

# AUTONOMOUS CONTROL OF A RECONFIGURABLE CONSTELLATION OF SATELLITES ON GEOSTATIONARY ORBIT WITH ARTIFICIAL POTENTIAL FIELDS

Garrie S. Mushet<sup>(1)</sup>, Camilla Colombo<sup>\*(2)</sup>, and Colin R. McInnes<sup>(3)</sup>

<sup>(1)</sup>*Advanced Space Concepts Laboratory, University of Strathclyde, Glasgow, G4 0LT, +44 (0)141 548 3897, garrie.mushet@strath.ac.uk*

<sup>(2)</sup>*Advanced Space Concepts Laboratory, University of Strathclyde, Glasgow, G4 0LT, c.colombo@soton.ac.uk*

<sup>(3)</sup>*Advanced Space Concepts Laboratory, University of Strathclyde, Glasgow, G4 0LT, colin.mcinnnes@strath.ac.uk*

**Abstract:** *This paper presents a method of controlling a constellation of small satellites in Geostationary Earth Orbit (GEO) such that the constellation is able to reconfigure - changing the angular position of its members relative to the Earth's surface in order to cluster them above particular target longitudes. This is enabled through the use of an artificial potential function whose minimum value corresponds to a state where the phase angle between each satellite and its intended target is minimised. By linking the tangential low-thrust acceleration of each satellite to this artificial potential function, the altitude of each satellite relative to the nominal GEO altitude is manipulated in order to achieve the required drift rate. A demonstration of the efficacy of the method is given through a simple test case in which a constellation of 90 satellites converge upon 3 equatorial targets, with each target requiring the attention of a varying number of spacecraft from the constellation. The constellation performance is analysed in terms of the time taken for the satellites to converge over their targeted longitudes and the  $\Delta v$  required to actuate the phasing maneuvers. This analysis is performed across a parameter space by varying the number of satellites in the constellation, the number of targeted longitudes, and a parameter representing the maximum acceleration of the thruster.*

**Keywords:** *Autonomous Constellation, Reconfiguration, Small Satellites, GEO.*

## 1. Introduction

In many space applications satellite constellations offer a number of advantages compared to single-satellite platforms. Some missions require multiple satellites by their very nature, for example GPS, which requires at least four satellites for accurate positioning on the Earth's surface, and many more to provide continuous global coverage [1].

Moreover, there is an increasing trend towards implementing constellations in mission applications which were traditionally performed with single-satellite platforms, such as Earth observation, space science and telecommunications, for example Disaster Monitoring Constellation (DMC) launched in November 2002 by Surrey Satellite Technology Ltd [2]. This is due to the reduced costs associated with the use of a great number of smaller and less massive satellites, which are becoming increasingly viable for implementation as advances in miniaturisation and mass production of space technology continue. Additionally, multi-satellite missions increase system reliability and

---

\*Currently at University of Southampton

robustness, as the failure or success of constellation missions is not dependent on the longevity and operational efficacy of a single satellite.

Traditionally, constellation design is driven by the minimisation of the number of satellites to meet the coverage requirements of the mission. Existing constellations, and those proposed for implementation in the near future involve multiple satellites with a fixed relative position, including Teledesic, Iridium, etc [3]. The propellant budget is defined to allow only small station-keeping maneuvers during the mission, as any other maneuvering can be expensive for these relatively large spacecraft. As such, much previous work on autonomous constellation control has centred on station-keeping and maintaining a constant relative position between satellites [3, 4, 5].

However, as miniaturised spacecraft technology advances, including high specific-impulse low-thrust propulsion systems, the notion of massively distributed constellations of small satellites with the ability to reconfigure to service real-time changes in coverage demand can be envisioned [6, 7, 8, 9]. This would enable enhanced coverage of specific regions of the Earth's surface. For example, a constellation of telecommunications satellites could respond to localised peaks in demand, which may be transient and unplanned, or may correspond to specific pre-planned events, for example the Olympics, World Cup, or other large scale events.

Traditional ground-based approaches for control and station-keeping for such large numbers of distributed spacecraft with high reconfigurability would prove prohibitively complex and expensive. For lower operational costs and increased system flexibility, on-board autonomy is preferred.

Hence, this paper presents a control algorithm based on the artificial potential field method for autonomous reconfiguration of a micro-satellite constellation in Earth-centered orbits for re-allocating satellite resources to match demand on the Earth's surface [10]. The potential function is defined such that its minimum value corresponds to the desired state - i.e. when the satellites have converged over their intended target and have returned to GEO altitude. The value of this potential is then linked to the tangential control acceleration, which is utilised to reconfigure the constellation and drive the potential to its minimum value using symmetrical continuous thrust profiles to actuate the rephasing maneuvers.

The remainder of the paper is organised as follows. Section 2 introduces the constellation model - a single circular ring of  $N$  spacecraft on GEO, and provides validation for the assumptions which underpin it. The artificial potential field controller is then developed. Section 3 provides a demonstration of the reconfigurability of the constellation in a simple test case with multiple targets on the Earth's surface. In section 4 a parameter analysis of the constellation efficacy in terms of its  $\Delta v$  requirement and convergence time is performed. Section 5 discusses the results and describes possible mission applications for which the reconfigurable behaviour could prove useful.

## 2. Constellation Model

### 2.1. Dynamics

The constellation is modelled as a ring of  $N$  satellite members, initially at geostationary altitude, and azimuthally equispaced, as described by Eq. 1,

$$\left. \begin{aligned} r_{i0} &= r_{\text{GEO}} \\ \theta_{i0} &= \frac{2\pi(i-1)}{N} \end{aligned} \right\} (i = 1, 2, \dots, N) \quad (1)$$

where  $r_{\text{GEO}}$  is the orbital radius of Geostationary Earth Orbit, 42,164.1 km, and  $\theta_i$  is the true anomaly of the  $i^{\text{th}}$  satellite.

The trajectory of each satellite is propagated according to the Gaussian form of the variation of Keplerian elements equations given in Eq. 2 [11].

$$\begin{aligned} \dot{a} &= \frac{2a^2 v}{\mu} a_t \\ \dot{e} &= \frac{1}{v} \left[ 2(e + \cos(\theta)) a_t - \frac{r}{a} \sin(\theta) a_n \right] \\ \dot{i} &= \frac{r \cos(u)}{h} a_h \\ \dot{\omega} &= \frac{1}{ev} \left[ 2 \sin(\theta) a_t + \left( 2e + \frac{r}{a} \cos(\theta) \right) a_n \right] - \frac{r \sin(u) \cos(i)}{h \sin(i)} a_h \\ \dot{\Omega} &= \frac{r \sin(u)}{h \sin(i)} a_h \\ \dot{M} &= n - \frac{b}{eav} \left[ 2 \left( 1 + \frac{e^2 r}{p} \right) \sin(\theta) a_t + \frac{r}{a} \cos(\theta) a_n \right] \end{aligned} \quad (2)$$

Where  $a$ ,  $e$ ,  $i$ ,  $\omega$ ,  $\Omega$ , and  $M$  are the standard Keplerian elements of semi-major axis, eccentricity, inclination, argument of perigee, right ascension of the ascending node and mean anomaly respectively.  $v$  is the orbital speed,  $r$  is the orbital radius,  $\theta$  is the true anomaly,  $u$  is the argument of latitude,  $p$  is the semi-parameter,  $b$  is the semi-minor axis,  $h$  is the specific angular momentum,  $\mu$  is the standard gravitational parameter of Earth and  $a_t$ ,  $a_n$  and  $a_h$  are the control accelerations in the directions tangential, normal and out-of-plane to the satellite velocity vector respectively.

It is assumed that the spacecraft only have the ability to thrust in the direction tangential to the orbital velocity vector, that the satellite orbits remain quasi-circular throughout the maneuvers, and that there will be no out-of-plane motion. Applying these assumptions, the equations of motion simplify to those given in Eq. 3, where the  $i$  notation is re-introduced to describe the multiple

satellite system, and is understood to run from 1 to  $N$ .

$$\begin{aligned}\dot{r}_i &= 2\sqrt{\frac{r_i^3}{\mu}}a_{ti} \\ \dot{\theta}_i &= \sqrt{\frac{\mu}{r_i^3}}\end{aligned}\tag{3}$$

A simple analytical solution to Eq. 3 exists and is given in Eq. 4.

$$\begin{aligned}r_i(t) &= \frac{\mu r_{i0}}{\sqrt{\mu} + \sqrt{r_{i0}}a_{ti}t} \\ \theta_i(t) &= \frac{a_{ti}\sqrt{r_{i0}}t\sqrt{\frac{(a_{ti}\sqrt{r_{i0}}t + \sqrt{\mu})^6}{\mu^2 r_{i0}^3}} - \sqrt{\mu}\sqrt{\frac{(a_{ti}\sqrt{r_{i0}}t + \sqrt{\mu})^6}{\mu^2 r_{i0}^3}} - \sqrt{\mu}\sqrt{\frac{\mu}{r_{i0}^3}}}{4a_{ti}\sqrt{r_{i0}}}\end{aligned}\tag{4}$$

In this work both numerical propagation of Eq. 3 and the analytical solution in Eq. 4 have been used. For the numerical propagation a Runge-Kutta method with relative tolerance of  $10^{-3}$  and absolute tolerance of  $10^{-6}$  has been employed.

## 2.2. Model Validation

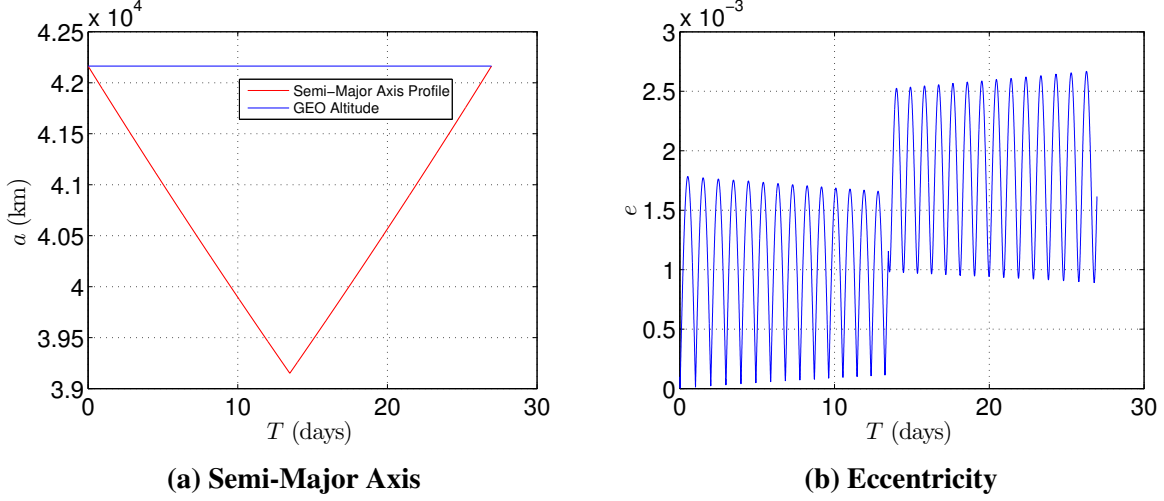
A typical maneuver from this work was also simulated using the full set of equations of motion given in Eq. 2 with the intention of proving that the eccentricity remains small, hence justifying the assumption that the satellite orbits will remain quasi-circular.

Figures 1a and 1b show the time-histories of the semi-major axis and eccentricity respectively during a typical low-thrust rephasing maneuver with a maximum acceleration of  $10^{-4}$  m/s<sup>2</sup> - the highest used throughout this work.

As can be seen in Fig. 1b, the eccentricity remains close to 0 as assumed.

## 2.3. Artificial Potential Field Controller

The constellation is controlled using a simple artificial potential field controller which is constructed in order to have a minimum corresponding to the constellation state in which the phase angle between each satellite and its target position,  $\theta_{tgt_i}$  is minimised, as well as the difference between each satellite's orbital radius, and the orbital radius of the nominal GEO orbit as given in Eq. 5,



$$V = \frac{1}{2} \sum_i^N (\alpha \phi_i^2 + \beta L_i^2) \quad (5)$$

where  $\phi_i = \theta_i - \theta_{\text{tgt}_i}$ ,  $L_i = r_i - r_{\text{GEO}}$ , and  $\alpha$  and  $\beta$  are control parameters.

According to Lyapunov stability theory, the convergence of the system to its state of minimum potential, i.e. where the constellation converges upon its target longitudes, can be assured if the rate of change of the potential function is negative definite [12]. The derivative of Eq. 5 is given as:

$$\dot{V} = \sum_{i=1}^N (\alpha \phi_i \dot{\phi}_i + \beta L_i \dot{L}_i) \quad (6)$$

It is clear from Eq. 6 that Lyapunov stability will be achieved if the control acceleration  $a_{ti}$  can be assigned such that Eq. 7 can be satisfied.

$$\begin{aligned} \text{sgn}(\dot{\phi}_i) &= -\text{sgn}(\phi_i) \\ \text{sgn}(\dot{L}_i) &= -\text{sgn}(L_i) \end{aligned} \quad (7)$$

However, upon consideration of these conditions, it can be noted that they have conflicting outcomes. Referring to the first condition, it suggests that in order to reduce the phase angle, a drift rate must be induced. This can only be achieved by changing the orbital radius with respect to the nominal GEO radius, with a higher orbital radius to induce a negative drift rate to correct a leading phase angle, and a lower orbital radius to induce a positive drift rate to correct a lagging phase angle. However, any change in orbital radius will increase the potential related to the second condition.

Hence, in order to avoid a conflict between the two terms of the potential function, it is necessary to operate them independently, by setting  $\beta = 0$  when the rephasing terms of the potential are active, and setting  $\alpha = 0$  when the altitude-related terms of the potential are active.

The following control law, in terms of the tangential acceleration, is proposed in order to best facilitate convergence of the system:

$$a_{ti} = \begin{cases} -\lambda \operatorname{sgn} \left( \phi_i - \frac{1}{2} \phi_{i0} \right) & |\phi_i| \geq \phi_{db} \\ -\lambda \operatorname{sgn} (L_i) & |\phi_i| < \phi_{db} \text{ and } |L_i| \geq L_{db} \\ 0 & |\phi_i| < \phi_{db} \text{ and } |L_i| < L_{db} \end{cases} \quad (8)$$

where  $\phi_{db}$  and  $L_{db}$  represent the dead-bands for the errors in phase angle and orbital radius respectively, and  $\lambda$  is a control constant which represents the maximum acceleration available from the spacecraft thruster.

This control law causes the spacecraft to thrust in one direction until the phasing angle has reached half of its original value, at which point the spacecraft begins to thrust in the opposite direction. Once the spacecraft phasing angle has reduced to within the deadband region, the control law switches to reduce errors in orbital radius, until they too have come within the deadband region.

The target longitude assigned to each satellite is slightly offset with respect to the longitude of the actual target in order to provide a minimum azimuthal separation distance between the satellites of 0.5 km.

The satellites are assigned to targets based on their initial proximity to the target longitudes. Future work will seek to implement this as part of the autonomous framework of the system using information dynamics through the use of consensus functions.

### 3. Reconfiguration Demonstration

In an initial test case of the control law, a constellation of 90 satellites are placed in GEO with the initial conditions described in Eq. 1. Three target longitudes corresponding to near-equatorial cities are then assigned on the Earth's surface, with each target requiring the attention of a varying number of satellites as described in Tab. 1.

**Table 1: Target Locations and Satellite Requirements**

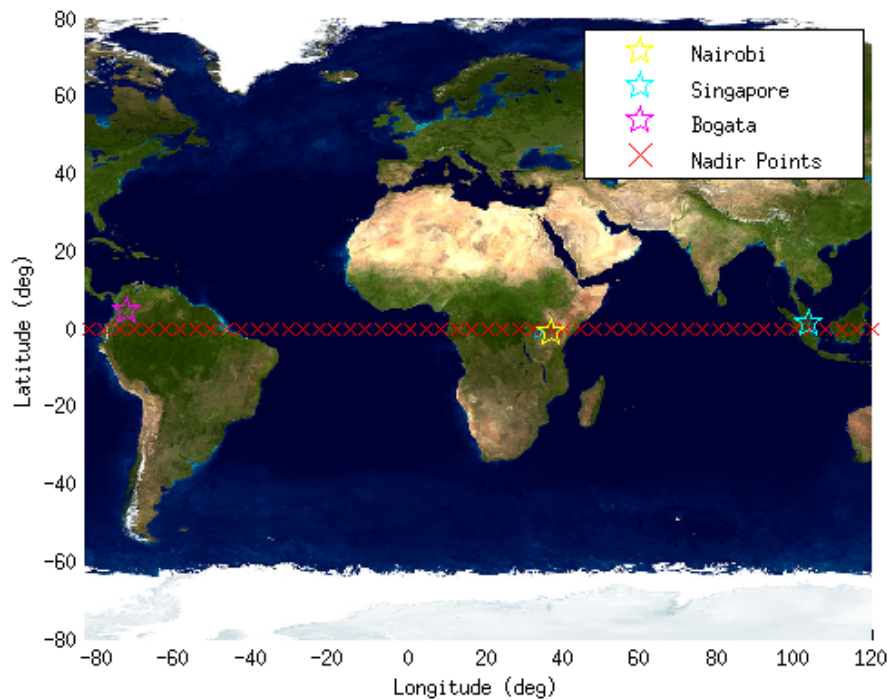
Target Longitude	Corresponding City	No. of Satellites Required
103.75° E	Singapore	10
74.08° W	Bogotá	12
36.82° E	Nairobi	18

The satellite maximum acceleration,  $\lambda$ , is equal to  $10^{-5} \text{ m/s}^2$ .

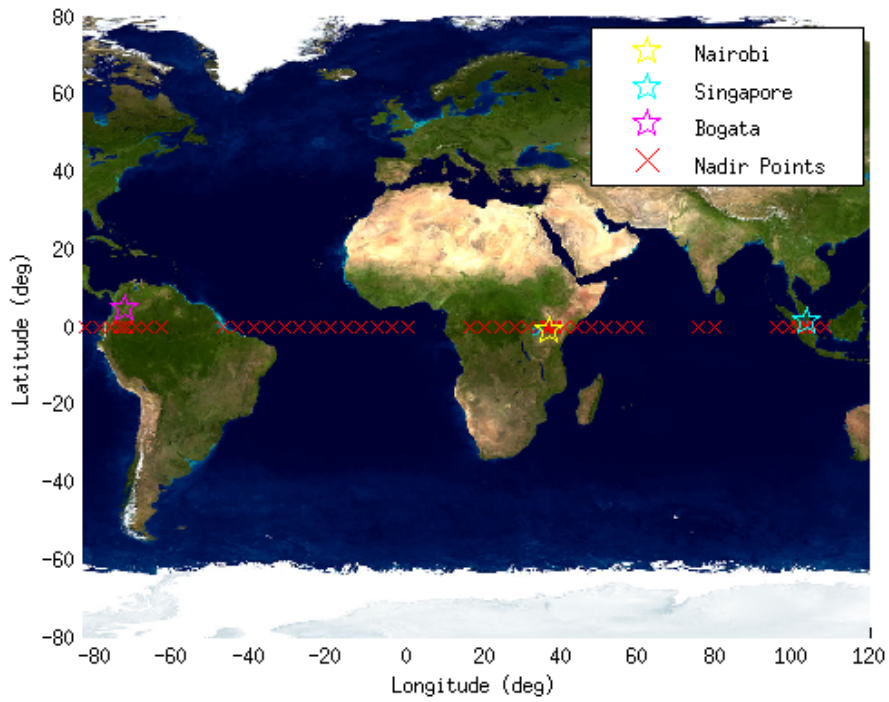
Figure 2 shows the equispaced position of the satellites via the longitude of their Nadir points on the Earth's surface at  $T = 0$  days. Figure 3 shows the position of the satellites at time  $T = 9$  days while the satellites are part way through their maneuver, and Fig. 4 shows their position once they have converged upon the targets. Figure 5 shows the time history of the longitude of the Nadir points of the satellites.

The time for all targets to have the required number of satellites above their position is 17.94 days, with a cumulative  $\Delta v$  for the maneuver being 0.45 km/s.

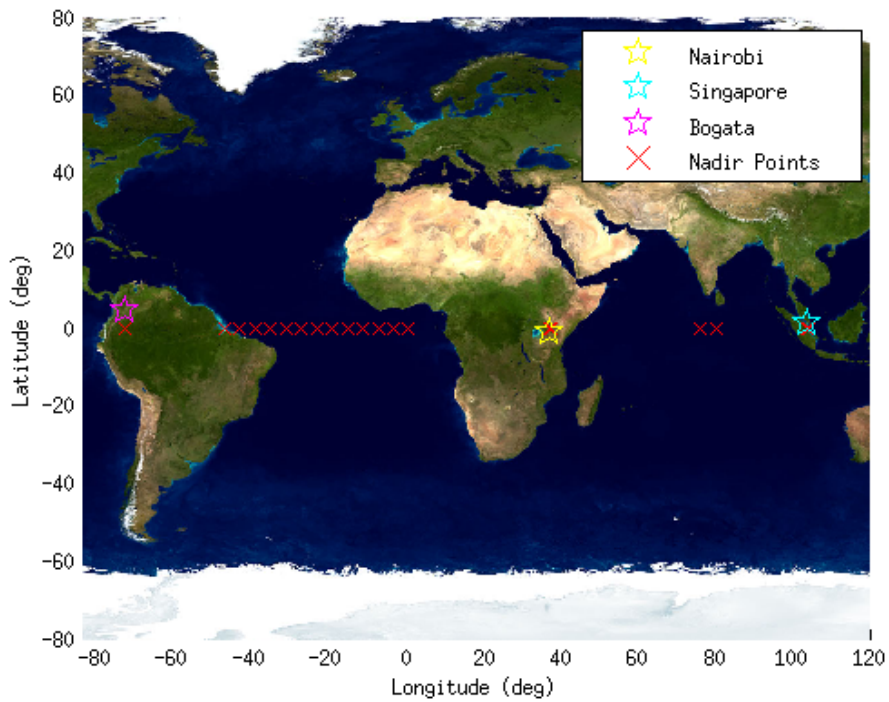
Figures 6a and 6b below show the how the  $\Delta v$  requirement and convergence time,  $T_{\text{conv}}$ , changes as the number of satellites in the constellation is increased, respectively.



**Figure 2: Satellite Nadir Point Longitudes at  $T = 0$  days**

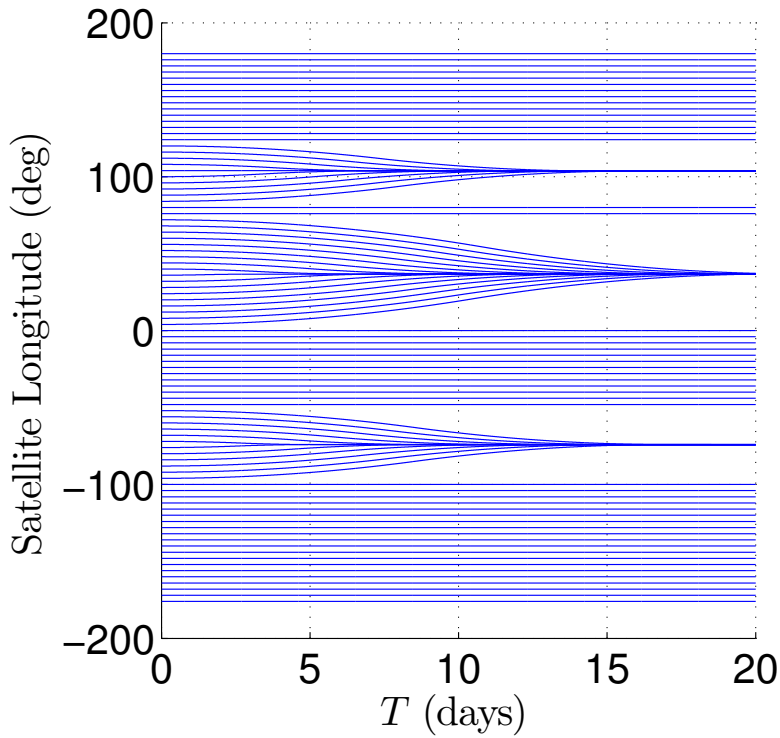


**Figure 3: Satellite Nadir Point Longitudes at  $T = 9$  days**

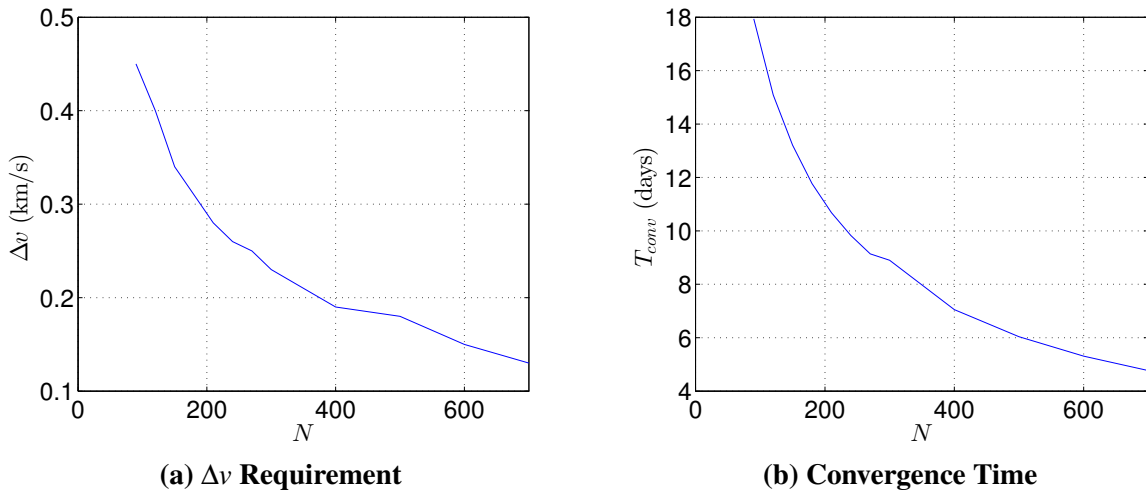


**Figure 4: Satellite Nadir Point Longitudes at  $T = 17.94$  days**





**Figure 5: Satellite Nadir Point Longitudes at  $T = 17.94$  days**



**Figure 6:  $\Delta v$  and  $T_{\text{conv}}$  as a Function of  $N$**

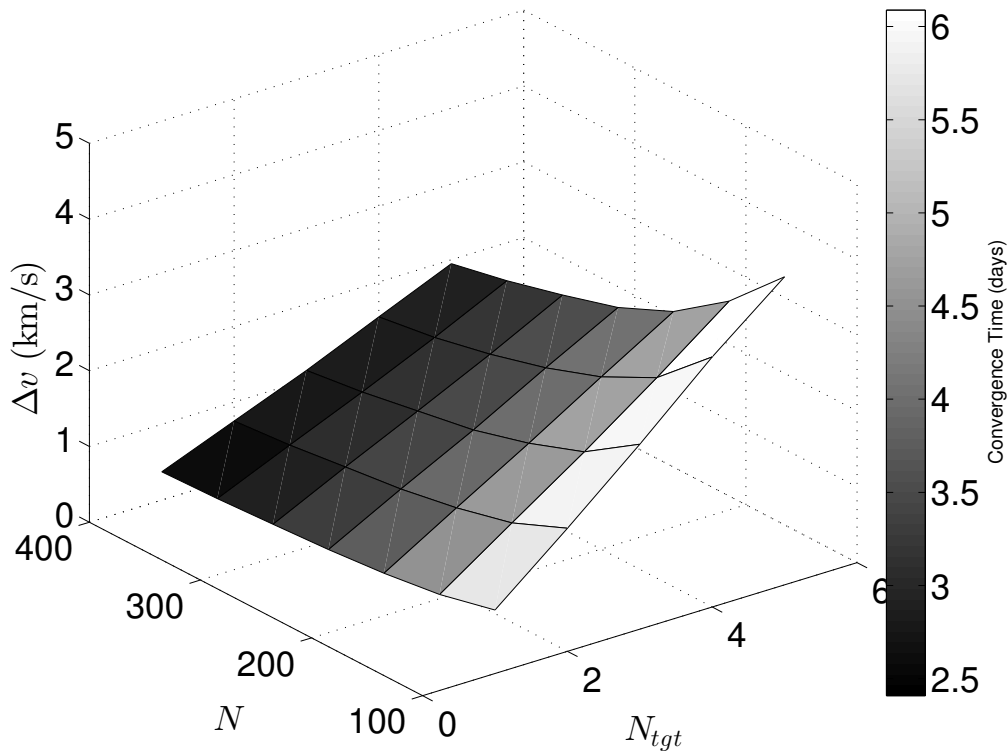
#### 4. Parameter Analysis of $\Delta v$ and Convergence Time

To understand the efficiency of the constellation reconfiguration method, a parameter analysis for maneuver  $\Delta v$  and  $T_{\text{conv}}$  was conducted for varying numbers of spacecraft,  $N$ , varying numbers of targets,  $N_{\text{tgt}}$ , and varying maximum tangential acceleration available to the satellites.

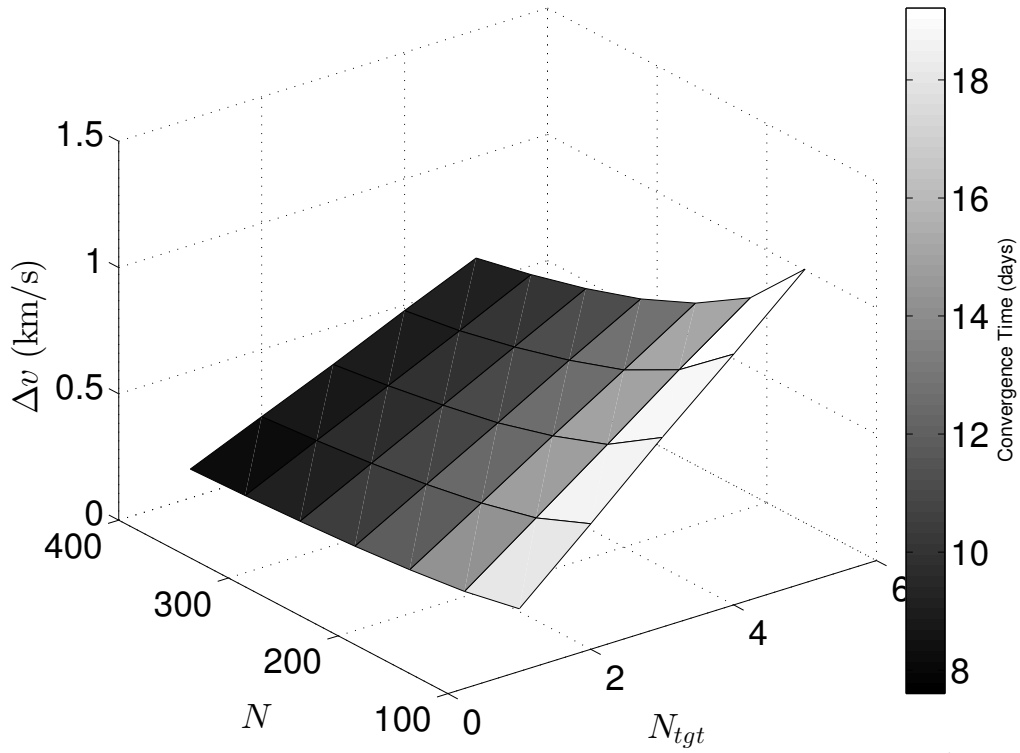
Simulations were run for  $N = 100, 150, 200, 250, 300, 350, 400$ ,  $N_{\text{tgt}} = 1, 2, 3, 4, 5$ , with  $\lambda = 10^{-6}$ ,

$10^{-5}$ ,  $10^{-4}$  m/s<sup>2</sup>. For the purpose of this study, it was assumed that each target required 20 satellites to converge above its position, however the method works for any number of targets, each with an arbitrary satellite requirement.

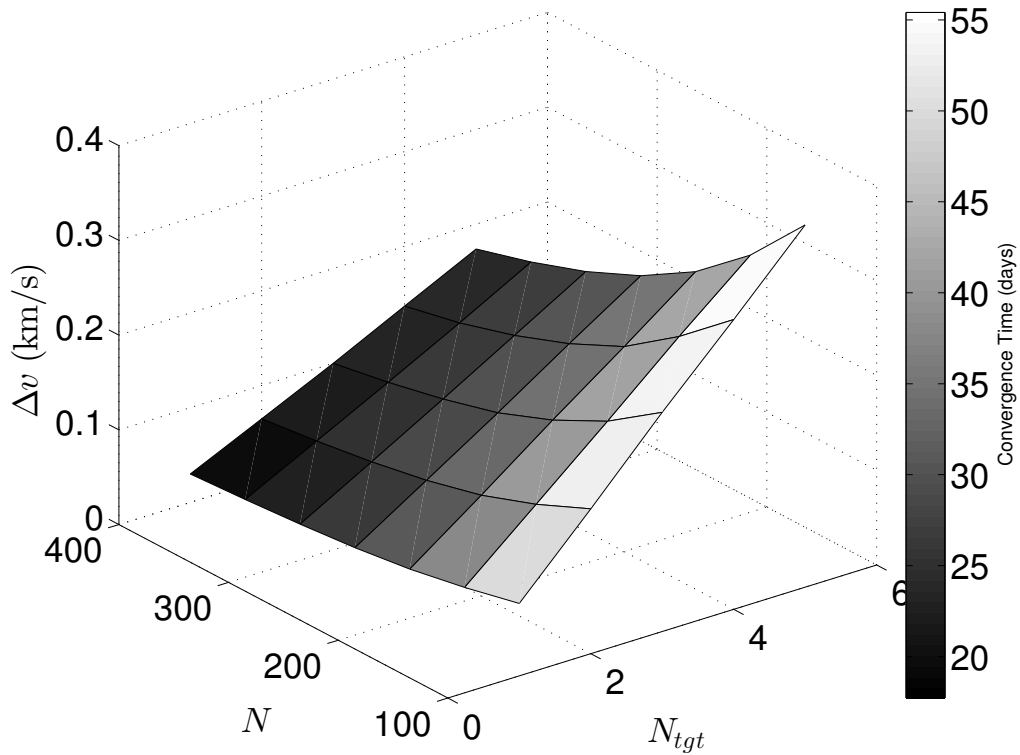
The results are shown in Fig. 7 - 9.



**Figure 7:**  $\Delta v$  and  $T_{conv}$  as function of  $N$  and  $N_{tgt}$  for  $\lambda = 10^{-4}$  m/s<sup>2</sup>.



**Figure 8:**  $\Delta v$  and  $T_{conv}$  as function of  $N$  and  $N_{tgt}$  for  $\lambda = 10^{-5} \text{ m/s}^2$ .



**Figure 9:**  $\Delta v$  and  $T_{conv}$  as function of  $N$  and  $N_{tgt}$  for  $\lambda = 10^{-6} \text{ m/s}^2$ .

It is clear from Fig. 7 - 9 that there is an intrinsic advantage of having a high number of satellites in the constellation, particularly in situations where a large number of targets are required to be visited.

As is expected, the  $\Delta v$  required for a maneuver increases linearly with the number of targets (the linear relationship is specific to the case where each target requires the same number of satellites), but as  $N$  increases, the  $\Delta v$  requirement reduces significantly, with the prominence of the reduction increasing as  $N_{\text{tgt}}$  is increased.

Additionally, the convergence time decreases rapidly as  $N$  is increased as shown by the color bars in Fig. 7 - 9.

With 5 targets, there is an average of a 40% saving in  $\Delta v$  and a 60% reduction in the convergence time between the case of  $N = 100$  and  $N = 400$  across the three different acceleration values. The  $\Delta v$  saving is more modest for lower number of targets, but is expected to increase as the number of targets increases, but the convergence time savings are always significant regardless of the number of targets.

## 5. Discussion

The parameter study of section 4 creates a basis for multiple mission applications for the reconfigurable constellation control method presented in this paper.

The parameter study displays the results for three different orders of magnitude of the satellite acceleration which gives rise to three different classes of  $\Delta v$  requirement and convergence time. These could correspond to difference classes of mission applications. For example, a telecommunications constellation which can reconfigure to meet pre-planned changes in local demand resulting from large scale sporting events like the olympics could employ a system of low  $\Delta v$  with high  $T_{\text{conv}}$ . This could also be useful for Earth observation or space science missions which wish to study effects or features associated with different seasons, etc.

Fast response systems, for example for Earth observation or telecommunications in response to disasters, would employ a high  $\Delta v$  low  $T_{\text{conv}}$  regime.

Naturally, the operational life-time of fast-response systems would be low compared to those with slower response times. However, as mass-fabrication of micro-satellite technology advances, the costs of replenishing such systems could make the notion of such constellations tenable.

In both cases, the study suggests that a high number of satellites is preferable as it reduces the convergence time and significantly reduces the  $\Delta v$  requirement when multiple targets are to be tracked.

## 6. Conclusion and Future Development

This paper has demonstrated simple method of controlling a constellation of small satellites in Geostationary Earth Orbit with the capability of reconfiguring to meet real-time changes in demand.

The feasibility of the control law, developed using the artificial potential field method, has been demonstrated through the use of simple test cases. The  $\Delta v$  and convergence time requirements for a variety of possible mission concepts has been analysed and multiple applications for which the method is particularly efficient have been highlighted, including disaster response and coverage for pre-planned large-scale events.

This work provides a basis for future work which will be performed, including the investigation of alternative control regimes including impulsive and continuous maneuvering with coast periods, multiple-ring constellations with reconfigurability and the exploitation of natural resonances in multiple-rings with varying number densities of satellites, extension of the controller to elliptical and 3D orbits where the natural clustering effect of the orbits can be exploited. Moreover, the effect of perturbations such as the  $J_{22}$  effect will be implemented and analysed to create resonances in GEO.

## 7. Acknowledgements

This work was funded by the European Research Council Advanced Investigator Grant - 227571: VISIONSPACE: Orbital Dynamics at Extremes of Spacecraft Length-Scale.

## 8. References

- [1] Vallado, D. A. Fundamentals of Astrodynamics and Applications. Microcosm Press, CA, USA, 2001.
- [2] Da Silva Curiel, A., Boland, L., Cooksley, J., Bekhti, M., Stephens, P., Sun, W., and Sweeting, M. "First results from the disaster monitoring constellation (DMC)." Vol. 56, pp. 261 – 271. 2005.
- [3] Johnston, A. G. Y. and McInnes, C. R. "Autonomous Control of a Ring of Satellites." "AAS/AIAA Space Flight Mechanics Meeting," AAS 97-104. AAS/AIAA, AAS, Huntsville, Alabama, February 1997.
- [4] McInnes, C. R. "Autonomous ring formation for a planar constellation of satellites." Journal of Guidance, Control, and Dynamics, Vol. 18, No. 5, pp. 1215 – 1217, 1995. ISSN 07315090.
- [5] Shah, N. H. Automated Station-Keeping For Satellite Constellations. Master's thesis, Massachusetts Institute of Technology, June 1997.
- [6] Warneke, B. A. and Pister, K. S. "MEMS for distributed wireless sensor networks." "Proceedings of the IEEE International Conference on Electronics, Circuits, and Systems," Vol. 1, pp. 291 – 294. Dubrovnik, Croatia, 2002.
- [7] Barnhart, D., Vladmirova, T., and Sweeting, M. "Satellite-on-a-Chip: A Feasibility Study." "Proc. 5th Round Table on Micro/Nano Technologies for Space," Nordwijk, The Netherlands, 2005.
- [8] Barnhart, D. J. Very Small Satellite Design for Space Sensor Networks. Ph.D. thesis, University of Surrey, June 2008.

- [9] Barnhart, D. J., Vladimirova, T., and Sweeting, M. N. “Very-Small-Satellite Design for Distributed Space Missions.” *Journal of Spacecraft and Rockets*, Vol. 44, pp. 1294–1306, Nov. 2007. doi:10.2514/1.28678.
- [10] Zhu, Q., Yan, Y., and Xing, Z. “Robot Path Planning Based on Artificial Potential Field Approach with Simulated Annealing.” *Intelligent Systems Design and Applications, 2006. ISDA '06. Sixth International Conference on,* Vol. 2, pp. 622 –627. oct. 2006. doi:10.1109/ISDA.2006.253908.
- [11] Battin, R. H. *An Introduction to the Mathematics and Methods of Astrodynamics*, Revised Edition. AIAA Education Series, Ohio, USA, 1999.
- [12] Wassim M. Haddad, V. C. *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. Princeton University Press, New Jersey, USA, 2008.