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Wei Shao, Peilin Zhou & Sew Kait Thong

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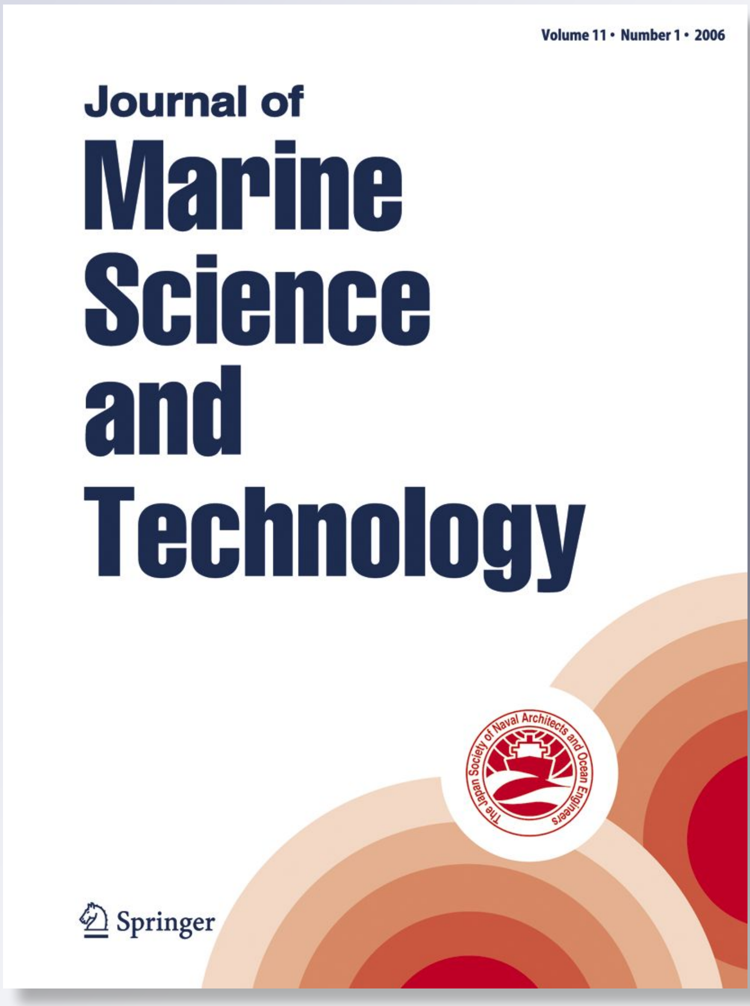

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ORIGINAL ARTICLE

Development of a novel forward dynamic programming method for weather routing

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Abstract This paper presents a novel forward dynamic programming method for weather routing to minimise ship fuel consumption during a voyage. Compared with traditional weather routing methods which only optimise the ship's heading, while the engine power or propeller rotation speed is set as a constant throughout the voyage, this new method considers both the ship power settings and heading controls. A float state technique is used to reduce the iterations required during optimisation and thus save computation time. This new method could lead to quasi-global optimal routing in comparison with the traditional weather routing methods.

Keywords Ship · Weather routing · Dynamic programming · Fuel saving

1 Introduction

Ship weather routing is defined as determining the optimum route of a ship that utilises the optimum engine speed and power for the ocean voyage based on weather forecasts, sea conditions and the individual characteristics of the ship. The term "optimum" means that ship safety and crew comfort are maximised, fuel consumption and duration of the voyage are minimised, or any desired combination of these factors. The accuracy of the optimum route is determined depends on the following three aspects:

- The accuracy of the prediction of the ship's hydrodynamic behaviour under different weather conditions
- The accuracy of weather forecasts
- The capability and practicability of the optimisation algorithm used

The focus of this study is on researching and developing an optimisation algorithm. Many optimisation algorithms have been developed to solve ship routing problems in which minimising ship fuel consumption and/or passage time is the objective. The most popular methods employed for this purpose include calculus of variations [1], the modified isochrone method [2, 3], the two-dimensional dynamic programming (2DDP) method [4, 5], and the isopone method [6, 7].

Calculus of variations treats ship routing as a continuous optimisation process. Inaccuracy of the solution may arise when second-order differentials are used in the optimisation process; errors can then expand to an unacceptable level by the end of the calculation.

The modified isochrone method is a recursive algorithm. The route with the minimum passage time is obtained by repeatedly computing isochrones (or time fronts), which are defined as the outer boundaries of attainable regions from the departure point after a certain time. This method yields the route with the minimum fuel consumption by first keeping the propeller revolution speed constant during the simulation and then applying the modified isochrone

W. Shao (✉) · P. Zhou
Department of Naval Architecture and Marine Engineering, University of Strathclyde, Glasgow, UK
e-mail: wei.shao@strath.ac.uk

P. Zhou
e-mail: peilin.zhou@strath.ac.uk

P. Zhou
College of Energy and Power Engineering, Wuhan University of Technology, Wuhan, China

S. K. Thong
APL Co Pte Ltd, Singapore, Singapore
e-mail: sew_kait_thong@apl.com

method to determine the minimum time of passage. By varying the propeller rotation speed, this method is able to determine the propeller rotation speed at which the minimum passage time satisfies the specified arrival time. This minimum-time route is then treated as the route of minimum fuel consumption. Thus, the fuel consumption of this route itself is not minimised.

The 2DDP method, which is based on Bellman's principle of optimality, is similar to the modified isochrone method. It also uses a recursive equation to solve the ship routing problem, expressed as a discrete optimisation problem. The accuracy of this method depends on the fineness of the grid system used. Compared with the modified isochrone method, an advantage of the 2DDP method is that it allows the operators to account for navigation boundaries by appropriately selecting the grid system. Both the modified isochrone and the 2DDP methods assume that the ship sails at a constant propeller rotation speed or a constant engine power for the entire voyage when optimising the route.

The isopone method is an extension of the modified isochrone method. An isopone is a plane of equal fuel consumption that defines the outer boundary of the attainable regions in a three-dimensional space (i.e. geographical position and time). This method enables the operators to consider different values for the ship engine power used during the optimised route. When it was first being developed, the weather routing software package named SPOS utilised the isopone method. However, although the isopone method is more mathematically elegant and theoretically offers better results than the modified isochrone method, the isopone method was replaced with the modified isochrone method in the final product. The main reason for this switch in the methods used in SPOS was that the isopone method seemed to be more difficult for the operators onboard ships to understand than the modified isochrone method.

Aside from the abovementioned methods, there are many other methods that have been used for weather routing in recent years, such as the iterative dynamic programming algorithm [8], the augmented Lagrange multiplier [9], the Dijkstra algorithm [10], the genetic algorithm [11], and so on.

Weather routing was first developed to determine the fastest routes (i.e. those that minimised the passage times) for shipping. However, nowadays shipping companies have began to show more interest in reducing fuel consumption while maintaining the time schedule specified in the chartering contract of a merchant vessel. In this paper, a new forward three-dimensional dynamic programming (3DDP) method is presented for minimising fuel consumption during a voyage. It is an extension of the traditional 2DDP method, but it allows the ship heading and speed to change with both time and geographic position, so it is able to achieve a quasi-global optimum result. Compared with the isopone method, the 3DDP method is more straightforward and easier to programme.

2 Problem description

The ship's engine power and its shipping course dictate its shipping route across the ocean. In this paper, P and ψ are chosen as the control variables to represent power delivered by the engine and shipping course measured from true north. The control variables are denoted as a control vector $\vec{U}$, which is defined by $\vec{U} = (P, \psi)$.

Ship position $\vec{X}$ is specified by longitude φ and latitude θ, $\vec{X} = (\varphi, \theta)$. Ship trajectory is represented by a sequence of pairs of ship position $\vec{X}$ and time t. $\vec{E}$ is used to denote weather conditions (speed and direction of wind, significant height, direction and peak frequency of waves and swells), which is a function of position $\vec{X}$ and time t, $\vec{E} = F\_E(\vec{X}, t)$. During a voyage, the ship route constraints $\vec{M}$ must be met to ensure ship and crew safety. The constraints include geographical constraints, control constraints and safety constraints.

Thus, the ship position $\vec{X}$ at time t can also be described by the function below:

$$\vec{X} = f_0(\vec{X}', \vec{U}', \vec{E}', \vec{M}'), \tag{1}$$

where $\vec{X}', \vec{U}', \vec{E}', \vec{M}'$ correspond to the preceding time t', and $t - t' = \Delta t$, where Δt is the time step used in the calculation.

Because $\vec{E}'$ is a function of $\vec{X}'$ and t', $\vec{X}$ can also be represented by

$$\vec{X} = f(\vec{X}', \vec{U}', t', \vec{M}'). \tag{2}$$

Equation 2 can be explained by noting that the ship will arrive at the present position $\vec{X}$ at the present time t, having travelled from $\vec{X}'$ under the control of the variables $\vec{U}'$ during the time step Δt, and the ship's operation during the period of time Δt satisfies the constraints $\vec{M}'$.

The instantaneous fuel consumption in kg/h q can be found via

$$q = q(\vec{X}, \vec{U}, t, \vec{M}). \tag{3}$$

Thus, the problem of minimising fuel consumption can be expressed as a constrained optimisation process to find the optimal control variables $\vec{U}$, which satisfies with the following conditions:

- Initial conditions: $\vec{X} = X_s(\varphi_s, \theta_s),\ t = t_s$
- Final conditions: $\vec{X} = \vec{X}_{end}(\varphi_{end}, \theta_{end}),\ t = t_{end}$
- The constraints $\vec{M}$, including geographical, weather, operational and safety constraints

3 Ship hydrodynamics

Fuel consumption is directly related to ship hydrodynamics. Thus, the accuracy of the model of the ship hydrodynamics is a critical influence on the accuracy of the estimation of fuel consumption, which subsequently affects the output quality of the weather routing optimisation.

3.1 Ship resistance and power delivered by the engine in calm water conditions

The total resistance for a ship can be decomposed into two main components: the total resistance in calm water and the total added resistance.

The total resistance in calm water is normally represented by a Froude number dependent component (i.e. wave resistance) and a Reynolds number dependent component (i.e. viscous resistance). In this study, the Holtrop method, an analytical regression method based on results obtained from model tests, is used to predict the total resistance in calm water.

To demonstrate the ship hydrodynamics used in the newly developed 3DDP method, a 54,000 DWT container ship is used as an example (a case ship) in this and subsequent sections. Figure 1 shows the predicted results for the total resistance in calm water obtained using the Holtrop method.

When the ship resistance and speed are known, the power delivered by the engine can be obtained using the propeller characteristics as follows, where the thrust, torque and propeller advance velocity are dimensionless quantities:

$$T = K_T \times \rho n^2 D^4 \tag{4}$$

$$Q = K_Q \times \rho n^2 D^5 \tag{5}$$

$$V_A = J \times nD. \tag{6}$$

Here, T is the propeller thrust (N), K_T is the thrust coefficient, ρ is the density of water (kg/m^3), n is the propeller rotation speed (rev/s), D is the diameter of the propeller (m), Q is the propeller torque in open water (N m), K_Q is the torque coefficient, V_A is the speed of advance, $V_A = V(1 - w)$ (m/s), V is the ship speed (m/s), w is the wake fraction, and J is the advance ratio.

According to Eqs. 4–6, the ratio K_T/J^2 can be obtained via

$$K_T/J^2 = \frac{T}{\rho V_A^2 D^2} = \frac{R}{\rho\,(1-\tau)(1-w)^2 V^2 D^2}, \tag{7}$$

where R is the resistance of the ship (N) and τ is the thrust deduction factor.

It is clear that K_T/J^2 can be calculated from the ship resistance R and ship speed V; K_Q can be obtained using the curves of K_T/J^2 versus J and $10K_Q$ versus J, as shown in Fig. 2. Hence, the power delivered by the engine can be determined from

$$P = \frac{2\pi n^3 D^5 K_Q}{\eta_G \eta_S \eta_R}, \tag{8}$$

where P is the power delivered by the engine (kW), η_G is the gearbox efficiency, η_S is the shaft efficiency, and η_R is the relative rotative efficiency.

The powers delivered by the engine for a series of ship speeds in calm water were saved in a database to be used by the 3DDP method.

3.2 Loss of ship speed under different weather conditions

The total added resistance can be further decomposed into many components, such as the added resistance due to the

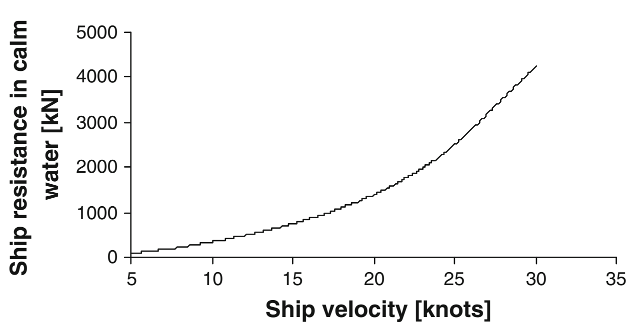

Fig. 1 Total resistance in calm water of the case ship, as predicted by the Holtrop method

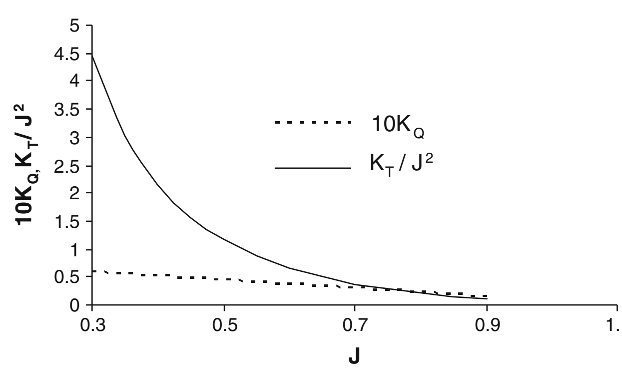

Fig. 2 Curves of K_T/J^2 and $10K_Q$ versus J

current, different fluid layers, ice loading, waves, wind, and so on. In the present study, only the added resistance due to waves and wind are considered, since waves and wind are the main phenomena experienced at sea. As a consequence of the added resistance due to the wind and waves, the ship speed is often lower than the service speed—a so-called involuntary speed reduction. An easy to use approximate method established by Kwon [12] can be used to predict this involuntary drop in speed in irregular waves and wind under the assumption that the engine of the selected ship is able to provide constant power output under different weather conditions.

The percentage loss of speed can be expressed as follows [12]:

$$\frac{\Delta V}{V_1}100\% = C_\beta C_U C_{Form} \quad (9)$$

$$\Delta V = V_1 - V_2 \quad (10)$$

$$V_1 = F_n\sqrt{L_{PP}g}, \quad (11)$$

where ΔV is the loss of ship speed (m/s), V_1 is the ship speed in calm water (m/s), C_β is the speed direction reduction coefficient, which is dependent on the direction of the weather and the Beaufort number BN, C_U is the speed reduction coefficient, which is dependent on ship's block coefficient C_B, the loading conditions and the Froude number Fn, C_{Form} is the hull form coefficient, which is dependent on the ship type, the Beaufort number BN and the ship displacement ∇ (m^3), V_2 is the ship speed under the selected weather conditions (m/s), Fn is the Froude number associated with the designed ship's operational speed V_1 in calm water conditions, L_{PP} is the ship length between perpendiculars (m), and g is the acceleration due to gravity (m/s^2).

Tables 1, 2 and 3 show the databases of the speed direction reduction coefficient C_β, the speed reduction coefficient C_U and the hull form coefficient C_{Form}, as taken from [12].

As can be seen from these tables, the application of Kwon's method involves the use of the following parameters: weather conditions (Beaufort number), ship type, ship displacement, ship's block coefficient, and ship speed in calm water. In addition, the ship load conditions (expressed as "normal", "loaded", or "ballast") are also taken into account. Using Kwon's method, the loss of speed loss can be predicted accurately and quickly without the need to perform complex hydrodynamic calculations.

Figure 3 shows the powers delivered by the engine under different Beaufort numbers in head sea conditions. For a given ship speed, more engine power is needed in rough sea conditions than in calm water. Consequently, more fuel will be consumed in a rough sea. Under certain sea conditions, the demand for engine power could be so high that it is beyond the characteristic envelope of the engine power and speed.

Table 1 Speed direction reduction coefficient C_β

Weather direction	Direction angle (with respect to the ship's bow)	Speed direction reduction coefficient C_β
Head sea (irregular waves) and wind	0°	$2C_\beta = 2$
Bow sea (irregular waves) and wind	30°–60°	$2C_\beta = 1.7 - 0.03(BN - 4)^2$
Beam sea (irregular waves) and wind	60°–150°	$2C_\beta = 0.9 - 0.06(BN - 6)^2$
Following sea (irregular waves) and wind	150°–180°	$2C_\beta = 0.4 - 0.03(BN - 8)^2$

Table 2 Speed reduction coefficient C_U

Block coefficient C_B	Ship loading conditions	Speed reduction coefficient C_U
0.55	Normal	$1.7 - 1.4Fn - 7.4(Fn)^2$
0.60	Normal	$2.2 - 2.5Fn - 9.7(Fn)^2$
0.65	Normal	$2.6 - 3.7Fn - 11.6(Fn)^2$
0.70	Normal	$3.1 - 5.3Fn - 12.4(Fn)^2$
0.75	Loaded or normal	$2.4 - 10.6Fn - 9.5(Fn)^2$
0.80	Loaded or normal	$2.6 - 13.1Fn - 15.1(Fn)^2$
0.85	Loaded or normal	$3.1 - 18.7Fn + 28.0(Fn)^2$
0.75	Ballast	$2.6 - 12.5Fn - 13.5(Fn)^2$
0.80	Ballast	$3.0 - 16.3Fn - 21.6(Fn)^2$
0.85	Ballast	$3.4 - 20.9Fn + 31.8(Fn)^2$

Table 3 Ship form coefficient C_{Form}

Type of ship	Ship form coefficient C_{Form}
All ships (except containerships) in loaded condition conditions	$0.5BN + BN^{6.5}/(2.7\nabla^{2/3})$
All ships (except containerships) in ballast condition conditions	$0.7BN + BN^{6.5}/(2.7\nabla^{2/3})$
Containerships in normal loading conditions	$0.5BN + BN^{6.5}/(22.0\nabla^{2/3})$

3.3 Calculation of the propeller rotation speed

When the predicted power delivered by the engine and the ship speed in a rough sea are known, the propeller rotation speed can be determined using the propeller characteristics.

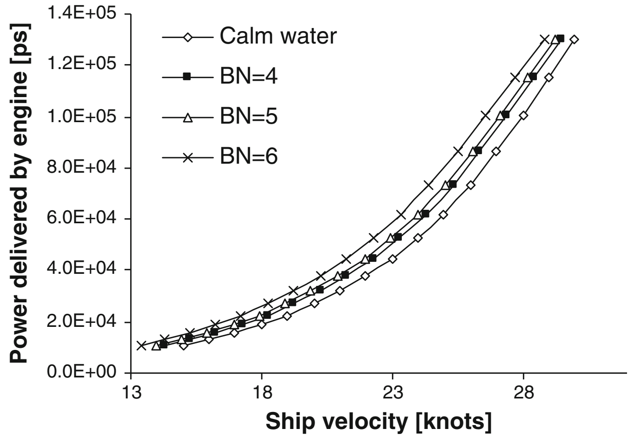

Fig. 3 Power delivered by the engine under different Beaufort numbers in head sea conditions

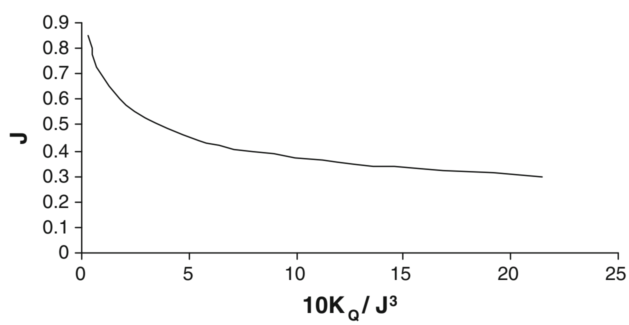

Fig. 4 $10K_Q/J^3$ versus J curve

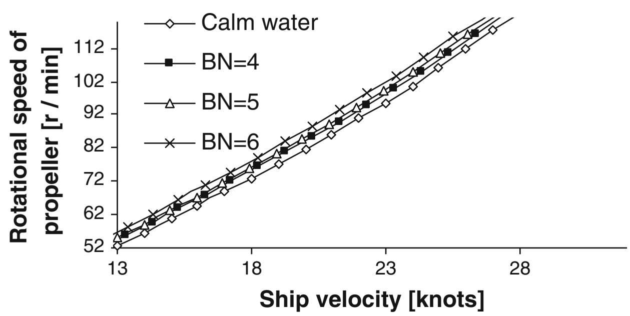

Fig. 5 Propeller rotation speeds under different Beaufort numbers in head sea conditions

The ratio $10K_Q/J^3$ can be expressed as

$$10K_Q/J^3 = \frac{10Q}{\rho V_A^3 D^2} = \frac{10P_P\eta_G\eta_S\eta_R}{V^3(1-w)^3(2\pi\rho D^2)} \tag{12}$$

$$n = \frac{V_A}{JD}. \tag{13}$$

The value of $10K_Q/J^3$ can be obtained when the power delivered by the engine P_P and the ship speed V are known. The advance ratio J can be calculated from the curve of $10K_Q/J^3$ versus J, as shown in Fig. 4. Figure 5 shows propeller rotation speeds under the different Beaufort numbers used in Fig. 3 in head sea conditions.

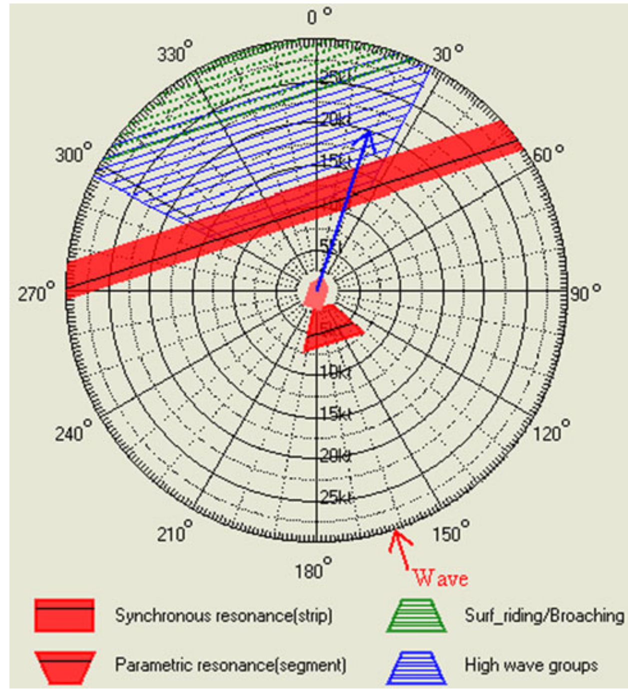

Fig. 6 The constraints specified by the IMO guidelines

3.4 Considering ship safety during a voyage

The IMO has published guidelines [13] for the safe operation of all types of merchant ships. These guidelines provide recommendations to prevent capsize and heavy roll motion due to

- Surf-riding and broaching-to
- A reduction of intact stability caused by riding on a wave crest of high wave groups
- Synchronous rolling motion
- Parametric rolling motion

Figure 6 represents the constraints specified by the IMO guidelines using a polar diagram that integrates weather conditions. The length and the direction of the blue arrow represent the ship speed and heading, respectively. Zones with different colours represent different hazardous conditions of ship operation corresponding to a given weather condition. If the arrow falls within a colour zone during the route optimisation process, it means that ship safety is at risk.

The route constraints in the simulation are only those included in the current IMO guidelines, which do not consider the effects of slamming, deck wetness, propeller racing, motion sickness and other hazards in adverse weather conditions. All of these factors should be treated as constraint conditions in future studies.

4 Dynamic programming

4.1 Advantages of the 3DDP method

Dynamic programming is a method that can solve complex problems by breaking them down into many simpler subproblems. The term "stage" is used in the 3DDP method to represent the divisions of a sequence of various subproblems of a system. The control vector is chosen at each stage. Subproblems are calculated stage by stage. Information from the preceding stage is used to determine the control variables of the next stage. The parameters used to describe a stage must be variables that monotonically increase as the ship voyage or route optimisation progresses. In ship routing problems, time, fuel consumption, and voyage progress monotonically increase during a voyage, so all of these three variables can be used to define a stage. The study reported in [14] uses time to describe its stages; Calvert and De Wit [4, 5] used voyage progress to describe stages; and in [6, 7] fuel consumption is utilised to describe stages. Each stage consists of many states, which are defined as a specifically measurable condition of the ship, such as time and geographic location. If time is chosen as the stage variable, the states of the stage can be defined by possible locations that the ship could pass. If voyage progress and fuel consumption are chosen as the stage variables, the states should be defined by time and possible geographical positions.

The 2DDP method uses voyage progress as the stage variable. It assumes during the route optimisation that ships sail at a constant propeller rotation speed or a constant engine power for the entire voyage. There is a one-to-one relationship between ship position and time. Thus, time is not needed as a variable to specify states in this method. Several authors [14–16] have already attempted to solve the weather routing problem by using 3DDP in which both engine power and shipping course are used as the control variables during a voyage. Aligne et al. [14] chose time as the stage variable and used the forward algorithm; Calvert [15] and Chen [16] employed voyage progress as the stage variable and used the backward algorithm. The 3DDP method presented in this paper employs voyage progress as the stage variable, together with the use of the forward algorithm.

In the forward algorithm, the initial departure time is fixed; the arrival time at a location at which the optimisation calculation has progressed to can be treated as a flexible parameter. This allows a set of routes with minimum fuel consumption to be obtained, corresponding to different arrival times at the current location that the optimisation has focused upon. This is a logical process in line with the ship operation along a voyage course.

Compared with the use of time as the stage variable, the advantage of using voyage progress as the stage variable is that a coarser grid system can be used, which will save much computation time. When voyage progress is chosen as the stage variable, the headings of the ship are predefined by the grid points, so the power delivered by the engine becomes the only control variable.

4.2 Grid design

Dynamic programming uses a grid system to specify the spatial layout of stages and states used in the algorithm calculation. Since the great circle is the optimum route through calm water from the departure point to the destination, it is chosen as the reference route when constructing the grid system used in the 3DDP route optimisation.

As described above, the states of a stage are of three-dimensional (i.e. time and geographical location, with a unit spacing of ΔY, perpendicularly away from the great circle). The farthest states of a stage from the great circle are locations that the ship may pass to avoid bad weather or sea conditions. Grids should be deleted when they cross islands/rocks en route. Unlike traditional dynamic programming, the time variable for states in the 3DDP is not fixed. It is determined as the optimisation procedure progresses.

Figure 7 shows an example of state projections on a longitude × latitude plane where 16 stages have been allocated $(1, 2, 3 \ldots 16)$ from the start to the destination of the shipping route. The distance between neighbouring stages is always ΔX. The total number of stages is determined by the total distance of the route and the available computing capacity.

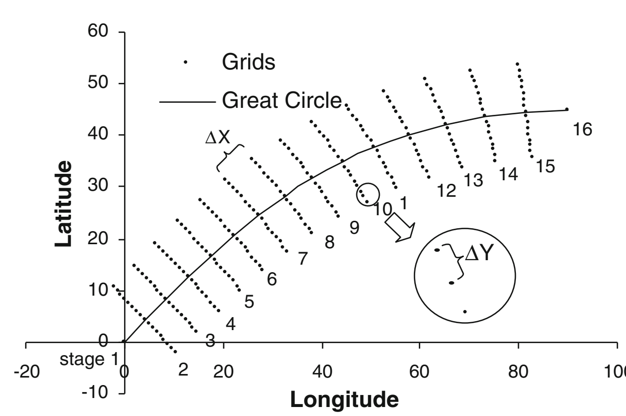

Fig. 7 Projections of the space grid system on a longitude × latitude plane

4.3 Algorithm description

Dynamic programming is performed based on Bellman's principle of optimality, which can be described as follows: "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" [17]. Dynamic programming usually uses a backward recursive algorithm, where a path is optimal if and only if, for any intermediate stage, the choice of the following path is optimum for this stage. However, for a weather routing problem, a forward algorithm offers more convenient programming. In contrast with the backward version, in the forward algorithm, a path is optimal if and only if, for any intermediate stage, the choice of the previous path is optimum for this stage. Using this principle, certain complex problems can be broken down into a sequence of simpler subproblems. The procedure of predicting fuel consumption between two stages can be considered as a subproblem to solve during the weather routing optimisation. The predicted optimum fuel consumption during an entire voyage is obtained by summing the fuel consumed between the stages along the entire route.

4.3.1 Notation definition

The notations used during programming are defined as follows:

K: Total number of stages.
$N(k)$: Total number of states projected on the latitude × longitude plane for stage k, where $k = 1, 2, 3\ldots k$. $N(1) = 1$, $N(K) = 1$.
$P(i,\ k)$: Positions of the states projected on the latitude × longitude plane for stage k, where $i = 1, 2, 3\ldots, N(k)$. The distance between $P(i,\ k)$ and $P(i+1,\ k)$ is Δy. $P(1,\ 1)$ is the departure position; $P(1,\ K)$ is the destination position.
J: Total number of intervals on the time axis for a stage.
$\Delta\bar{t}$: Time interval between two adjacent grids on the time axis for a stage.
Δt: Time step for calculating the fuel consumption between two stages.
$X(i, j, k)$: State of stage k, where $i = 1, 2, 3\ldots, N(k)$, $j = 0, 1, 2\ldots, J-1$, $k = 1, 2, 3\ldots, K$. The geographical position of the state $X(i,\ j,\ k)$ is defined by $P(i,\ k)$ for stage k, the time variable for a state is $t_{i,j,k}$, $\bar{t}_j \leqq t_{i,j,k} \leqq \bar{t}_{j+1}$, $\bar{t}_j = \Delta\bar{t} \times j$, $\bar{t}_{j+1} = \Delta\bar{t} \times (j+1)$. Between point $(P(i,\ k), \bar{t}_j)$ and $(P(i,\ k), \bar{t}_{j+1})$, there is only one state that corresponds to minimum fuel consumption. A state $X(i,\ j,\ k)$ of stage k has a floating status during the iteration until the point corresponding to minimum fuel consumption is found. This is then chosen as state $X(i,\ j,\ k)$ before proceeding with the following calculation.
$X(1,\ 0,\ 1)$: Initial state. The time variable $t_{1,0,1}$ of the initial state is 0.
$X(1,\ j,\ K)$: States of the final stage K, $j = 0, 1, 2\ldots, J-1$. The positions of these states on the latitude × longitude plane is $P(1,\ K)$.
$F_{\mathrm{opt}}(i,\ j,\ k)$: The minimum fuel consumption from the initial state to the state $X(i,\ j,\ k)$.
$u(m)$: Ship speed through calm water, where $m = 1, 2, 3\ldots, M$, $u(m+1) - u(m) = \Delta u$, and M is the total number of discrete speeds between two stages.

4.3.2 Subproblem solution

When executing the 3DDP, the power delivered by the engine is taken to be constant between two stages once it has been optimised, except in situations where the ship must slow down due to certain weather conditions to ensure ship safety. Since it is convenient to use ship speed in hydrodynamic analysis, when determining resistance and the loss of ship speed, etc., the ship speed in calm water (instead of the power delivered by the engine) is used as a stage control variable in the following discussion. Once the ship speed in calm water has been determined, the ship power can be obtained from the one-to-one relationship between engine power and ship speed described in Sect. 3.1. Thus, fuel consumption can be obtained indirectly once the ship speed is known. The selected calm-water ship speed between two stages is not allowed to be larger than the value corresponding to the maximum continuous rating (MCR) of the main engine. The procedure for determining the fuel consumption of a given ship speed in calm water between two stages is as follows:

Step 1 Set the calm-water ship speed.
Step 2 If the calm-water ship speed is smaller than the minimum allowed value, skip the calculation for the given heading. This means that the navigation for this given heading is not available under this calm-water ship speed or power delivered by the engine.
Step 3 Calculate the ship speed. The loss of ship speed is calculated based on the Kwon's method.
Step 4 If ship operations violate the set constraints, the ship speed in calm water is reduced by Δu and the procedure returns to step 2.
Step 5 Calculate the propeller rotation speed. The propeller rotation speed is determined by the method described in Sect. 3.3.
Step 6 Calculate the ship position and fuel consumption for the next time interval Δt.

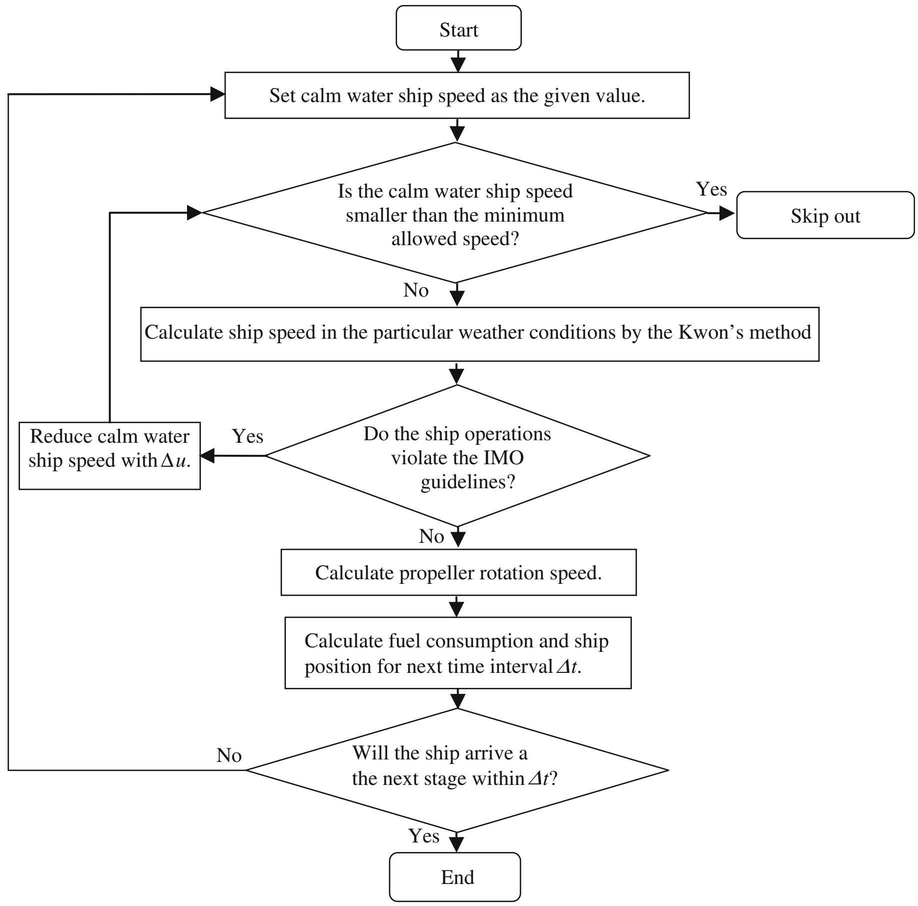

Fig. 8 Estimating the fuel consumption between two stages

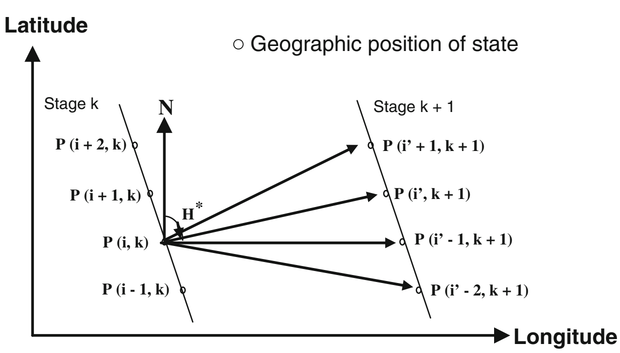

Fig. 9 Calculating the ship heading between two stages

Step 7 Execute steps 1–6 repeatedly with a fixed time interval Δt between two stages until the ship (simulation step) arrives at the next stage or the final destination.

The time interval Δt for the calculation is normally chosen to be the frequency of receipt of weather forecasts, which is usually every 3 or 6 h. Figure 8 shows the procedure described above.

4.3.3 *Recursive procedure for forward dynamic programming*

The recursive procedure for forward dynamic programming can be described as follows:

Step 1 Set the stage variable $k = 1$. $F_{\text{opt}}(1, 0, 1) = 0$ and all other $F_{\text{opt}}(i \geq 1,\ j \geq 0,\ k \geq 2)$ are set to infinity.

Step 2 Iterate steps 3–7 below for each state $X^* = X(i, j, k)$ of stage k, where $i = 1, 2, 3\ldots, N(k)$, $j = 0, 1, 2\ldots J - 1$. Parameters with the symbol * are used as interim parameters at stage k when performing iterations. If a state $X^* = X(i, j, k)$ is unattainable due to constraints, the calculation of the state is abandoned and the next state is calculated.

Step 3 Calculate the ship heading H^* from position $P(i, k)$ of state X^* to the position of the next stage $P(i', k + 1)$, as shown in Fig. 9, where $i' = 1, 2, 3\ldots, N(k + 1)$. A rhumb line is used as the ship route between positions $P(i, k)$ and $P(i', k + 1)$. Iterate steps 4–7 for each H^*. If a certain

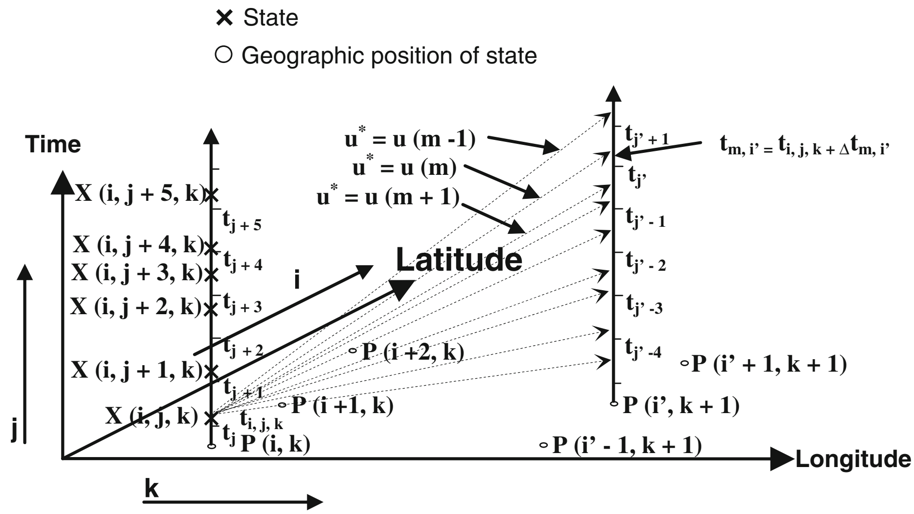

Fig. 10 Calculating the possible states of stage $k + 1$

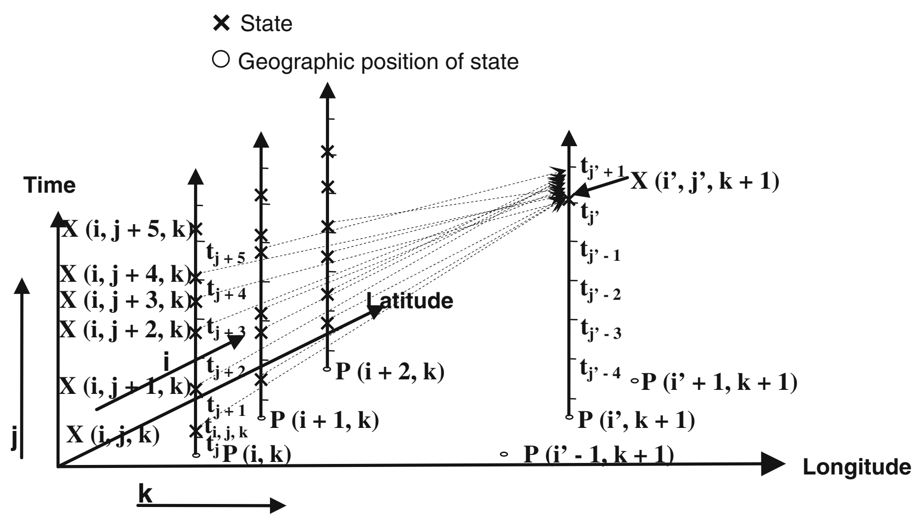

Fig. 11 Determining the state

ship heading H^* violates the set constraints, abandon the calculation for this chosen heading and go to the next heading calculation.

Step 4 Iterate steps 5–7 for each $u^* = u(m)$, where $m = 1, 2, 3\ldots, M$.

Step 5 Calculate the fuel consumption $\Delta f_{m,i'}$ and the voyage time $\Delta t_{m,i'}$ between state X^* of stage k and a state of stage $k + 1$. If a ship speed u^* violates the set constraints, abandon the calculation of the chosen speed and go to the calculation for the next speed. $t_{m,i'}$ is the arrival time (simulation progression) at position $P(i', k + 1)$ from the initial state $t_{m,i'} = t_{i,j,k} + \Delta t_{m,i'}$, as shown in Fig. 10. The position $X' = (P(i', k + 1), t_{m,i'})$ represents a possible new state at stage $k + 1$. The total fuel consumption from the departure point to X' is f', $f' = F_{\mathrm{opt}}(i, j, k) + \Delta f_{m,i'}$. Parameters with the symbol $'$ are used as interim parameters at stage $k + 1$ when performing iterations.

Step 6 Calculate the j' which satisfies $\bar{t}_{j'} \leqq t_{m,i'} \leqq \bar{t}_{j'+1}$. If $f' < F_{\mathrm{opt}}(i', j', k + 1)$, then replace the old $X(i', j', k + 1)$ and $F_{\mathrm{opt}}(i', j', k + 1)$ by X' and f', respectively, to give new $X(i', j', k + 1)$ and $F_{\mathrm{opt}}(i', j', k + 1)$ terms, as shown in Fig. 11. It is clear that only one possible state that corresponds to minimum fuel consumption between the times $\bar{t}_{j'}$ and $\bar{t}_{j'+1}$ at position $P(i', k + 1)$ of stage $k + 1$ will be treated as the state $X(i', j', k + 1)$. The time variable of the state $X(i', j', k + 1)$ is not fixed. The benefit of using an nonfixed time for possible (floating) states is that it eliminates the need for interpolation, which is usually used in dynamic programming problems. This saves computation time. When the weather during $\Delta\bar{t}$ does not change very much, employing the

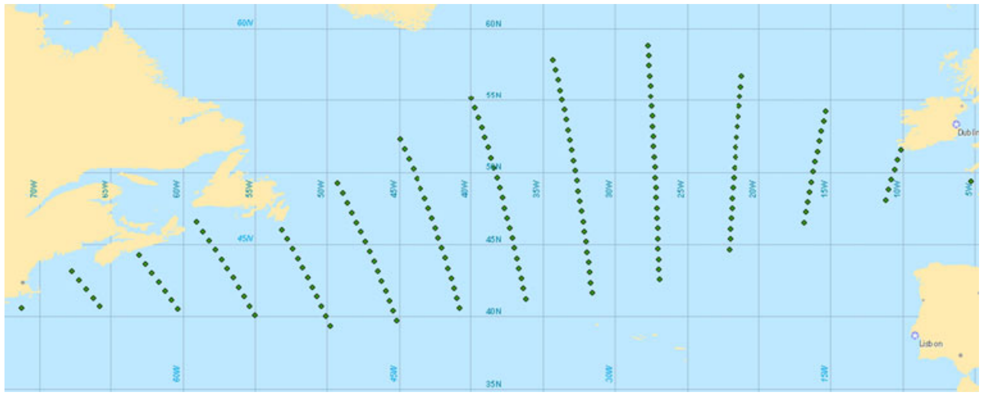

Fig. 12 The grid system

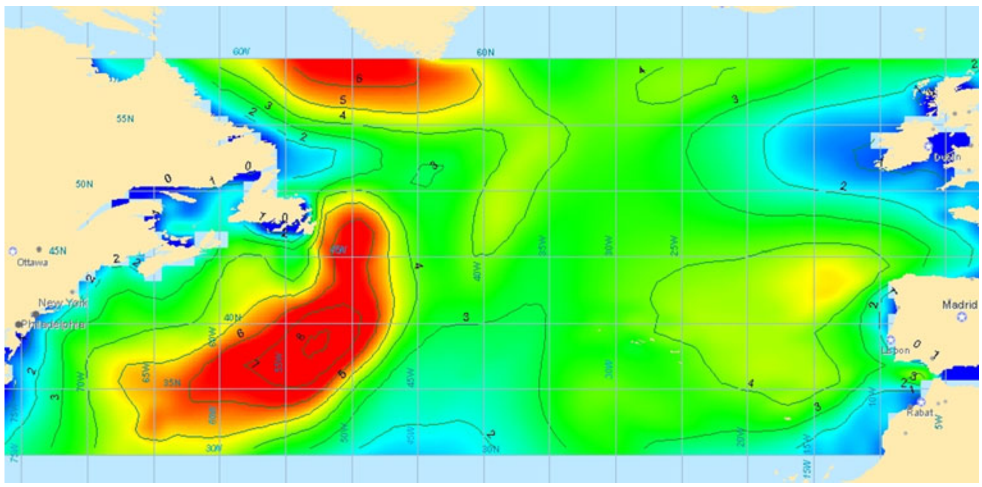

Fig. 13 Weather forecast at 3 a.m. 25/01/2011 for the North Atlantic region

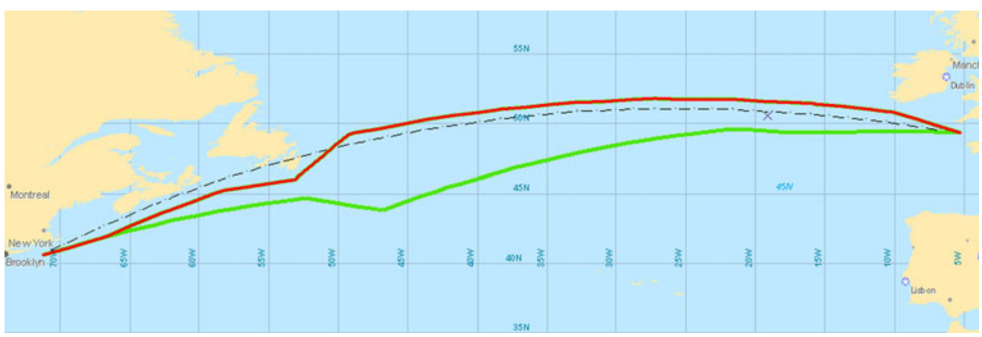

Fig. 14 Optimised route

floating technique will not influence the accuracy of the optimised results.

Step 7 The state X^* of stage k from which the ship departs (simulation step) before arriving at the state $X(i', j', k+1)$ of stage $k+1$, and the corresponding control variables, are saved in order to allow the optimum route to be traced by a backward procedure at the end of the calculation.

Step 8 Let $k = k + 1$, and go to step 2 until $k = K$.

Once the state of the final stage K has been obtained, a backward calculation procedure is used to identify the route with the optimum fuel consumption. This route is the optimised route obtained from the 3DDP.

The optimisation procedure described above is repeated each time the weather forecast or ship position is updated at sea.

5 Case study

We now present the following data for the case ship, route and weather conditions (http://nomads.ncep.noaa.gov/cgi-bin/filter_wave.pl):

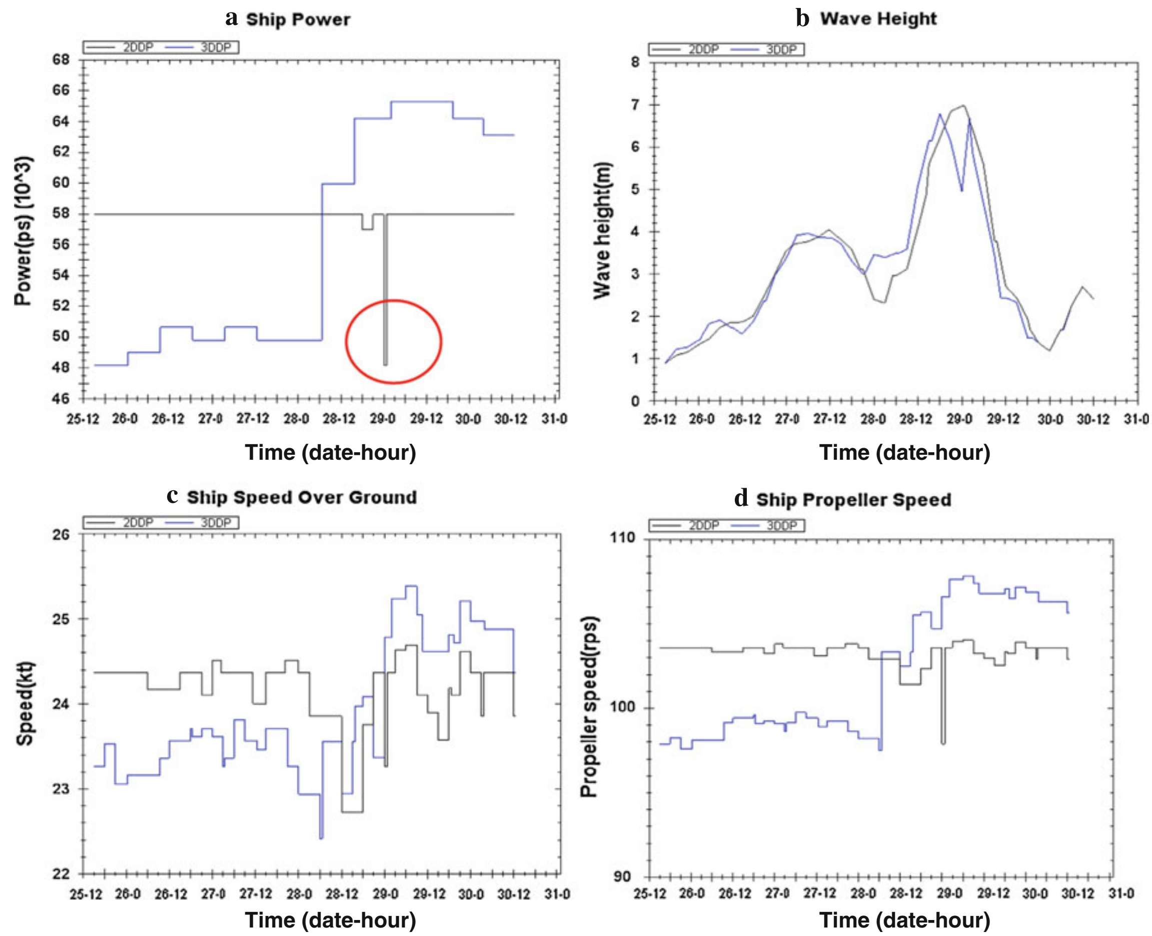

Fig. 15 Simulation results

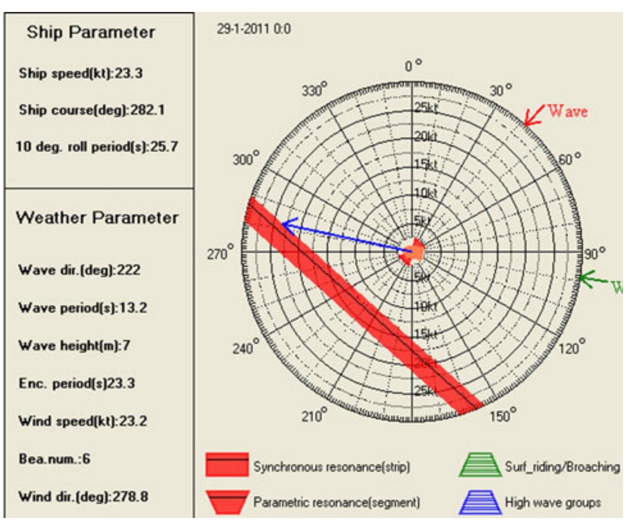

Fig. 16 Polar diagram corresponding to the red circle area of Fig. 15a

Case ship: 54,000 DWT container ship
Departure from Le Havre
Arrival at New York
Departure time: 03:00 p.m. 25/01/2011
Schedule arrival time: 00:30 p.m. 30/01/2011
Time interval between states of a stage: $\Delta\bar{t} = 1$ h
Time step for calculating the fuel consumption between two stages $\Delta t = 6$ h
Calm-water ship speed: $u = 5$–25.4 knots
Increments of ship speed: $\Delta u = 0.1$ knots
Total number of stages: $K = 14$
Size of a stage: $\Delta X = 212$ miles
Size of a state: $\Delta Y = 46$ miles
Maximum continuous rating of the main engine (MCR): 48598 kW (corresponding to 25.4 knots in calm water)
Specific fuel oil consumption: 170 g/kWh
Safety constraints: the maximum wave height is 7 m and the ship must comply with the IMO guidelines [13].

Figure 12 shows the grid system of the simulation. Figure 13 illustrates the weather forecast, with a significant wave height of 8 m, at 3 a.m. 25/01/2011 in the North Atlantic region. Stormy weather is moving in a northeasterly direction. Significant waves with heights of >10 m occur during the storm. The ship must therefore avoid this area with stormy weather for safe operation.

The above ship operation data and route constraints together with the weather forecast are used as input for the 2DDP and 3DDP methods, respectively, in order to perform route optimisation. Figure 14 shows the optimised routes calculated by both methods: the red line represents the route optimised by the 3DDP method and the green line is from the 2DDP method. The dash-dotted line is the great circle between the departure and the destination. The simulation results indicate that the total distance of the optimised route from the 3DDP method is 2807.0 nautical miles, with a total fuel consumption of 823.6 tons, whereas the optimised route from the 2DDP method is 2838.2 nautical miles in length and uses 849.5 tons of fuel. The route chosen by the 3DDP method is closer to the great circle and 31.2 nautical miles shorter than that chosen by the 2DDP method. Figure 15 shows the simulation results for the ship speed over the ground, ship power and propeller speed during the voyage. As discussed early, the 2DDP method treats the power delivered by the engine as a constant during the voyage, aside from when the vessel reduces her speed upon encountering certain storm weather conditions, as shown by the red circle in Fig. 15a. Figure 15b shows the wave heights encountered during the shipping routes optimised by the 2DDP and 3DDP methods, respectively. The engine power of the route optimised by the 3DDP method is variable during the voyage. As shown in Fig. 15c, the ship speed (power) is initially lowered until about 6 a.m. on 28 January to allow the stormy weather to pass by. Once the bad weather has passed, the ship speeds up to catch up for lost time, and thus arrives at the destination on time. As a result, 25.9 tons of fuel are saved. Figure 16 shows the constraints specified by the IMO guidelines on a polar diagram. In Fig. 16, if the ship increases speed, it will enter the dangerous synchronous resonance zone, in which the ship could capsize. The programming must ensure that the ship speed keeps it out of the danger zone.

The simulation results have clearly demonstrated that the optimised route and ship operation parameters obtained by the 3DDP method lead to a fuel saving of 3.1% as compared with the fuel used during the optimised route obtained with the 2DDP method using the same input data and constraints.

6 Conclusion

A novel forward dynamic programming method for weather routing has been presented. The newly developed 3DDP method offers quasi-global optimal routing. Compared with other traditional weather routing programmes, the newly developed method allows changes of shipping course and power delivered by the engine during the optimisation process. The newly developed method is easier to understand. Its programming is straightforward and utilises a forward strategy. A case study with real weather conditions showed that significant fuel savings can be achieved by following the optimised route and ship operation profile provided by the newly developed method rather than a traditional method.

The simulation results indicated that a fuel saving of up to 3.1% can be achieved by following the route and operation profiles afforded by the newly developed 3DDP method rather than the 2DDP method. As the price of fuel oil continues to increase and emission regulations are becoming more and more stringent, fuel savings from shipping are of significant importance to shipping companies and have a long-term impact on the global environment. The newly developed weather routing programme is able to contribute to the international push to save fuel and reduce greenhouse gas emissions.

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