Nondemolition Measurement of the Vacuum State or its Complement

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Measurement is integral to quantum information processing and communication; it is how information encoded in the state of a system is transformed into classical signals for further use. In quantum optics, measurements are typically destructive, so that the state is not available afterwards for further steps. Here we show how to measure the presence or absence of the vacuum in a quantum optical field without destroying the state, implementing the ideal projections onto the respective subspaces. This not only enables sequential measurements, useful for quantum communication, but it can also be adapted to create novel states of light via bare raising and lowering operators.

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At first glance, measuring the vacuum is trivial, a perfect photodetector will reveal the vacuum state upon the nonoccurrence of a click. However, the converse, i.e., measuring the nonvacuum, ideally should preserve this sector for further interrogation—something which is difficult to achieve with direct photodetection. Formally, we would like to implement the following measurement projectors, $\{|0\rangle\langle 0|, \mathbb{1} - |0\rangle\langle 0|\}$, where the latter nonvacuum outcome removes the vacuum component without affecting the relative amplitudes or coherences of the other Fock states. This is crucial for sequential measurement schemes [1–3], and rules out other projective schemes such as quantum nondemolition measurements of photon number [4–7]. Developing methods for nondestructive measurements on optical fields [8,9] is therefore important for quantum information processing systems.

Measurement is also a key element in performing non-Gaussian operations, e.g., for entanglement purification of continuous variable states [10–12]. Recent examples include the implementation of the quantum optical creation and annihilation operators, both of which rely on postselection [13–15]. Extending the type of possible operations is crucial for the production of tailored states in quantum information systems. Our method can be simply extended to provide a first realization of the bare photon addition and subtraction operators.

We consider a single mode of an optical cavity in an arbitrary quantum state, ρ, as our system to be measured. To perform the measurement, we introduce a probe which consists of a three level atom in the Λ configuration [see Fig. 1(a)]. The cavity mode can be coupled controllably to transition B whereas transition A interacts with an externally applied laser field [16,17]. In these papers, the general adiabatic mapping of atomic levels to cavities was introduced. Our particularly simple configuration is insensitive to all field amplitudes other than the vacuum.

The Hamiltonian of the combined system can be written in the rotating wave approximation (RWA) as

$$\begin{aligned} H^{\mathrm{RWA}} = {} & \hbar\Delta|e\rangle\langle e| + \hbar\gamma_A(t)(|e\rangle\langle g_A| + |g_A\rangle\langle e|) \\ & + \hbar\gamma_B(t)(|e\rangle\langle g_B|a + |g_B\rangle\langle e|a^\dagger), \end{aligned} \tag{1}$$

where the coupling constants γ_A and γ_B between the atom and the two fields depend on the strength of the respective fields at the point where the atom is located. An optional detuning Δ can be applied to both fields in order to suppress single-photon resonance effects as long as we maintain the two-photon resonance condition,

$$E_{g_B} - E_{g_A} = \hbar(\omega_B - \omega_A). \tag{2}$$

The situation is similar to the V-STIRAP scheme for producing single photons [18] where a cavity evolves from $|0\rangle \rightarrow |1\rangle$ through a dark state adiabatic evolution of an atom $|g_A\rangle \rightarrow |g_B\rangle$.

In our measurement procedure we run the V-STIRAP sequence in reverse: the initial state of the atom is $|g_B\rangle$, and the order of the A and B couplings is switched [see Fig. 1(b)]. If the cavity field initially contains at least one photon, at the end of the sequence the atom is left in $|g_A\rangle$ and the field has one photon subtracted. However, if the cavity was originally in the vacuum state, the atom stays in $|g_B\rangle$ and the cavity is left unchanged. An initial superposition of the cavity evolves as

$$|g_B\rangle \sum_{n=0}^{\infty} \alpha_n |n\rangle \rightarrow |g_B\rangle \alpha_0 |0\rangle - |g_A\rangle \sum_{n=1}^{\infty} \alpha_n |n-1\rangle. \tag{3}$$

The state of the atom is now entangled with that of the cavity. By measuring the atomic state in either $|g_A\rangle$ or $|g_B\rangle$, we have determined whether the initial cavity state had at least one photon or none. By coherent rotations of the ground states before a population measurement, projections onto more general subspaces are also possible.

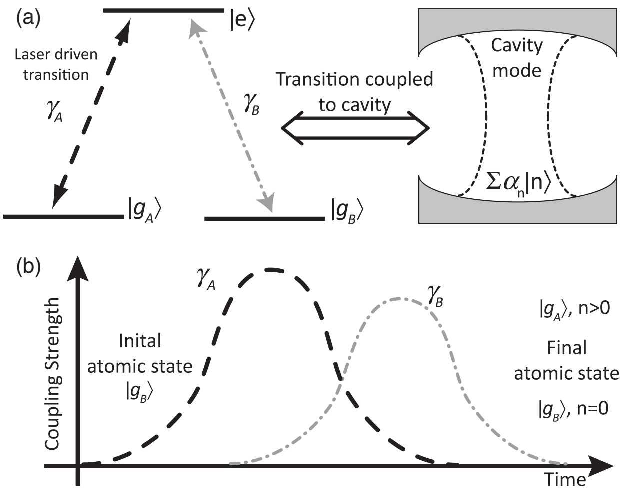

FIG. 1. (a) Lambda atomic system coupled to cavity. A three level atom with two ground states ($|g_A\rangle$, $|g_B\rangle$) and a single excited level ($|e\rangle$) can be used to probe for the vacuum component of an optical field with annihilation operator a. The A transition is driven by a STIRAP laser and the B transition can be controllably coupled to the cavity mode to be measured. Initially, the atom is in the state $|g_B\rangle$. The final state of the atom $|g_A\rangle$ or $|g_B\rangle$ depends on the presence or absence of photons in the field, respectively. (b) Counterintuitive pulse sequence for the couplings. Consider the case where there are $n \geq 1$ photons in the field. With the initial state of the combined atom-mode system $|g_B, n\rangle$, coupling A is turned on first. As coupling B is slowly increased, the atom-cavity state adiabatically follows the dark-state manifold $\sin\theta|g_A, n-1\rangle - \cos\theta|g_B, n\rangle$, $\theta = 0 \rightarrow \pi/2$. Coupling A is now turned off, followed by coupling B, leaving the final state of the system as $|g_A, n-1\rangle$. If the cavity was originally vacuum ($n = 0$), the final state of the atom remains as $|g_B\rangle$.

If the atom is found in $|g_A\rangle$, the field amplitudes have been shifted by one. The ideal projection ($\mathbb{I} - |0\rangle\langle 0|$) results if we replace the subtracted photon, this is simply achieved by running the V-STIRAP procedure forwards. Note that this does not require the initial cavity state to be vacuum, we can add a photon to an arbitrary state of the field. As discussed later, the shifting property of the procedure can be exploited to perform novel operations and generate nonclassical states of light.

The key aspect of the adiabatic process is that the evolution of the system does not rely on the dynamics of the Hamiltonian, provided that the conditions of adiabatic transition are satisfied. In this way, the state of the ancilla atom can be made asymptotically insensitive to the cavity photon number, except for the critical case of the vacuum. In the usual Jaynes-Cummings scenario, the dynamics in each of the combined Fock subspaces proceeds at a rate proportional to the square root of the photon number, leading in general to different states of the atom. In our scheme, the atom does not distinguish between different photon numbers $n = 1, 2, 3, \ldots$, which allows us to perform the ideal projection onto the complement of the vacuum, in contrast to previous proposals for quantum nondemolition measurements of the optical field [6,7].

We can extend the method to project onto the joint n-mode vacuum state or complement, as required in the decoding scheme of [2]. This requires a probe atom with $n + 2$ levels in an $(n + 1)$-pod configuration. Let $|g_0\rangle$ denote the initial state of the atom, the remaining ground states be denoted $|g_n\rangle$ for $n = 1, \ldots, n$, and $|e\rangle$ be the excited level. The $|e\rangle \leftrightarrow |g_n\rangle$ $(n > 0)$ transitions are driven by lasers with strength Γ_j and the $|e\rangle - |g_0\rangle$ transition is selectively coupled in turn to each of the n modes with strength γ_j. In real atoms, this may be difficult but it may be simpler in engineered systems, e.g., superconducting qudits coupled to transmission lines. We apply in turn the same procedure as for the single mode measurement by sequential pairwise adiabatic variation of $\{\Gamma_j, \gamma_j\}$, $j = 1, \ldots, n$, after which the population of $|g_0\rangle$ is determined. If the atom is detected in $|g_0\rangle$, then the n modes are projected onto the joint vacuum state $|00\ldots 0\rangle$, otherwise the atom and n modes are left in a (generally entangled) state where the $|g_0\rangle|00\ldots 0\rangle$ state has been truncated. To disentangle the atom and add back subtracted photons, running the sequence of couplings backwards and in reverse order returns the atom to $|g_0\rangle$ which erases any information of the photon number distribution of the n modes.

The experimental setup for V-STIRAP [18] can be easily adapted to perform our vacuum measurement (Fig. 2). We

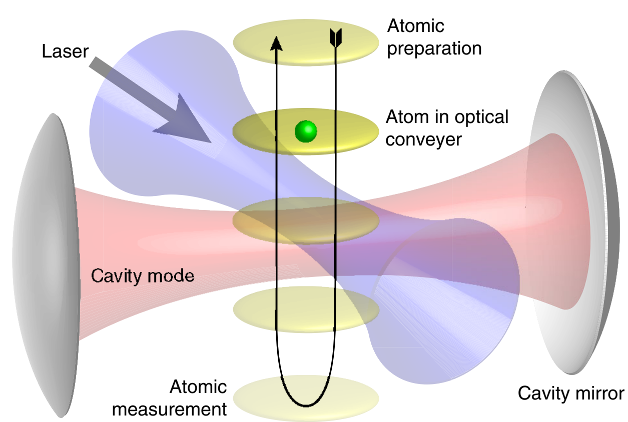

FIG. 2 (color online). Cavity QED vacuum measurement. The configuration of the cavity mode, driving laser, and trajectory of the atom are similar to the V-STIRAP scheme, except that the laser beam is encountered before the cavity mode. An optical lattice traps and controls the position, hence the coupling, of the atom with both the driving laser and cavity mode. State preparation in $|g_B\rangle$ is performed before the atom is transported into the cavity. After the atom has crossed the cavity and adiabatically interacted with laser and mode, it is measured, e.g., by fluorescence shelving [41] or cavity enhanced detection [42], to discover which ground state it is in. To perform the ideal ($\mathbb{I} - |0\rangle\langle 0|$) operation in the case of the $|g_A\rangle$ result, the motion of the atom is reversed in order to replace the photon extracted from the cavity.

use a cavity with a long storage time to reduce leakage and decoherence. We also introduce preparation and readout zones for the atom before and after the cavity, respectively. The motion of the atom is reversed in the case of measuring the atom in $|g_A\rangle$ in order to replace a subtracted photon. There are several experimental challenges, mainly the lifetime of the field compared to the time required to implement the measurement. The cavity field must last long enough for the atom to be adiabatically transported, measured, and returned. We can modify the scheme to allow for a second atom prepared in $|g_A\rangle$ to immediately replace the photon in parallel with the probe atom measurement. In the case of a $|g_B\rangle$ result, we can either subtract the photon again or let natural cavity decay return the cavity to the vacuum state.

Most important for the scheme is the ratio of the cavity damping κ to maximum atom-cavity coupling $g = \max\gamma_B$. Both these rates depend upon the effective mode volume of the cavity and balancing these factors will be system dependent. To examine the performance of the protocol under nonideal conditions, we have simulated the measurement of a lossy cavity with finite sweep times with immediate photon replacement, the results displayed in Fig. 3. Full details of the simulation can be found in the Supplemental Material [19].

A straightforward application of this measurement is in sequential decoder schemes as discussed in [3]. In the protocol of [2], the state of an n-mode system has to be identified in order to successfully decode the symbol being sent. The state is taken from an ensemble of products of coherent states, $\{|\alpha_1^k, \alpha_2^k, \ldots, \alpha_n^k\rangle\}$. A sequence of displacements and projections onto the n-mode vacuum or its

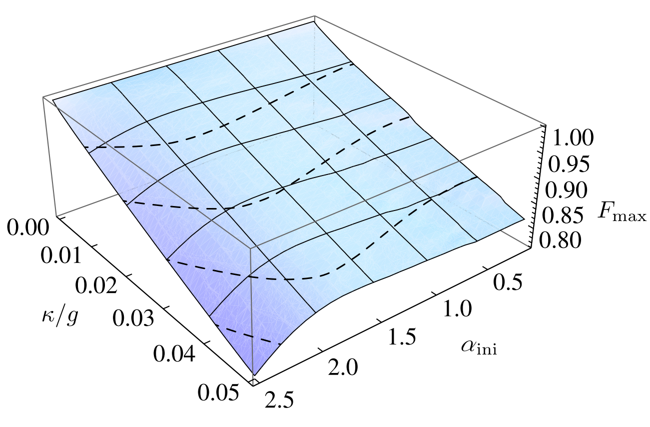

FIG. 3 (color online). Fidelity of the full measurement procedure, conditioned on probe result $|g_A\rangle$. The initial cavity field is in a coherent state of amplitude $\alpha_{\rm ini}$. The detuning Δ was zero and the $|e\rangle$ decay rate was 1% of the maximum cavity coupling $g = \max\gamma_B$. The coupling constants γ_A and γ_B were modulated as $\cos^2 t$ and $\sin^2 t$, respectively. Parallel photon replacement and probe readout is assumed. Using knowledge of the cavity properties and initial state, the transition time and laser coupling at each data point were optimized for fidelity. Contours (dashed) indicate equal optimum fidelity.

complement has been shown to decode the message successfully in the $n \to \infty$ limit as long as the rate of transmission is below the Holevo bound.

We can also use the photon number altering properties of our procedure to enact bare raising and lowering operations, in contrast to the creation $a^\dagger$ and annihilation operators a as usually considered. The non-Hermitian $a^\dagger$ and a operators represent non-Gaussian operations and have been realized probabilistically in experiments [13–15]. "Subtracting" a photon from squeezed light can produce an approximate cat state [20–23], and both processes have been used in superoptimal optical amplification protocols [24–27].

Due to Bosonic enhancement however, the $a^\dagger$ and a operators do not simply add and subtract photons, but also modify the state amplitudes with $\sqrt{n}$ factors. Pure addition and subtraction of photons are represented by bare raising and lowering operators [28], sometimes known as photon number shifting operators [29],

$$E^+ = \sum_{n=0}^{\infty} |n+1\rangle\langle n|, \qquad E^- = \sum_{n=1}^{\infty} |n-1\rangle\langle n|. \quad (4)$$

These can produce nonclassical states of light, for example, any state which has E^+ applied to it must violate the Klyshko criterion [29]. Applying E^+ to a coherent state produces a state with sub-Poissonian statistics, whereas applying E^- makes the state super-Poissonian.

There has been little study of the bare operators and their effects, mainly because they have not been realized experimentally [30]. Implementing E^+ and E^- requires cancellation of the $\sqrt{n}$ Bose enhancement factors inherent in a and $a^\dagger$. The nature of the $(\mathbb{I} - |0\rangle\langle 0|)$ projection and the adiabatic process that we have described does not alter the relative weights of the amplitudes corresponding to different photon numbers, in contrast to other schemes which rely on $a^\dagger$. The V-STIRAP process therefore implements E^+ and the reverse process realises E^-. In addressing the problem of quantum optical phase the measurement of moments of bare operators was proposed using a basic scheme similar to that considered here, without a detailed analysis of the effect of reachable experimental parameters [31]. An implementation of the photon subtraction operator has been suggested in superconducting systems [32].

We can also perform a reverse quantum scissors. In the original quantum scissors [33], photon numbers higher than one are truncated from a state, $|\psi\rangle = \sum_{j=0}^{\infty} c_j|j\rangle \to c_0|0\rangle + c_1|1\rangle$, up to normalization. This has been extended to make the cut at higher photon numbers [34,35]. In contrast, applying our measurement n times *without photon replacement* truncates the first n amplitudes, conditioned on not observing the vacuum, $|\psi\rangle = \sum_{j=0}^{\infty} c_j|j\rangle \to \sum_{j=n}^{\infty} c_j|j-n\rangle$. By adding n photons, we return the state to its original form but without the first n terms, $\sum_{j=n}^{\infty} c_j|j\rangle$. The probability that this will occur is $1 - \sum_{j=0}^{n-1} P_j$, where $P_j = |c_j|^2$ is the probability of

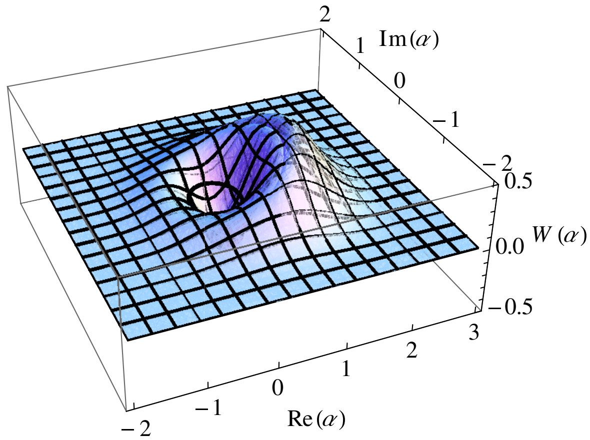

FIG. 4 (color online). Wigner function of a simulated vacuum-stripped $|\alpha = 1\rangle$ coherent state obtained using an $n = 1$ reverse quantum scissors (thick mesh) in comparison to that of the idealised output state $(\mathbb{1} - |0\rangle\langle 0|)|\alpha = 1\rangle$ (thin mesh). A loss rate of $\kappa = 0.033g$ and an atomic decay rate of $\Gamma = 0.217g$, as reported in [36], were used in the simulation, along with the other parameters and pulse shapes specified in Fig. 3. The values of time of transition $T = 21.43g^{-1}$ and ratio of the amplitudes of $\max\gamma_A{:}\max\gamma_B = 1.675$ were obtained by optimization of the output fidelity. The simulated state shares all the features of the idealised output state, including negativity (the intersection with $z = 0$ is drawn). This state is obtained from $|\alpha = 1\rangle$ with a probability of 0.52. The theoretical probability of measuring the vacuum $|\langle\alpha = 1|0\rangle|^2$ is 0.37, and an error of 0.11 is caused by photon loss during the process. The fidelity between the two considered states is 87%.

observing j photons. Trivially, we can also use the protocol to resolve photon number in this way. Figure 4 illustrates the results of an $n = 1$ reverse quantum scissors producing a vacuum stripped coherent state.

The ability to implement ideal projections on a field opens up new possibilities for quantum communication and computation. Adiabatic evolution in our method avoids the $\sqrt{n}$ factors in dynamical schemes and achieves the unusual nonlinearity required. Existing experiments, though not optimized for our measurement, already possess parameters sufficient for a proof of principle demonstration [36]. Though it will be challenging to engineer systems with even better κ/g ratios, recent advances in ultrastrong coupling in microwave systems [37], supermirror coatings [38], high-finesse cavities [39], and microresonators [40] all give grounds for optimism for achieving greater fidelities (Supplemental Material [19]). The simplicity and utility of the system described here for implementing several quantum optical information protocols should be significant drivers towards this goal.

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