective functions and constraints are used here, approaching the environmental limitations, the emissions trading system, as well as the stability and the reliability of the grid.

2.2 Time dependent investments under uncertainty

The analysis of power sector portfolios under multiple uncertainties may depend on the evolution of various stochastic variables, which determine the efficiency of energy investment opportunities. Various computational algorithms have been recently applied for the representation of stochastically evolving variables. The autoregressive models [29] are mainly used for the simulation of stationary and non-stationary time-series, whilst the random-walk algorithms [30], including Ito and Ornstein-Uhlenbeck processes, may be considered as particular cases of
autoregressive models when dealing with discrete time. In computational practice, the above two processes are applied through the Geometric Brownian Motion (GBM) and the mean reverting (MR) models respectively. The Cox Ingersoll Ross (CIR) and the Vasicek models [31, 32, 33] are mean-reverting derivatives mainly used for the prediction of interest and inflation rates. The accuracy and the computational cost of the above mentioned random-walk algorithms may depend on the utilised solvers which are basically comprised of Euler-Marujama routines [34, 35]. Moreover, they may depend on the number of a Monte-Carlo solver trials [36] used to average the multiple lognormal solutions set.

Among the stochastic variables commonly seen in the analysis of energy investments, the electricity prices, the fuel and the CO2 allowance prices as well as the energy demand, represent the most significant business, and/or climatic uncertainties [17, 37, 38, 39, 40, 41]. GBM and MR models have been recently applied for the representation of the evolution of stochastic electricity prices [17, 21, 42]. The discounting factor and more specifically the interest rates may introduce additional uncertainties in the model. Ingersoll and Ross (1992) [33] and Tolis et al. (2010) [43] suggest that the interest rate volatility may be an important factor for the investment decision. In the review of Dias and Shackleton (2005) [44] switching between the options to invest or disinvest is analysed considering different methods of stochastically evolving interest rates: the (CIR) model and the Vasicek model. In more recent studies [45] it is argued that the simulation of interest rates may contribute to the elimination of arbitrary risk premium assumptions, mainly within the framework of real-options models. The capital costs may decrease over time due to the global experience on similar projects, introducing further time-dependent characteristics in the analysis of energy investments [21, 46, 47].

3. Modelling approach

3.1 Uncertainties modelling and assumptions

In the present research, forward contracts have been considered for the prices of electricity selling to the grid. Forward prices may not contain information about the very short term variations of the underlying spot prices as the latter are averaged over discrete time intervals [48]. Both spot
and forward prices are characterised by normally distributed variations (noise) as observed in the raw data sets retrieved by the Greek System Operator [49]. Based on the fact that the GBM and the MR processes require normally distributed Brownian differentials, it is assumed that:

A- The evolution of forward prices is represented through random walk processes (GBM/MR).

GBM and MR models have been adopted in several past researches for the representation of the evolution of forward electricity prices, thus considering the absorption of jumps and spike processes over time [16, 50, 51]. Either endogenous [17] or exogenous [21, 42] modelling of the evolution of spot electricity prices through GBM or MR processes have been adopted in past studies, indicating a wide range of applications. It has been shown that the impact of investors’ actions on electricity prices is minimised [48, 52] when multiple investors are considered, provided that the following two assumptions are additionally made:

B- Market equilibrium is considered.

C- The investors’ profit is a direct function of electricity prices.

In the present work the above assumptions have been also adopted and therefore, the effect of multiple investors’ actions on the evolution of forward prices has been considered to be minimal. However, the dynamics of the electricity prices evolution may additionally depend on the evolution of the remaining stochastic commodities for which uncertainty may be introduced: fuel and CO2 allowance prices, electricity demand as well as interest and inflation rates. For this reason, a correlation of all the participating Brownian differentials has been performed, thus permitting some form of endogenous modelling of the uncertainties. The required statistical factors of data correlation have been extracted from the historical data of the stochastic variables whilst a similar procedure has been followed for the mean drift and the volatility parameters as in Clewlow and Strickland (2000) [53].

The evolution of the stochastic variables is based on the numerical solution of the corresponding Stochastic Differential Equations (SDE). GBM models are used when the past data fail to converge to a long run mean, whilst MR models are used in the opposite case. The evolution of the interest rates is represented through a CIR model, which is an MR derivative, suitable for
producing non-negative forecasts, as required in real world interest rates. The entire Monte-Carlo approximation of the underlying uncertainties is comprised of the following two steps: a) producing multiple SDE solutions using an Euler-Maruyama solver and b) averaging the multiple solution sets; the average path of each variable is further introduced as input in the optimisation algorithm. The above process precedes the optimisation algorithm, thus allowing the reduction of the floating point operations during its iterative numerical processes. Alternatively, one may run the optimisation code for each one of the multiple input combinations of SDE paths, requiring an excessive number of complete optimisation cycles. However, this option, namely a stochastic programming algorithm with recourse [13, 42] has not been the approach of the present work. Various reasons like the numerous variables, comprising of long-term investment horizon (>40 years) for various technologies, as well as numerous non-linear constraints, promoted the decision for the final simplified approach, namely a stochastic programming algorithm without recourse.

3.2 An overview of the case study

The Greek power sector is the market under uncertainty for which an optimal structure is sought, in terms of energy production and fuel sources. A variety of technologies are examined to form the optimal portfolio. Existing base-load technologies (mainly fossil fuelled) and competing emerging technologies (mainly based on renewable sources) are considered as potential contributors to the power generation mix. The objective function is comprised of the aggregate benefit from electricity selling to the grid, subject to environmental constraints and State regulations. The stability of the grid, the total power demand, and the availability of energy sources are additional targets to be met. Four different scenarios are investigated in order to assess the impact of electricity prices evolution:

a) electricity prices regulated: evolving according to inflation rates,

b) electricity prices deregulated: forecasted according to a stochastic procedure and based on historical data,
c) electricity prices semi-regulated: prices of conventional producers are deregulated and stochastically evolving according to past data, while the prices for renewable energy producers are contractually fixed and evolving according to inflation rates.

d) electricity prices semi-regulated: prices of conventional producers are deregulated and stochastically evolving according to past data, while the prices for renewable energy producers are contractually fixed and follow the drift of the conventional forward prices.

The above electricity prices are further processed in order to form the forward prices evolution paths, required for the optimisation model. Apart from the electricity price uncertainty, a non-stationary behaviour has been modelled for the CO2 allowance prices, by assuming two discrete scenarios:

I) CO2 allowance prices regress around their recent observations slightly increased in the long term,

II) CO2 allowance prices regress around a higher long run mean (31 Euros/tn CO2) with a moderate mean-reverting speed.

3.3. Mathematical formulation

The context of the study is the existing power sector, in which the prices of electricity, fuel and CO2 allowances as well as the inflation rates are assumed to evolve through random walk (GBM) processes, as formulated by the following equation 1:

\[ dA_t = \mu(t) \cdot A_t \ dt + D(t, A_t) \cdot V(t) \ dW_t \]  

(1)

where by \( A_t \), the above mentioned stochastic variables are denoted using a generic notation. The evolution of the risk-free interest rates is represented through the Cox-Ingersoll-Ross (CIR) model (equation 2), resulting to non-negative forecasts. This is ensured by the component \( r_t^{1/2} \) which is included in the diffusion vector function (D):

\[ dr_t = \mu(t) \cdot [L(t)-r_t] \ dt + D(t, r_t^{1/2}) \cdot V(t) \ dW_t \]  

(2)
The use of stochastic interest rates contributes to the endogenous modelling of the multivariate problem by avoiding arbitrary assumptions and this may have direct implications on the financial results, as explained in [43]. Investing in the electricity market may incorporate some risk. An approximation of the risk uncertainty may be derived from the combined simulations of the underlying fiscal uncertainties, thus reflecting the difference between a stochastic NPV calculation (with optimised investment entry times) and a traditional DCF calculation. In the present study, the discounting factor \( D_z \) is calculated using the above non-constant, risk-free interest rates (equation 2) and a discrete discounting formulation:

\[
D_z = \prod_{t=1}^{z} (1 + r_t)^{-1}
\] (3)

The independent variables of the optimisation problem are comprised of the capacities ordered in the year \( v \) (\( X_{i,v} \) in MW) and the load factor \( \theta_{i,z} \) of the operational power plants during year \( z \) (\( z \neq v \)), for power plants of technology \( i \). \( \theta_{i,z} \) is a dimensionless factor representing the usage intensity of power plants i.e. the actual operating time over the annually available time.

A detailed modelling of the capacities installed in different time spots is further required. Thus, the capacity additions (either ordered in the past or planned for the future) may be linked with the currently installed capacity (\( L_{i,z} \) in MW\textsubscript{el}) using the following formulation:

\[
L_{i,z} = \sum_{v=1}^{z-T_i} X_{i,v} + \sum_{v=40}^{0} C_{i,v} - \sum_{v=1}^{z} \overline{X}_{i,v} - \sum_{v=40}^{0} \overline{C}_{i,v} \quad \forall i \quad 0 \leq z \leq Y
\] (4)

where:
- \( \overline{X}_{i,v} = 0 \) \quad \( v + T_{i,v} \leq z \leq v + T_{i,v} + T_{o_i} \)
- \( \overline{X}_{i,v} = X_{i,v} \) \quad \( v + T_{i,v} + T_{o_i} \leq z \)
- \( \overline{C}_{i,v} = 0 \) \quad \( v + T_{i,v} \leq z \leq v + T_{i,v} + T_{o_i} \)
- \( \overline{C}_{i,v} = C_{i,v} \) \quad \( v + T_{i,v} + T_{o_i} \leq z \)

The dashed symbols denote the capacity additions that have already exceeded their operational life time. With the above relationship it is ensured that the plants exceeding their operational lifetime, may not contribute anymore to the power production process. The electricity energy production \( P_{i,z} \) (\( P_{i,z} \) in MWh\textsubscript{el}) represents the aggregate energy produced by the operational power plants of type \( i \) during year \( z \):
This is further used in the discrete optimisation as a factor of the incomes and the expenses contributors. The objective function represents the NPV of the system and includes the aggregate incomes (electricity selling and emissions trading) and expenses (fixed costs, variable costs, emission allowances, investment costs etc.) in present values. Its mathematical formulation may be represented by the following equation:

\[
\text{NPV}(X_{i=1,z}, \ldots, X_{i=1,z}, \theta_{i=1,z}, \ldots, \theta_{i=1,z}) = \\
\max \left\{ \sum_{i=1}^{l} \sum_{z=1}^{v} p_{r} D_{r}^{z} P_{r}^{z} - \sum_{i=1}^{l} \sum_{z=1}^{v} C_{r}^{z} D_{r}^{z} P_{r}^{z} - \sum_{i=1}^{l} \sum_{z=1}^{v} C_{v}^{z} D_{v}^{z} I_{v}^{z} - \sum_{i=1}^{l} \sum_{z=1}^{v} I_{i}^{z} X_{i}^{v} \right\}
\]

(6)

where the factor \(D_{z}\) is calculated using equation 3. A more detailed expression of equation 6 can be seen in Appendix A (equation A5) where all the different contributors of the NPV function are analytically expressed in relationship with the requested capacity orders. One may notice that the maximisation of the aggregate system NPV is required instead of the minimisation of the aggregate power production cost, thus allowing the estimation of the anticipated profits or losses.

Concerning the energy produced in excess to the demand, it is noted that this condition may occasionally occur during the numerical iterations of the optimisation algorithm until it converges to an optimal solution. In the present research, any excessive energy produced is assumed to have zero (income) value, as it cannot be exploited by the system. On the other hand, the production costs of the non-served energy, contribute (being a part of the expenses) in the NPV objective function and therefore they are always calculated (equation 6). As a result, the optimisation algorithm tends to eliminate any excess energy production, by iteratively attempting to reduce the overall system costs.

The last two terms in equation 6 represent the emissions trading costs and revenues respectively. The expenses of the required emission allowances for conventional power plants are represented by the left term with the (-) sign. The revenues are represented by the right term with the (+) sign.
and correspond to the incomes from trading the emission allowances generated by using renewable energy sources. They result from the multiplication of the renewable energy generated with the emissions factor of the current conventional generating mix and the allowance prices. The emissions factor of the current conventional mix, denoted by the fractional term

\[ \sum_{i=1}^{I-RE} \frac{E_{CO_2i}}{E_{i,z}} \]

implies the assumption that the renewable energy generated, replaces energy that would otherwise be produced by the conventional generation mix of technologies of the country.

The fixed costs as well as the logistical fuel costs (only for those fuels with no available historical data) are approximated by compounding their present values in the future using the stochastically (GBM) evolving inflation rate:

\[ C_{f,z} = \left[ \prod_{t=1}^{z} \left(1 + r_{in,t} \right) \right] \cdot C_{f,0} \quad \forall i \quad (7) \]

\[ C_{v,z} = \left[ \prod_{t=1}^{z} \left(1 + r_{in,t} \right) \right] \cdot C_{v,0} \quad \forall i \quad (8) \]

In this study the past capacity additions of the last 40 years are taken into account due to unavailability of older data. The unitary investment costs \( I_{i,v} \) of a power plant type (i) ordered in the year (v) depend on the technical advances arising from long periods of cumulative experience on construction of such power production units in the domestic market. This can be mathematically formulated as follows:

\[ I_{i,v} = I_{i,0} \cdot \left[ \sum_{k=1}^{v} X_{i,k} + \sum_{v=40}^{0} \frac{C_{i,v}}{\sum_{v=40}^{0} C_{i,v}} \right] \quad \forall i \quad (9) \]

The capital costs of the year 2009 are used as a reference value (\( I_{i,0} \)) for each technology (i).

The objective NPV function of equation 6 is subject to the following constraints:
1) Non-negativity constraints:

The factor \((\theta_{i,z})\) may take values in the range: \(0 \leq \theta_{i,z} \leq 1\) \((\theta_{i,z} = 0\) means a non-operational power plant whilst \(\theta_{i,z} = 1\) implies the maximum operational time in the year \(z\)). Moreover, the annual capacity orders for new power plants \(X_{i,v}\) should be positive numbers: \(X_{i,v} > 0, \forall i,v\).

2) Natural resource availability:

The plants from each technology may not produce more cumulative power than the maximum energetic potential of the corresponding natural resources, domestically available:

\[
I_{i,z} \leq E_{i,z} \quad \forall i, z
\]  

(10)

where the aggregate installed capacity of technology \((i)\) \((I_{i,z})\) is calculated using equation 4.

3) Demand Constraints

Meeting the demand target is a basic condition ensuring social acceptance of power production at a national level. The rationale for this constraint was based on the assumption that a possible generic failure of power supply (system black-out) would result to an excessive social cost. This case should be avoided in the expense of additional capacity orders causing a system NPV reduction. Therefore the average annual electricity energy production is assumed to be at least as much as the simulated annual electricity energy demand projection. This constraint can be mathematically formulated as follows:

\[
\sum_{i=1}^{I} P_{i,z} \geq d_{z,f} \Rightarrow \sum_{i=1}^{I} \left( \sum_{v=1}^{V} X_{i,v} + \sum_{v=0}^{V-40} C_{i,v} - \sum_{v=0}^{V-40} \overline{C}_{i,v} \right) \theta_{i,z} a_{i,z} a_{i,z} \cdot 8760 \geq d_{z,f} \quad \forall z
\]

(11a)

An additional reliability requirement may be imposed by stating that the installed capacities should be able to serve the peak power demand. In that case the availability and the intensity factors have been considered to be equal to 1, meaning full-load operating conditions with guaranteed fuel availability. However, this may not be the case for the wind turbines and the photovoltaic (PV) plants as they certainly depend on meteorological conditions, which are unpredictable in the medium term. Based on past observations, the peak-power spots generally
follow a linear trend and therefore have been linearly projected to the future. An additional safety factor has been further multiplied to the projected peak-power demand in order to account for the possibility of some plants’ failure or fuel unavailability [13, 21]. The mathematical formulation of this constraint is represented in equation 11b:

\[
\sum_{i=1}^{n} a_{z,i} \cdot L_{z,i} \geq P_{c,z} \cdot (1 + m_z) \quad \forall z, \quad a_{z,i} = 1, \quad \forall i \text{ except wind farms \& PVs for which } a_{z,i} \leq 1 \quad (4) \Rightarrow \\
\sum_{i=1}^{n} a_{z,i} \left( \sum_{\nu=1}^{\nu_{-d}} X_{t,z} + \sum_{\nu=0}^{0_{LB}} C_{t,z} - \sum_{\nu=0}^{0_{LB}} C_{t,z} \right) \geq P_{c,z} \cdot (1 + m_z)
\]

(11b)

The above two-fold demand constraints (11a - 11b) might be able to model any possible case of the system’s reliability limits (either total energy or peak power) depending on which constraint proves to be stricter.

4) Grid Stability

Some power generation technologies, which strongly depend on weather conditions, might constitute a base-load solution in the long term (i.e. photovoltaic and/or wind farms). However, despite their certain advantages (short setup periods, zero emissions, zero fuel requirements) they often suffer from unavailability of natural resources. These –occasionally unpredictable– conditions might impact the stability of the national grid and the reliability of power supply. Also, despite the fact that there is no consensus on the maximum allowable percentage of renewable energy sources to secure the grid stability, scientists agree that there is currently an upper limit on renewable power infusion to the grid [54]. For this reason a constraint is imposed ensuring that the total energy production from these specific types may not exceed 50% of the total energy demand:

\[
\sum_{i=1}^{Rc} P_{t,i} \leq 0.5 \cdot d_{x,f} \quad \forall z
\]

(12)

5) Environmental constraints
These are obligatory constraints applied in every country participating in the Kyoto Protocol.

Greece is required to meet the following targets imposed by the EU directive 2001/77/EC [55]:

5a) The renewable energy technologies should obtain a share of more than 20.1% of the total domestic electricity production by the year 2010. Hence:

\[
\sum_{i=1}^{RE} P_{i,z} > (0.201) \cdot \sum_{i=1}^{1} P_{i,z} \quad \forall z \geq 2010
\]  

5b) The renewable energy technologies should obtain a share of more than 30% of the total domestic electricity production by the year 2020. Hence:

\[
\sum_{i=1}^{RE} P_{i,z} > (0.3) \cdot \sum_{i=1}^{1} P_{i,z} \quad \forall z \geq 2020
\]

4. The numerical algorithm

4.1 Inputs of the model

The numerical algorithm used for the determination of the optimal power generation mix comprises of the following discrete steps: a) Retrieving technological and economical inputs, b) retrieving historical time-series of stochastic variables, c) simulating the evolution of stochastic variables and d) optimisation routine. The inputs used in the present study are shown in table 1.

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<Insert table 1 about here>

The historical data of the stochastically fluctuating variables have been acquired by various sources. The past demand loads and SMPs have been acquired by the national grid operator [49]. The historical data of fuel prices have been provided by the Greek State [56], the Statistical Service [57], the natural gas provider [58] and the International Energy Agency [59]. The recent
CO2 allowance prices were retrieved from Point-Carbon [60]. For the case of biomass, lacking price historical data, their current gate fees were estimated using a holistic Activity Based Costing procedure [61]. After the current supply chain costs are estimated, they are post-projected using equation 7 together with the stochastically evolving inflation rates (SDE equation 1). A similar procedure has been followed for the estimation of the hard-coal’s logistic costs. In that case, fuel imports are considered as there are only few non-exploitable domestic reserves.

4.2 Stochastic variables

The uncertainty is introduced for all the stochastic variables: forward electricity selling prices, electricity demand, CO2 allowance and fuel prices. Their evolution paths constitute the inputs for the optimisation algorithm as described in the mathematical formulation. The historical data of the various stochastic variables were available in different time intervals. Therefore, prior to the optimisation iterations, the data structures have been averaged in order to create uniform arrays over time, as adopted by Fleten and Maribu (2007) [37]. The selected granularity was based on the widest common time intervals, comprising of monthly periods. Concerning a forward electricity contract with maturity $T_1$ and monthly delivery period $[T_1, T_2]$, an approximation of monthly averages $F(T)$ (forward prices) can be made. Daily prices $f(t)$ within daily intervals $dt$ may thus be averaged as represented in the following equation 15 [62]:

$$F(T) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t) dt \quad T \in [T_1, T_2]$$ \hspace{1cm} (15)

This way, a time-average of peak and off-peak prices is performed. The above process was not the case for the demand loads. Spot demand observations were modified by multiplying with their occurrence time interval $dt$, and aggregating the products within each month thus resulting to total monthly energy demand estimations ($D(T)$ in $\text{MWh}_{el}$):

$$D(T) = \int_{T_1}^{T_2} d(t) dt \quad T \in [T_1, T_2]$$ \hspace{1cm} (16)

When in the same granularity form, the stochastic variables were mathematically correlated using correlation factors derived from their past history, in a similar vein to [53]. Actually, the Brownian differentials of fuel prices, electricity prices and electricity demand were mutually
correlated. They were also correlated with the corresponding Brownian differentials of the interest and the inflation rates. The CO2 allowance prices have been correlated with the electricity prices as in [17]. The above mentioned Brownian differentials were processed through a multivariate Monte-Carlo algorithm in which an Euler-Maruyama solver was used to produce multiple lognormal solutions of GBM (or MR) SDEs. The resulting paths were extracted by averaging the multiple solutions set as shown in the following figures 1 and 2:

<Insert figure 1 about here>

<Insert figure 2 about here>

In figure 1 (upper) the anticipated electricity demand is presented and in figure 1 (lower) the electricity price scenarios are displayed. The forecasts of scenarios C and D represent the prices foreseen for renewable producers, while the prices of conventional electricity generation, in the same scenarios, retain the path of scenario B. In figure 2 (upper) the fuel price forecasts are presented (years 11-50) together with their historical data (years 1-10), while in figure 2 (lower) the inflation and the interest rates are presented -with the same time distribution-. The Monte-Carlo simulation, being a variation reduction technique, succeeded to smooth-out the time-path of many variables whose historical data were characterised by relatively low volatilities. On the contrary the oil and the natural gas price forecasts as well as the electricity prices are characterised by non-stationary profiles, due to high recent variations. The remaining fuel prices are characterised by negative slopes. In particular the lignite’s ascending price may be in line with the domestic reserves depletion, which in turn might lead to inefficient mining in the expense of increasing fuel costs. In figure 3, the two assumed scenarios for the CO2 allowance price evolution (I and II) are presented.

<Insert figure 3 about here>
It is noted that a relatively slow mean-reverting speed has been selected in the scenario II, assuming a slow convergence to the long run mean (31 Euros/tn CO2). The correlations matrix derived by the historical data of fuel prices is presented in table 2:

<Insert table 2 about here>

Moreover, the statistical analysis indicated a significant correlation between lignite and electricity prices (0.77) while the correlation between natural gas and electricity prices is slightly weaker (0.71). A moderate correlation has been recorded for the Brownian differentials of CO2 allowance and electricity prices (0.62).

5. Numerical results

5.1 Results presentation and discussion

Various numerical experiments were conducted in order to reveal the influence of the electricity and the CO2 allowance prices in the economics of the expansion mix and the sustainability. In figure 4, the electricity and CO2 allowance price scenarios (A, B, C, D – I, II) are compared in respect of the optimal system NPVs derived from the optimisation.

<Insert figure 4 about here>

The NPVs vary proportionately with the forward electricity prices, as expected. The scenario D reflects the most profitable case due to the highest prices foreseen for renewable energy forward contracts. A two-fold explanation may hold: (i) high prices constitute an attractive condition for hypothetical investors (more capacities are ordered) and therefore the anticipated yields of electricity selling to the grid are more promising (ii) the renewable energy technologies are characterised by low or zero fuel costs and moreover their neutral emission rates contribute to the minimisation of the anticipated CO2 penalties.
The increased CO2 allowance prices (scenario II) contribute to the reduction of the system NPV, leading to lower CO2 emissions per unit of energy generated over time, as shown in figure 5.

The optimisation algorithm succeeds to reduce the aggregate CO2 emissions of the Greek Power Sector from the current levels (0.75 tn CO2 eq. / MWh el) to the levels of 0.60 tn CO2 eq. / MWh el at the end of the investigated time period. This considerable reduction (~20%) may be attributed to the high CO2 allowance prices (Scenario II), which seem to act as barriers for conventional plant investments. The average differences between the two scenarios (I and II) in terms of CO2 emissions is close to 20 Kg CO2 eq. / MWh el, which is not negligible considering the aggregate electricity production. The numerical experiments of Scenario D-II resulted to the lowest average emissions compared to any other scenario. More specifically, the emissions produced by scenario D-II were approximately 50 Kg CO2/ MWh el lower than the average emissions of the remaining class-II scenarios and almost 70 Kg CO2/ MWh el lower than the average emissions of class-I scenarios. This might also be attributed to the higher electricity prices foreseen for renewable producers in D scenarios, which in turn favour their increasing share in the domestic market, resulting in lower aggregate systemic emissions over time.

The capacity additions that should be ordered on a yearly basis vary between the investigated scenarios. In the graphs of figures 6 and 7 the optimal capacity orders and plant usage intensities are presented for the scenario D-I. The annual energy production –corresponding to the above capacities and plant loads–is presented in figure 8. Capacities, intensities and cumulative energy production for scenario D-II are presented in figures 9, 10, 11 respectively. As stated before, the D-scenarios represent the most profitable cases irrespective of the CO2 allowance price levels and therefore, they have been selected in the following graphical presentations:
The expansion mix of Scenario D-II include more capacity orders compared to D-I, especially concerning the renewable technologies (hydroelectric and biomass plants, as well as wind farms). The significant investment costs of photovoltaic plants seem to act as inhibitors for solar-PV projects. On the other hand, the natural gas plants prove to be the most intensive in terms of productive hours, since their output $\theta$ factor was the highest among the competing technologies (figures 7 and 10). They were proved to operate for more than 90% of the available operating hours annually. For comparison reasons, the corresponding $\theta$ factors of renewable energy sources (except biomass) were imposed to be equal to 1, thus meaning operational readiness depending on favorable weather conditions. The lignite and the natural gas fuelled plants determine the base-load workhorse technology in all the scenarios, due to the high availability factors and their relatively moderate fuel prices. However, as the lignite costs gradually increase, the lignite fired plants should progressively decrease their productivity ($\theta$ factor) to the levels of 65%. The high emission rates and the relatively lower power production efficiency of the lignite fired plants might further justify the progressive reduction of lignite-based electricity, as reflected on the load intensity of these plants (figures 7 and 10). By comparing those graphs it is also noted that the average $\theta$ factor is lower in Scenario D-II than in D-I. This may be attributed to the optimisation algorithm, which on the one hand attempted to meet the electricity demand target (by increasing
lignite plant orders) but on the other hand attempted to minimise the emission costs (by lowering the plants’ usage intensity). This counterbalancing effect proves to be stronger in Scenario D-II, as reflected in its lower $\theta$ factor, by considering that the emission costs are prone to be relatively higher, due to the higher assumed CO2 allowance prices of Scenario D-II.

The above arguments may be further justified through the analysis of figures 8 and 11, where the annual electricity production is presented (Scenarios D-I and D-II respectively). Some problems may be identified regarding a hypothetical broadening of renewable energy penetration in electricity market. The long construction periods of hydro-electric plants and the relatively low capacity factors of photovoltaic, hydroelectric and wind plants induce the requirement for very high installed capacities in order to meet the demand targets. On the other hand, the investment costs of the above mentioned technologies –and in particular of solar energy technologies– are significantly high –despite their State subsidisation– and may not favour renewable energy investments. The above arguments are reflected to the annual capacity orders resulting from the optimisation algorithm, which are not as high as one might expect (figures 6, 9). Moreover, the renewable energy generation (shown in figures 8 and 11) barely manages to meet its lower bound share in the electricity market (> 30% after the year 2020 as stated in the EU directive 2001/77/EC). It also stays far away from reaching the upper bound target determined by the requirements for the grid stability (50% of total electricity demand as described in equation 12). Therefore, the renewable energy technologies are challenged to the power sector domination race and may not gain a higher market share unless the experience acquired in the distant future contributes to the further reduction of their investment costs. Despite their obvious environmental advantages, the renewable energy technologies may not be promoted under the current CO2 allowance prices, which might have to be strengthened, thus forming an attractive condition for investors willing to engage in this market. The reduction of investment costs due to the expected experience from future constructions or market competition may be a matter of time, while on the other hand, the determination of CO2 allowance prices may not depend only on allowances’ market dynamics but it may rather be a consequence of environmental policy interventions. Nonetheless, the renewable energy technologies are currently favoured by their low (or zero) fuel cost, which constitutes their sole significant promoting factor.
Of particular interest might be the exploitation of biomass feedstocks, which are cheap and readily available in the domestic fuel market. Agricultural crops like cotton, corn, wheat etc. which are cultivated and grow in many rural areas of Greece might become an alternative to the conventional energy production. Nonetheless, the complicated supply chains as well as life-cycle, logistical and organisational issues should prior be resolved. Probably they could not replace natural gas as a base-load fuel, but their neutral CO2 emissions and the relatively high capacity factors render this technology as a candidate alternative needing additional focus for the oncoming decade (figures 6, 9).

The required policy interventions should be incorporated in the future power generation licence calls released by the State. They should aim at high percentage of renewable energies (hydro-plants, wind turbines and biomass) in the first 3-4 years of implementation of the program in order to put a basis for meeting the environmental directive targets. Some kwh subsidising policies should probably be considered for the promotion of renewable energies during these first years of the program. From this point forward, the distribution of capacity orders for power plants based on combustion technologies (natural gas, lignite and biomass) should be balanced, whilst the capacity orders of solar PVs, wind parks and hydro plants (either of hydro pumped storage or of medium and high head hydro plants) should be reduced. As the total installed capacities gradually rise over time, the environmental constraints impose the requirement for decreased usage of conventional lignite plants (characterised by high emission factors), thus enabling the minimisation of aggregate system emissions. The above rationale should be followed in the future licensing procedures, irrespective of the pricing model. Nonetheless, individual power plants should take it into account, in order to remain competitive and keep up with current market trends -as far as their own potential expansion planning is concerned-. It is noted that the pricing model seems to impact, basically, the comparative scale of the interventions but not their qualitative characteristics.

5.2 Dependency from uncertainties and installed generation mix

The expansion planning may depend on the multiple effects of the uncertainties as well as on the diversification of the installed generation mix. More specifically the results of the model, which
essentially depend on the input state, may not be valid for the entire future horizon. This problem might be attributed to the progressively increasing uncertainty characterising long-term random walk simulations. Moreover, the effect of the long times required for the construction, setup and commissioning of each plant type should be accounted, thus complicating the determination of the valid time-period of the results. Therefore, despite the fact that the scope of the work spans to the next 25 years, the optimisation algorithm was allowed to run for an extended time-period of 42 years, in an attempt to smoothly absorb the above mentioned boundary timing effects. Furthermore, the uncertainties’ modelling may not represent the real future evolution of the corresponding stochastic variables. It rather projects their past behaviour by sampling the underlying noise through probability distributions determined by recent historical data. This inherent limitation should definitely be accounted during any decision making process.

Interestingly, non-smooth variations of the capacity orders have been derived from the optimisation, despite that the opposite might be expected for progressive time spots. Moderately smooth distributions of optimal plant loads ($\theta$ factors) were also produced over time. The variations of the corresponding graphs may be partially attributed to the non-stationary evolution of the forward electricity and fuel prices, as indicated by their simulated projections. In a similar vein, the volatile evolution paths of the CO2 allowance prices and of the discounting factors can further justify the above non-smooth distributions. The capacity orders required for the displacement of obsolete plants (mainly of the workhorse lignite-fired plants), also contribute to non-uniform results, as long as the end of their operation life signals massive capacity orders to meet the demand targets, thus resulting to non-smooth orders’ distribution. This may be characterised as a moderate drawback of discrete -or quasi-continuous treatment-, especially when expansion planning is modelled on an annual basis.

6. Statistical analysis

The proposed method could be validated by comparing the corresponding results from several studies. However, on the one hand there is no similar research concerning the Greek power sector; on the other hand, there were no readily available input data from other countries, which
would allow the processing of their portfolio planning and/or the comparison with other power sectors. Moreover, it might be hard enough to establish solid qualitative or quantitative criteria which may potentially allow a direct comparison with other similar researches referring to power sectors with essentially different structures. Although the proposed model may be able to capture the different characteristics (or structure) of various power sectors, a statistical analysis of the results is used instead, for the assessment of the suggested approach.

6.1 Convergence history of the optimisation algorithm

The convergence history of the algorithm is comprised of the objective function differences (between successive iterations) until a very small value ($\varepsilon$) is obtained. Indicatively, the convergence history of the scenario A-I is presented in figure 12. An initial state has been introduced in the algorithm, based on a moderate evolution of the current generation mix. In the first steps of the iterative procedure, the suboptimal NPV values deviate significantly from the optimal value. This is finally converged to the level of $52\times10^9$ Euros (also shown in figure 4) indicating a significant improvement compared to the initial NPV. Negative NPV values are progressively iterated as the algorithm was built to seek for minimum values. Therefore, if a maximisation is required, the negative of the NPV objective function should be derived.

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6.2 Deviation from the constraints

The cumulative energy production shown in figures 8 and 11 indicates a minimal deviation from the energy demand as imposed in the relevant constraints (equations 11a and 11b). Their coincidence ensures meeting the demand target with minimal aggregate costs.

6.3 Error analysis

The comparison of the simulated with the actual data was performed using the median absolute deviation (MAD) as a measure of error. The MAD indicator is defined as follows:

\[
\text{MAD} = \text{median}(d_{z,f} - \text{median}(d_{z,a}))
\]  

(17)
The analysis was performed using the daily recorded data of energy demand for the years 2000-2009. This period was split in two discrete time-frames: 2000-2005 and 2006-2009. The first set was used as a sample set needed for the random walk simulation while the second was used for the validation of the resulting forecasts for the corresponding years (2006-2009). The comparison lead to an error with MAD=11.5% of the average observations recorded in the validation data-set, thus reflecting a relatively accurate forecast. It is noted though that the random walk forecast is characterised by progressively increasing statistical uncertainty. In order to calculate this uncertainty a statistical analysis is required. Primary objective of this statistical analysis is the determination of the statistical distribution of the original (historical) data. The demand data may be considered as normally distributed, as shown in the observations statistical fitting (figure 13).

The determination of the uncertainty is possible by assuming normally distributed Brownian differentials and a requirement for a confidence interval (CI) equal to 95%. According to theory [36], when the Brownian differentials are normally distributed the resulting GBM simulation comprises of log-normally distributed solutions. These are characterised by progressively increasing confidence limits over time -for a given CI- and may significantly deviate from the forecast especially in the distant future. The confidence limits of the aggregate annual demand forecasts are plotted in figure 1 (assuming CI=95%). The lognormal distribution fitting of an indicative time-point of the GBM solution is shown in figure 14.

The mean value is extracted by the lognormal distribution for this time point, forming an average forecast. The same procedure is performed for the calculation of the confidence limits of the remaining stochastic variables. It is noted that adequate iterations of this Monte-Carlo procedure are required until the confidence limits cannot be further reduced for an assumed CI.
7. Conclusions

In this article, the impact of electricity and CO2 allowance prices is investigated within the framework of electricity production portfolios. The numerical experiments conducted, focused on the optimisation of the power sector structure over time. The Greek Power Sector was introduced as the case-study market under multiple uncertainties. The electricity demand, the forward prices of electricity, the CO2 allowance prices as well as the fuel prices were considered as stochastically evolving. The evolutions of interest and inflation rates were represented through stochastic processes as well, thus minimising the requirement for arbitrary risk assumptions. The iterative optimisation algorithm attempted to maximise the system NPV while being subject to various social, supply and demand constraints. Constraints related with the availability of natural resources, the stability of the national grid operation and the environmental directives were additionally imposed. A stochastic programming approach without recourse was used, incorporating an SQP numerical solver.

The results of the optimisation focus on the annual capacity additions, the plant loads and the corresponding energy production required from each technology. A significant improvement may be identified in respect of the iterated system NPVs and the anticipated CO2 emissions over time. More specifically, the algorithm tends to maximise the system NPV by iteratively attempting to reduce the overall system costs and the CO2 emission costs in particular, thus minimising the anticipated CO2 emissions as well. The resulting generation mix (suggested for the medium-term expansion planning of the Greek Power Sector) shows that the CO2 emissions per unit of electricity might be reduced by 30% within the next 25 years. The analysis of the results indicates that higher electricity prices may be proportionally beneficial for the financial yields of the power sector. On the other hand, the CO2 allowance prices may be inversely proportionate with the expected yields and with the anticipated CO2 emissions over time, thus forming an inhibitor for capacity orders from conventional technologies.

The combined influence of the various commodities under uncertainty may marginally be explained in an empirical way as the results are derived through complex algorithmic optimisation processes. Nonetheless, they could provide useful information by focusing on specific time spots, thus identifying particularly interesting investment opportunities. If these
opportunities are supposedly bypassed (investments realised in suboptimal time-windows) there may be fewer chances for individual players to be profitable. Therefore, the hypothetical investors might have to be aware of the upper bound share of each technology. Their subsequent decisions should be probably reconsidered according to current market trends, fiscal conditions and competition. Moreover, their decisions should be further evaluated in conjunction with standard licensing practices, periodically applied by the State during expansion planning, thus complying with the relevant policy interventions, which determine the bounds of capacity shares. This strategy might potentially contribute to the reduction of the social cost as well as to more favourable effects on the sustainability of the system.

Appendix A.

A general optimisation problem may be formulated as follows:

$$\begin{aligned}
\min_{\mathbf{x}} & \quad f(\mathbf{x}) \\
\text{subject to} & \quad C_i(\mathbf{x}) = 0, \quad \forall i = 1,...,k_i \\
& \quad C_i(\mathbf{x}) \leq 0, \quad \forall i = k_i + 1,...,k
\end{aligned} \tag{A1}$$

where $\mathbf{x}$ is the independent variables vector of length $k$, $f(\mathbf{x})$ is the objective function, which returns a scalar value, and the vector function $C(\mathbf{x})$ returns a vector of length $k$ containing the values of the equality and inequality constraints evaluated at $\mathbf{x}$. The Karush-Kuhn-Tucker equations (KKT) can be described by the following:

$$\begin{aligned}
\nabla f(\mathbf{x}) + \sum_{i=1}^{k} \lambda_i \nabla C_i(\mathbf{x}) &= 0 \\
\lambda_i \cdot C_i(\mathbf{x}) &= 0, \quad \forall i = 1,...,k_i \\
\lambda_i \geq 0, \quad \forall i = k_i + 1,...,k
\end{aligned} \tag{A2}$$

It is noted that the KKT equations include the constraints set of equation A1. The principal idea for convergence in the SQP algorithms is the construction of a quadratic sub-problem based on a quadratic approximation of the Lagrangian function:
\[ L(\mathbf{x}, \lambda) = f(\mathbf{x}) + \sum_{i=1}^{k} \lambda_i \cdot c_i(\mathbf{x}) \]  \hspace{1cm} (A3)

By assuming that boundary constraints have been expressed as inequality constraints, the QP sub-problem may be obtained by linearising the nonlinear constraints:

\[
\begin{align*}
\min_{\mathbf{x}, \lambda} & \quad \frac{1}{2} \mathbf{d}^T \mathbf{H}_x \mathbf{d} + \nabla f(\mathbf{x}_k) \mathbf{d} \\
\text{s.t.} & \quad \nabla c_i(\mathbf{x}_k) \mathbf{d} + c_i(\mathbf{x}_k) = 0, \quad \forall i = 1, \ldots, k \\
& \quad \nabla c_i(\mathbf{x}_k) \mathbf{d} + c_i(\mathbf{x}_k) \leq 0, \quad \forall i = k + 1, \ldots, k
\end{align*}
\]  \hspace{1cm} (A4)

where \( \mathbf{H}_x \) denotes the Hessian matrix of the quadratic approximation and \( \mathbf{d} \) denotes the updates solutions vector. The objective function of the present research may be formulated by combining equations (4), (5) and (6) (described in the main text):

\[
\begin{align*}
\text{NPV}(X_{i,v-1,v}, \ldots, X_{i,v-1,v}, \theta_{v-1,v}, \ldots, \theta_{1,v}) &= \\
& \left[ \frac{1}{2} \sum_{i=1}^{1} \sum_{z=1}^{1} \nabla_{x_i} D_i \left( \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 - \\
& \quad - \sum_{i=1}^{1} \sum_{z=1}^{1} \nabla_{c_i} D_i \left( \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 - \\
& \quad - \sum_{i=1}^{1} \sum_{z=1}^{1} \nabla_{y_i} D_i \left( \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \text{max} \left[ \sum_{i=1}^{1} X_{i,v} X_{i,v} - \\
& \quad - \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 + \\
& \quad + \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right) \\
& \quad \sum_{i=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \\
& \quad \sum_{i=1}^{1} \sum_{z=1}^{1} \sum_{v=1}^{v} X_{i,v} + \sum_{v=0}^{v} C_{i,v} - \sum_{v=0}^{v} \bar{X}_{i,v} - \sum_{v=0}^{v} \bar{C}_{i,v} \right) \theta_{i,v} a_{i,v} a_{i,z} \cdot 8760 \right] \
\end{align*}
\]  \hspace{1cm} (A5)

It represents an analytical relationship of the final NPV with the requested optimal capacity orders and the usage intensities of the power plants during the entire period of investigation. A double sweeping in respect of time is implemented: the first one scans the lead-times and the second sweeps over the operational life-time. An additional third sweeping direction through the competing technologies is additionally implemented. The uncertainties of NPV function are
comprised of the forward electricity prices \( (p_{ez}) \), the electricity demand \( (d_{z,f}) \), the CO2 allowance prices \( (p_{co2,z}) \) and the fuel prices \( (Cf_{i,z}) \). The corresponding paths are calculated using the stochastic models described by equation 1, whilst the discounting factors \( (D_z) \) are calculated using the stochastic differential equation 2 combined with equation 3. The above uncertainty sets are introduced as inputs in the optimisation problem A1-A5 using the objective function (equation A5).

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Fig. 1. Electricity demand (up) and forward prices – scenarios A, B, C, D (down)
Fig. 2. Anticipated fuel prices (up) and fiscal rates (down)

![Graph showing anticipated fuel prices and fiscal rates]

Fig. 3. The CO2 allowance price evolution

![Graph showing CO2 allowance price evolution]

Fig. 4. The comparison of the optimal system NPV for the various scenarios

![Bar chart comparing optimal system NPV for various scenarios]
Fig. 5. Expected CO2 emissions Scenario I (up), Scenario II (down)
Fig. 6. Optimal Capacity orders (Scenario D-I)

Fig. 7. Plants load intensity (Scenario D-I)

Fig. 8. Aggregate electricity production (Scenario D-I)
Fig. 9. Optimal Capacity orders (Scenario D-II)

Fig. 10. Plants load intensity (Scenario D-II)
Fig. 11. Aggregate electricity production (Scenario D-II)

Fig 12. Convergence history for scenario A-I

Fig 13. Normal distribution fitting of actual electricity demand data (daily basis)
Fig 14. Log-normal distribution fitting of the multiple GBM solutions for a future time-point
### Nomenclature table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>Generic notation for stochastic variables evolving according to a GBM model</td>
</tr>
<tr>
<td>$a_{a,i}$</td>
<td>Availability factor for plant (i) (%)</td>
</tr>
<tr>
<td>$a_{c,i}$</td>
<td>Capacity factor for plant (i) (%)</td>
</tr>
<tr>
<td>$b_{i(i)}$</td>
<td>Learning rate of plant (i) construction</td>
</tr>
<tr>
<td>$C_{f,i,z}$</td>
<td>Fuel cost during year (z) for plant (i) (€/MWh$_{el}$)</td>
</tr>
<tr>
<td>$\bar{C}_{v,i,v}$</td>
<td>Capacity installed in the past (year v before the initiation of the investigated time-period) for plants (i) (MW)</td>
</tr>
<tr>
<td>$C_{i,v}$</td>
<td>Capacity installed in the past (year v before the initiation of the investigated time-period) for plants (i) (MW)</td>
</tr>
<tr>
<td>$C_{V,i,z}$</td>
<td>Fixed (Operational and Maintenance) cost of electricity production for plant (i) in year (z) (€/MW)</td>
</tr>
<tr>
<td>$D$</td>
<td>Diffusion vector function (-)</td>
</tr>
<tr>
<td>$\text{dW}_t$</td>
<td>Wiener (Brownian Motion) Vector Differential, Normally Distributed: $\sim iN(0,1)$</td>
</tr>
<tr>
<td>$\text{D}_z$</td>
<td>Discounting factor of year z</td>
</tr>
<tr>
<td>$d_{a,z}$</td>
<td>Aggregate electricity demand for year (z) (actual data in MWh$_{el}$)</td>
</tr>
<tr>
<td>$d_{z,f}$</td>
<td>Aggregate electricity demand for year (z) (simulated projection in MWh$_{el}$)</td>
</tr>
<tr>
<td>$E_{CO2}$</td>
<td>Emissions generated by using a specific technology (i) (tons CO2 equivalent)</td>
</tr>
<tr>
<td>$E_{n_i}$</td>
<td>The annual maximum potential of natural resources for technology (i) (MW)</td>
</tr>
<tr>
<td>$f_{CO2}$</td>
<td>CO$_2$ emission rate of fuel type (i) (tons CO2 equivalent/MWh)</td>
</tr>
<tr>
<td>$I$</td>
<td>Number of electricity production technologies</td>
</tr>
<tr>
<td>$I_{i,v}$</td>
<td>Investment Cost for orders of plant type (i) made in year (v) (€/MW)</td>
</tr>
<tr>
<td>$L_{i,z}$</td>
<td>Installed Capacity during year (z) for plant category (i) (MW)</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Power Reserve Margin</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Power production efficiency for plant (i) (%)</td>
</tr>
<tr>
<td>$\text{NPV}$</td>
<td>Net Present Value of a project (€)</td>
</tr>
<tr>
<td>$P_{c,z}$</td>
<td>Projected Peak Power demand</td>
</tr>
<tr>
<td>$p_{CO2,z}$</td>
<td>Price of CO$_2$ allowances for the year (z) (€/tons CO$_2$ equivalent)</td>
</tr>
<tr>
<td>$p_{e,z}$</td>
<td>Forward electricity price for year (z) (selling to the grid in €/MWh)</td>
</tr>
<tr>
<td>$P_{i,z}$</td>
<td>Aggregate electricity energy production during year (z) for plants (i) (MWh$_{el}$)</td>
</tr>
<tr>
<td>$RE$</td>
<td>Number of electricity production technologies based on renewable energy sources</td>
</tr>
<tr>
<td>$r_{in}$</td>
<td>Inflation Rate (%)</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Interest Rate (%)</td>
</tr>
<tr>
<td>$T_{i}$</td>
<td>Construction, setup and commissioning time of plant type (i) (years)</td>
</tr>
<tr>
<td>$T_{0,i}$</td>
<td>Operational life-time for each plant type (i) (years)</td>
</tr>
<tr>
<td>$V$</td>
<td>Volatility vector function (-)</td>
</tr>
<tr>
<td>$v$</td>
<td>Counter of the years of investment decisions (capacity orders)</td>
</tr>
<tr>
<td>$X_{i,v}$</td>
<td>Orders made in year (v) for plant (i) (MW)</td>
</tr>
<tr>
<td>$\bar{X}_{i,v}$</td>
<td>Capacities ordered in year (v) for plant (i) and whose operational life time has ended (MW)</td>
</tr>
</tbody>
</table>
Table 1. The assumed inputs of the numerical algorithm

<table>
<thead>
<tr>
<th></th>
<th>Hard-coal</th>
<th>Oil</th>
<th>Natural Gas</th>
<th>Lignite</th>
<th>Biomass</th>
<th>Solar PV</th>
<th>Wind turbines</th>
<th>Hydroelectric</th>
<th>Hydro pumped storage</th>
<th>Geothermy</th>
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</thead>
<tbody>
<tr>
<td>Investment cost (€ / KWel) for the reference year 2009</td>
<td>1350</td>
<td>1300</td>
<td>450</td>
<td>1800</td>
<td>1300</td>
<td>4500</td>
<td>1100</td>
<td>1600</td>
<td>3400</td>
<td>1050</td>
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<tr>
<td>Fixed cost of year 2009 (Operational, maintenance, insurance etc.) (in € / KWel)</td>
<td>35</td>
<td>38</td>
<td>14</td>
<td>39</td>
<td>19</td>
<td>12</td>
<td>23</td>
<td>25</td>
<td>50</td>
<td>32</td>
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<tr>
<td>Availability factor</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.85</td>
<td>0.75</td>
<td>0.9</td>
<td>0.9</td>
<td>0.85</td>
<td>0.92</td>
<td>0.7</td>
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<tr>
<td>Capacity factor</td>
<td>0.8</td>
<td>0.8</td>
<td>0.65</td>
<td>0.75</td>
<td>0.8</td>
<td>0.15</td>
<td>0.25</td>
<td>0.34</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Learning rate</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.15</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Commissioning time (Years)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>9</td>
<td>2</td>
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<tr>
<td>Efficiency Factor</td>
<td>0.42</td>
<td>0.31</td>
<td>0.57</td>
<td>0.37</td>
<td>0.35</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Fuel CO2 emissions (tn CO2 / MWhnet)</td>
<td>0.4</td>
<td>0.3</td>
<td>0.21</td>
<td>0.41</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>Operational Life-Time (Years)</td>
<td>40</td>
<td>45</td>
<td>35</td>
<td>45</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>45</td>
<td>45</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 2. Stochastic Differential Correlation of fuel prices

<table>
<thead>
<tr>
<th></th>
<th>Hardcoal</th>
<th>Oil</th>
<th>Natural Gas</th>
<th>Lignite</th>
<th>Biomass</th>
<th>Geothermal</th>
<th>Hydroelectric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardcoal</td>
<td>1</td>
<td>-0.267</td>
<td>-0.253</td>
<td>-0.366</td>
<td>-0.869</td>
<td>0.925</td>
<td>0.616</td>
</tr>
<tr>
<td>Oil</td>
<td>-0.267</td>
<td>1</td>
<td>0.931</td>
<td>0.334</td>
<td>0.311</td>
<td>-0.031</td>
<td>0.061</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>-0.253</td>
<td>0.931</td>
<td>1</td>
<td>0.343</td>
<td>0.311</td>
<td>-0.031</td>
<td>0.061</td>
</tr>
<tr>
<td>Lignite</td>
<td>-0.366</td>
<td>0.334</td>
<td>0.343</td>
<td>1</td>
<td>0.583</td>
<td>-0.183</td>
<td>0.059</td>
</tr>
<tr>
<td>Biomass</td>
<td>-0.869</td>
<td>0.311</td>
<td>0.311</td>
<td>0.583</td>
<td>1</td>
<td>-0.645</td>
<td>-0.194</td>
</tr>
<tr>
<td>Geothermal</td>
<td>0.925</td>
<td>-0.031</td>
<td>-0.031</td>
<td>-0.183</td>
<td>-0.645</td>
<td>1</td>
<td>0.812</td>
</tr>
<tr>
<td>Hydroelectric</td>
<td>0.616</td>
<td>0.061</td>
<td>0.061</td>
<td>0.059</td>
<td>-0.194</td>
<td>0.812</td>
<td>1</td>
</tr>
</tbody>
</table>