CHIRP-BASED MULTICARRIER MODULATION

Ahmed A. A. Solyman, Stephan Weiss and John J. Soraghan

Department of Electronic & Electrical Engineering, University of Strathclyde, Glasgow, Scotland, UK
{ahmed_amin,stephan,j.soraghan}@eee.strath.ac.uk

ABSTRACT

In this paper we demonstrate that in doubly-dispersive environments, a multicarrier (MC) system based on a fractional Fourier transform (FrFT) can achieve a better concentration of power near the main diagonal of the equivalent channel matrix compared to standard orthogonal frequency division multiplexing (OFDM). The resulting inter-symbol and inter-carrier interference in such a chirp- based MC system can therefore be suppressed with a reduced complexity equaliser. Simulations show that compared to an equalised OFDM system, the equalised FrFT-MC approach can either significantly reduce complexity or enhance performance in a time-varying environment.

***Index Terms*—** Multicarrier transmission; fractional Fourier Transformation; time varying channel; equalisation.

1. INTRODUCTION

The popularity of orthogonal frequency division multiplexing (OFDM) systems is based on the diagonalising property to the discrete Fourier transform (DFT) for circulant matrices. This can be fully exploited under stationary conditions, but in the presence of e.g. carrier frequency offset [1] or Doppler spread [2, 3, 4, 5, 6] both the circulant property of the effective channel matrix and therefore the optimality of OFDM through the introduction of inter-channel interference (ICI) are lost.

OFDM is often required for transmission over doubly-dispersive channels carrying Doppler spread, such as e.g. for digital video broadcast (DVB) to handheld devices (DVB-H). To mitigate ICI, MMSE equalisation schemes have been introduced [7, 8], whereby the inversion of matrices whose dimension matches the number of subcarriers [7] — up to 2^{13} for DVB-H — has led to low-complexity schemes [8], where only a band-limited matrix structure is inverted.

As an alternative to DFT-based OFDM, a multicarrier system using the fractional Fourier transform (FrFT) has been introduced in [9]. In stationary conditions DFT-OFDM since it does not entirely decouple subcarriers. However, in doubly-dispersive channels reduced ICI has been reported [9] and further improved by MMSE equalisation in [10].

In this paper we present a low complexity MMSE equaliser for FrFTOFDM systems that depends on the $\mathbf{LDL}^{\mathrm{H}}$ factorization [11, 12]. The proposed system exploits the enhanced energy concentration close to the main diagonal of the equivalent channel matrix compared to what can be obtained on DFT-based OFDM. The complexity of the FrFT-OFDM system is approximately the same as for classic DFT-OFDM and the equaliser complexity [9] is linear with the number of subcarriers which exhibits a lower complexity compared to the block MMSE. It will be demonstrated that the performance of the proposed low complexity system is approximately the same as traditional OFDM systems employing a complete MMSE equaliser.

2. FRFT MULTICARRIER SYSTEM MODEL

2.1. Fractional Fourier Transform

The FrFT is a generalised form of the Fourier transform, which analyses signals w.r.t. chirps rather than complex exponentials. A factor $a \in [-1, 1]$ determines the chirp rate and therefore selects a representation between the time ($a = 0$) and frequency domains ($a = 1$) [13]. Using the discretisation proposed in [13] leads to the transformation of a signal $x[n]$ defined on the interval $n \in [0, (N-1)]$,

$$X_a[k] = \sum_{0}^{N-1} x[n] K_a[n,k] \ , \quad k = 0 \ldots (N-1), \quad , \quad (1)$$

whereby $K_a[n,k]$ are mutually orthonormal chirp signals spanning the basis of the discrete FrFT. The FrFT coefficients

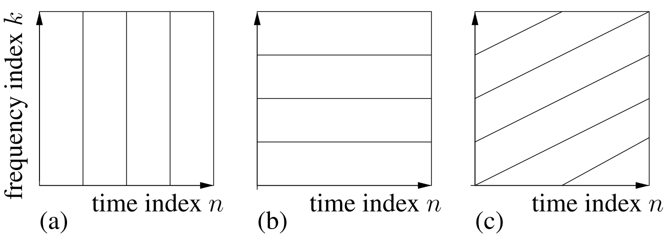

Fig. 1. Time=frequency tilingsof the TFrFT bases functions $K_a[n,k]$ for (a) $a = 0$ (time division duplex), (b) $a = 1$ (frequency division duplex), and (c) $0 < a < 1$ (FrFT).

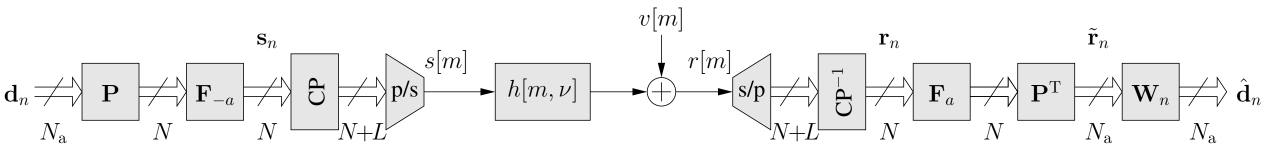

Fig. 2. FrFT-OFDM system using N subcarriers and a cyclic prefix of length L to transmit over a doubly-dispersive channel $h[m,\nu]$, whereby $\mathbf{P}$ maps symbols onto $N_{\mathrm{a}} \leq N$ active subcarriers and $\mathbf{F}_a$ is the FrFT matrix.

$X_a[k]$ can therefore be obtained by a matrix operation

$$\mathbf{y}_a = \begin{bmatrix} X_a[0] \\ X_a[1] \\ \vdots \\ X_a[N-1] \end{bmatrix} = \mathbf{F}_a \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} = \mathbf{F}_a \mathbf{x} \quad , \tag{2}$$

with $\mathbf{F}_0 = \mathbf{I}$ an identity and $\mathbf{F}_{\pm 1} = \mathbf{T}^{\pm 1}$ based on the normalised DFT matrix $\mathbf{T}$. Since an N-point FrFT matrix $\mathbf{F}_a$ is unitary, the inverse discrete FrFT (IFrFT) is defined as $\mathbf{x} = \mathbf{F}_a^{\mathrm{H}} \mathbf{y}_a = \mathbf{F}_{-a} \mathbf{y}_a$, where $(\cdot)^{\mathrm{H}}$ denotes Hermitian transpose. These discrete forms will be referred to as FrFT and IFrFT in the following. The time-frequency tilings of the basis functions $K_a[k,n]$ are sketched in Fig. 1, where the FrFT bases are shown for different values of a, creating time-division or frequency division duplex bases in the extreme cases of Fig. 1(a) and (b) and a linear chirp basis in the case of Fig. 1(c).

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2.2. FrFT-Based Multicarrier System

A conventional OFDM systems applies an inverse discrete Fourier transform (IDFT) matrix to a data vector $\mathbf{s}_n$ and introduces a cyclic prefix (CP) prior to multiplexing the OFDM symbol across a doubly dispersive channel $h[m,\nu]$ corrupted by additive white Gaussian noise $v[m]$. After demultiplexing the received signal $r[m] = \sum_{\nu=0}^{\infty} h[m,\nu] s[m-\nu] + v[m]$ and removal of the cyclic prefix, a DFT matrix reconstructs the transmitted data vector $\hat{\mathbf{s}}_n$.q In a DFrFT-based multicarrier system, the DFT matrix is replaced by a FrFT matrix $\mathbf{F}_a$ [9, 10] as shown in Fig. 2.

The signal $\mathbf{r}_n = \mathbf{H}_n \mathbf{s}_n + \mathbf{v}_n$ at the output of the demultiplexer in Fig. 2 is characterised by th equivalent channel matrix $\mathbf{H}_n$ with elements

$$[\mathbf{H}_n]_{i,j} = \begin{cases} h[n-L+i, i-j] & i \geq j, \\ h[n-L+i, L+i-j-1] & i < j. \end{cases} \tag{3}$$

In stationary conditions, $\mathbf{H}_n$ is circulant, and can be decoupled by $\mathbf{F}_a$ with $a = \pm 1$, whereby the case $a = 1$ represents the conventional OFDM system.

We additionally introduce a binary matrix $\mathbf{P} \in \mathbb{Z}^{N \times N_{\mathrm{a}}}$, which assigns a data vector $\mathbf{d}_n \in \mathbb{C}^{N_{\mathrm{a}}}$ to N subcarriers, of which only N_{a} are active according to

$$\mathbf{P} = \left[\mathbf{0}_{N_{\mathrm{a}} \times (N-N_{\mathrm{a}})/2} \;\; \mathbf{I}_{N_{\mathrm{a}}} \;\; \mathbf{0}_{N_{\mathrm{a}} \times (N-N_{\mathrm{a}})/2} \right]^{\mathrm{T}} \quad , \tag{4}$$

where $\mathbf{0}_{L \times M}$ is an $L \times M$ matrix with zero entries, and $\mathbf{I}_L$ an $L \times L$ identity matrix. The equaliser matrix $\mathbf{W}_n \in \mathbb{C}^{N_{\mathrm{a}} \times N_{\mathrm{a}}}$ in the receiver operates on the input

$$\begin{aligned} \tilde{\mathbf{r}}_n &= \mathbf{P}^{\mathrm{H}} \mathbf{F}_a \mathbf{H}_n \mathbf{F}_{-a} \mathbf{P} \mathbf{d}_n + \mathbf{P}^{\mathrm{H}} \mathbf{F}_a \mathbf{v}_n \\ &= \mathbf{C}_n \mathbf{d}_n + \tilde{\mathbf{v}}_n \quad , \end{aligned} \tag{5}$$

with a system matrix $\mathbf{C}_n \in \mathbb{C}^{N_{\mathrm{a}} \times N_{\mathrm{a}}}$. The purpose of the binary matrix $\mathbf{P}$ is not only to reduce out-of-band emissions, but also to eliminate components in the upper right and lower left corners of $\mathbf{C}_n$.

3. LOW COST EQUALISATION

3.1. MMSE Equaliser

Assuming perfect knowledge of the channel matrix $\mathbf{H}_n$, the approach in [8] can be extended to the system in Fig. 2. In the ideal case, a linear block MMSE equaliser is defined based on the system matrix $\mathbf{C}_n$. Below, we restrict the calculation of $\mathbf{W}_n$ to the first Q sub- and super-diagonals of $\mathbf{C}_n$ by means of a binary masking matrix $\mathbf{M}$ with elements

$$[\mathbf{M}]_{ij} = \begin{cases} 1 & 0 \leq |i-j| \leq Q, \\ 0 & Q < |i-j| < N_{\mathrm{a}}. \end{cases} \tag{6}$$

The shape of this matrix is imprinted on the masked matrix $\mathbf{B}_n = \mathbf{M} \odot \mathbf{C}_n$, where $\odot$ represents element-wise multiplication. Based on the masked matrix, analogously to [8] the MMSE equaliser can be defined as

$$\mathbf{W}_{n,\mathrm{MMSE}} = \mathbf{B}_n^{\mathrm{H}} (\mathbf{B}_n \mathbf{B}_n^{\mathrm{H}} + \gamma^{-1} \mathbf{I})^{-1} \quad , \tag{7}$$

where γ is the signal to noise ratio (SNR) at the input to the equaliser, assuming corruption by white Gaussian noise. The matrix inversion in (7) requires $\mathcal{O}(N^3)$ flops which is not practical for high values of N_a, such as found in DVB-T.

The masking is justified since the equivalent system matrix in the fractional domain is approximately banded [9]. This enables a low-cost approach to equalisation, which is introduced next.

3.2. Equalisation Using Band $\mathrm{LDL}^{\mathrm{H}}$ Factorization

The band structure of $\mathbf{B}_n$ with Q off-diagonal terms below and above the main diagonal leads to a band structure for $(\mathbf{B}_n\mathbf{B}_n^{\mathrm{H}})$, where only the first $2Q$ off-diagonal terms above and below the diagonal contain finite elements. This can simplify the calculation of the MMSE equaliser in (7). However, since (7) is time-dependent, we will below calculate $\hat{\mathbf{d}}_n = \mathbf{W}_{n,\mathrm{MMSE}}\tilde{\mathbf{r}}_n$ without explicitly determining $\mathbf{W}_{n,\mathrm{MMSE}}$.

The $\mathbf{LDL}^{\mathrm{H}}$ factorisation of the Hermitian band matrix $\mathbf{B}_n\mathbf{B}_n^{\mathrm{H}} + \gamma^{-1}\mathbf{I} = \mathbf{LDL}^{\mathrm{H}}$ is numerically straightforward [11], and leads to

$$\hat{\mathbf{d}}_n = \mathbf{B}_n^{\mathrm{H}}(\mathbf{LDL}^{\mathrm{H}})^{-1}\tilde{\mathbf{r}}_n = \mathbf{B}_n^{\mathrm{H}}\mathbf{x}_n \quad . \tag{8}$$

Instead of calculating the inverse in (8), the system

$$\mathbf{L}\underbrace{\mathbf{D}\overbrace{\mathbf{L}^{\mathrm{H}}\mathbf{x}_n}^{\mathbf{x}_{1,n}}}_{\mathbf{x}_{2,n}} = \tilde{\mathbf{r}}_n \tag{9}$$

is solved by forward substitution to obtain $\mathbf{x}_{2,n}$ via the lower left triangular matrix $\mathbf{L}$ and a rescaling by the diagonal matrix $\mathbf{D}^{-1}$ to calculate $\mathbf{x}_{1,n}$. Finally backsubstitution with the upper right triangular $\mathbf{L}^{\mathrm{H}}$ yields $\mathbf{x}_n$, which can be inserted into (8) in order to determine $\hat{\mathbf{d}}_n$.

The overall complexity for obtaining $\hat{\mathbf{d}}_n$ is $(8Q^2 + 22Q + 4)N_{\mathrm{a}}$ multiply-accumulates (MACs) [8], such that the selection of the parameter Q involves a trade-off between accuracy and cost.

4. SIMULATIONS AND RESULTS

4.1. Simulation Setup

To assess the performance of the proposed system, we assume an OFDM transmission with $N = 128$ subcarriers of which $N_{\mathrm{a}} = 96$ are active, a cyclic prefix of length $L = 8$, and QPSK modulation. Simulations are performed over an ensemble of 10^4 Rayleigh fading channels defined by an exponential power delay profile with an RMS delay spread of three sampling periods. This channel model uses the same statistics as in [14], including a maximum Doppler spread Ω_{D} equal to 15% of the carrier spacing.

4.2. Power Concentration

To assess the impact of the masking level Q, we compare the ratio between the power of components in the original system matrix $\mathbf{C}_n$ and in the reduced matrix $\mathbf{B}_n$ after masking by $\mathbf{M}$. Averaged over the ensemble and using the trace operator $\mathrm{tr}\{\cdot\}$, this power ratio is defined as

$$\rho[Q] = \mathcal{E}\left\{\frac{\mathrm{tr}\{\mathbf{B}_n\mathbf{B}_n^{\mathrm{H}}\}}{\mathrm{tr}\{\mathbf{C}_n\mathbf{C}_n^{\mathrm{H}}\}}\right\} \tag{10}$$

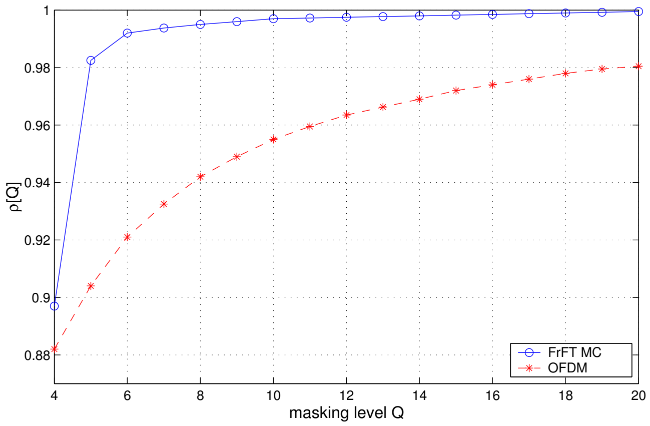

Fig. 3. Percentage of power of $\mathbf{C}_n$ contained in $\mathbf{B}_n$, measured by $\rho[Q]$ in dependence of the number of off-diagonal elements Q considered by $\mathbf{M}$, comparing FFT-OFDM ($a = 1$) and FrFT-OFDM ($a = 0.2$).

with $0 \leq \rho[Q] \leq 1$.

Results in Fig. 3 clearly indicate that FFT-OFDM experiences a spread of energy away from the main diagonal due to Doppler fading which is not limited to nearby off-diagonals, hence requiring a high value for Q to capture most of the power contained in $\mathbf{C}_n$. FrFT-OFDM does not manage to diagonalise $\mathbf{C}_n$, but in contrast to FFT-OFDM, the leaked power is contained in neighbouring off-diagonal element, and a much lower value of Q suffices to capture most of the components of $\mathbf{C}_n$ in $\mathbf{B}_n$.

4.3. BER Performance

Motivated by the above power concentration, we perform MMSE equalisation for simulations based on channels with a maximum Doppler frequency equivalent to 15% of the subcarrier spacing. The low complexity equaliser introduced in Sec. 3 is operated with $Q = \{5, 96\}$, whereby the second setting is equivalent to a standard block MMSE equaliser. The system uses chirp rates $a = \{0.2, 1\}$, which for the second setting is identical to standard FFT-OFDM.

Performance results in terms of bit error ratio are shown in Fig. 4. The FFT-OFDM curves match those reported in [8]. The proposed FrFT system with $Q = 5$ shows a degradation over the full scheme at high SNR, but only requires 3.4% of the computational cost in terms of MACs and still outperforms FFT-OFDM even with a full block MMSE equaliser with a computation cost of $\mathcal{O}(N_{\mathrm{a}}^3)$ MACs.

5. CONCLUSIONS

We have proposed a low cost equaliser for an FrFT-OFDM system, which generalises an FFT-OFDM system when assuming a chirp rate $a = 1$. Equalisation is based on a limitation of the considered system matrix to the first Q off-

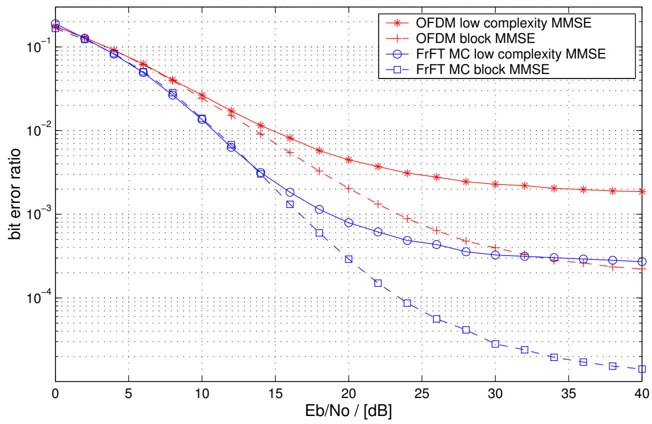

Fig. 4. Bit error ratio for MMSE equalisation using block ($Q = N_a = 96$) and low-cost ($Q = 5$) approaches for FrFT-($a = 0.2$) and FFT-OFDM ($a = 1$).

diagonal terms and uses a numerically efficient $\mathbf{LDL}^{\mathrm{H}}$ factorisation. The fact that in a doubly-dispersive scenario the FrFT-based multicarrier system concentrates the power in its system matrix closer to the main diagonal than a standard FFT-OFDM scheme leads to a significantly improved performance w.r.t. standard OFDM, and a considerable cost reduction w.r.t. previous FrFT equalisation approaches. This performance mildly depends on a, whose optimisation based on channel parameters is the subject of on-going research.

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