

Asynchronous QKD on a Relay Network

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Abstract

We show how QKD on a multi-user, multi-path, network can be used to establish a key between any two end users in an asynchronous fashion using the technique of bit-transport. By a suitable adaptation of our previous secret-sharing scheme we show that an attacker has to compromise all of the intermediate relays on the network in order to obtain the key. Thus, two end users can establish a secret key provided they trust at least one of the network relays.

1 Introduction

The elegant and startlingly original theoretical idea of Quantum Key Distribution (QKD) [1] has developed into a mature technology [2] with commercial systems readily available. Nevertheless, for a variety of reasons, it still remains something of a curiosity amongst security professionals. There is a sense in which the technology, however beautiful, addresses a non-existent problem since the threat models used by security professionals rarely put key distribution at the top of the list, with good reason. Existing key distribution mechanisms are considered to be more than adequate to address the perceived risk. Furthermore, with suitable key-expansion algorithms, there is little in practice that QKD can achieve that a conventional classical system cannot.

The security of QKD is based on different principles, however, and conventional security techniques largely rely on unproven (but reasonable) assumptions [3]. So, for example, the security proof for a block cipher used in a suitable mode for key expansion, rests on the assumption that the cipher is a pseudorandom permutation. Whether we choose a QKD system or a conventional classical system for our key distribution, we still have to rely to some extent on our confidence in the underlying principles on which the security is based.

Another difficulty with QKD is the limitation imposed by the nature of the technology. The technique relies on the transmission of single quanta, or at least a reasonable approximation to them. Any network element which is too lossy,

or actively processes the signal in some way, will destroy the capability of the quantum channel to transmit keys. Thus, installing the technology on realistic networks poses something of a technical challenge. Whilst stable and tested solutions exist for point-to-point links, extending this to a network application is not straightforward and relies on the introduction of additional trusted network elements to enable the system to span reasonable distances and to route the signal between the required end points of the network. Good progress, however, has been made in developing the basic technique to work on more realistic communication networks [4,5].

The above comments notwithstanding it is likely that QKD will find application as part of an overall security solution for some situations and networks. Furthermore, the current threat model to key distribution will significantly alter as more progress is made towards the development of a working quantum computer that can process strings of qubits of sufficient size to pose a threat to existing public-key mechanisms [6,7]. Whilst classical key distribution techniques based on symmetric cryptography can address the threat posed by quantum computation, it is by no means certain that these will be an obvious natural choice over a QKD solution should the need arise for a widespread overhaul of the existing key distribution techniques based on public-key cryptography.

In this paper we look at how the bit-transport technique for QKD [8] can be used on a network in an *asynchronous* fashion to establish keys between any end-users of the network. The technique requires that the network relays act as intermediaries to correlate various QKD transmissions together. We show that with a suitable arrangement of relays an attacker has to compromise all of the relays on any particular channel in order to obtain the key. Thus, instead of having to trust *all* of the relays on a channel, the end users only have to trust *at least one*. We achieve this by a suitable adaptation of our ‘drop-out’ technique [9] for single QKD channels.

2 A Single-Relay QKD Channel

A relay on a QKD channel is used, primarily, to increase the distance. The conventional way of achieving this is for Alice and Bob to each establish separate quantum¹ keys with the intermediate relay. In an obvious notation Alice (A) establishes a key with the relay (R) which we label QK_{AR} . Bob establishes a different key QK_{RB} where we have used the order of the indices here to denote the ‘direction’ of key establishment which we take to be the direction in which the quanta are transmitted. The final key, K , between Alice and Bob can be established in a linkwise fashion.

If one relay is not enough to span the distance between A and B with a quantum key transmission then, clearly, we can use any number of intermediate relays which each establish a separate quantum key $QK_{R_j R_k}$. Once all of the

¹We use the term ‘quantum key’ here merely to describe the process by which the key has been established, that is, by quantum key distribution. There is, of course, nothing quantum about the key itself!

keys between the various network entities have been established, the final key between Alice and Bob can again be established in a linkwise fashion.

In our previous work [8] we showed how we can use intercept/resend relays [10] to establish an end-to-end key over an extended distance with no loss of effective final key size. The primary concern with any relay system is that the relays have to be trusted intermediaries. In conventional, or intercept/resend, operation the compromise of a single relay compromises the entire channel. In [9] we showed how it was possible to modify the transmission protocol by adding a single relay (at least) so that an attacker needs to compromise *all* of the relays on a channel in order to obtain the key. This technique employs the notion of quantum secret sharing developed for multi-path networks in [11] in which we create distinct logical paths on a single channel by randomly dropping out the relays from the channel.

2.1 Asynchronous Bit-Transport on the Channel

Let us consider a channel over which Alice and Bob desire to establish a quantum key. Further, let us suppose that a single relay is required to span the distance so that the channel is of the form Alice \longrightarrow Relay \longrightarrow Bob. There are two ways in which the relay can be operated; in link-by-link mode, or in intercept/resend mode to establish an end-to-end key. We have shown how bit transport can be used to establish an end-to-end key with intercept/resend relays [8], we'll now consider how bit transport can be used to establish a key between Alice and Bob in an asynchronous fashion with the help of the relay. The relay establishes an independent QKD channel with Alice and Bob, respectively. On each channel the bits are sifted with public announcement of the coding basis and the 'bad' channels discarded. At the end of this process Alice and the relay, and Bob and the relay, possess a set of data that should, in an ideal world and in the absence of an eavesdropper, be in perfect respective agreement. The 4 sets of data can be checked for errors. If the error rate is not too high then the data sets can be saved and labelled.

After many such transmissions Alice and the relay have n sets of data $S_{AR}^{(k)}$ and $\bar{S}_{AR}^{(k)}$ ($1 \leq k \leq n$), where the bar denotes the relay's data set which could differ slightly from the corresponding set of Alice if there are errors on the channel. The relay and Bob have similar sets of data, $S_{RB}^{(k)}$ and $\bar{S}_{RB}^{(k)}$ and we assume that they have m such sets. The elements of each set consist of a tuple (t, b) where t is the timeslot and b is the bit value. So for a given pair of sets, $S_{AR}^{(k)}$ and $\bar{S}_{AR}^{(k)}$, we would have elements (t, b) for Alice and elements (t', b') for the relay. We would have $t = t'$ but in the presence of errors we would have $b = b'$ for most, but not all, of the timeslots. Let us suppose that Alice and the relay now conduct a secure error-correcting process so that at the end of this, and with a suitable re-labelling of the timeslot values, we have that $t = t'$ and $b = b'$ for all elements. Let us label the sets after error-correction by Σ . We further suppose that Bob and the relay perform the same process on their sets so that they also have identical sets with elements (τ, β) .

Now if Alice and Bob wish to establish a key then the relay can choose one of the error-corrected sets $\Sigma_{RB}^{(k')}$ and one of the error-corrected sets $\Sigma_{AR}^{(k)}$ essentially at random (or select from those which initially had a lower error rate, for example). The relay can then choose the timeslots, at random, from the sets such that $b = \beta$ and simply announce the respective timeslots to both Alice and Bob. Alice and Bob will then share the same set of bits which can subsequently be used as a key. An eavesdropper has to have collected all of the data between Alice, Bob and the relay in order to have any chance of getting any information about the key, because she cannot know in advance which sets of data the relay will choose. Of course, once the timeslots have been announced then Alice and Bob need to perform a privacy amplification [2,7] on their data in order to eliminate the possible information the eavesdropper could have gleaned to a negligible level².

There are variations on this protocol. For example, the error-correction need not be done before the linking of the timeslots by the relay (although this will increase the effective error rate on the final data). The participants on the channel could, with a collection of error-corrected sets, decide to XOR these sets together to reduce the potential information of the eavesdropper. With the necessity to establish only a short key and with a potentially large number of sets to choose from this technique could reduce the eavesdropper's information to negligible levels in a similar fashion to the standard privacy amplification procedure. Alternatively, the privacy amplification could be done on the error-corrected sets Σ , individually, before the bit-transport by linkage of the timeslots. Furthermore, the linkage on this channel need not be initiated by the relay. Alice, for example, could begin the linkage by announcing the timeslots she wishes to use which then get correlated by the relay to suitable timeslots of Bob's. The linkage need not be restricted to the selection of a single respective set of the participants. Indeed, elements from different sets could be chosen at random provided that they have the same bit value. The main limitation of this asynchronous key-establishment, whichever protocol variation is adopted, is that the relay has to be entirely trusted by Alice and Bob.

3 QKD Channels with Multiple Relays

It is clear that this process can be extended to channels which require multiple relays. Let us consider a channel between Alice and Bob that requires 2 relays to span the distance. Thus we have a channel of the form :

$$A \longrightarrow R_1 \longrightarrow R_2 \longrightarrow B$$

Independent QKD transmissions are run on the channels $A \longrightarrow R_1, R_1 \longrightarrow R_2$ and $R_2 \longrightarrow B$ so that at the end of many such runs, and after sifting and

²Although privacy amplification can be performed after the bit-transport (as we have discussed here) this is not optimal because the linkage of the timeslots gives the attacker a greater number of bits of the key that she knows with certainty.

error-correction, Alice and R_1 share n sets $\Sigma_{AR_1}^{(i)}$, the two relays share m sets $\Sigma_{R_1R_2}^{(j)}$, and R_2 and Bob share l sets $\Sigma_{R_2B}^{(k)}$. For convenience we shall assume that all of these sets are of the same size with cardinality N . The timeslot index is just an integer identifying a transmission instance, thus each of these sets consists of elements of the form (t, b) with $1 \leq t \leq N$ and $b \in \{0, 1\}$. Each set is therefore an ordered list of N bit values. An example for $N = 10$ is given below, in which Alice and the first relay have selected³ the 4th set from their list of n sets, the relays have selected the 2nd set from their list of m sets, and the second relay and Bob have selected the 7th set from their list of l sets. In practice N will be orders of magnitude greater than 10.

N	$\Sigma_{AR_1}^{(4)}$	$\Sigma_{R_1R_2}^{(2)}$	$\Sigma_{R_2B}^{(7)}$
1	0	1	0
2	1	1	0
3	1	1	1
4	0	0	1
5	1	1	0
6	1	1	1
7	0	1	0
8	0	0	1
9	1	0	1
10	0	1	1

Let us suppose that Alice and Bob wish to establish a key of length 4 bits. We'll consider the case where the relays select the key to be used. R_1 and R_2 communicate and agree on 4 elements chosen at random from their set $\Sigma_{R_1R_2}^{(2)}$. For example, we suppose that they agree on the following list (6,1,9,3) giving the key 1101. Now R_1 chooses, at random, the indices from the set $\Sigma_{AR_1}^{(4)}$ that will give this key and communicates the list of chosen indices to Alice. So, for example, in order to communicate the key 1101 to Alice, R_1 might transmit the list (3,2,8,9). It is important that once an index value has been selected it is eliminated from any subsequent choice. The relay R_2 performs the same process with Bob and, for example, might transmit the index list (4,9,2,10). At the end of this process both Alice and Bob will share the key 1101.

As we have noted above, the selection of the bit values need not be restricted to a single set. These values could be chosen randomly from all available sets. In this case each bit value index must be accompanied by another integer which indexes the set from which it is taken. Thus a list of tuples must be transmitted. So, for example, the first relay could send Alice the list [(12, 4), (2, 1), ... (34, 10)] which would indicate that the first bit of the key is the 4th element of their set $\Sigma_{AR_1}^{(12)}$ and so on.

Of course, more sophisticated schemes for key establishment can be envisaged, rather than just the straight linkage of the timeslots. For example, Alice

³We assume that such a selection has been made for the purposes of explanation of the asynchronous bit-transport technique.

and the first relay could partition the data in their error-corrected sets into 10 bit blocks (say), where the elements of each block are selected at random by publicly agreeing on a random sequence. In effect, a common random permutation is applied to the set. The bit values of the key can then be established by announcing a block and determining the parity. This procedure combines an element of privacy amplification into the key establishment.

Again, the main limitation on this technique, from a security perspective, is that each network element knows the final key and so each network element (that is, the relays) needs to be trusted. If any one relay is compromised then the key between Alice and Bob can be determined by the attacker. We can adapt our previous secret sharing technique [9] to alleviate this problem so that an attacker has to compromise *all* of the relays on the channel. Let's look at an example of how this works.

3.1 Securing a Multiple-Relay Channel

Let us consider the following channel

$$A \longrightarrow R_1 \longrightarrow R_2 \longrightarrow R_3 \longrightarrow B$$

Now let us suppose that a successful QKD transmission can be performed between network elements that are, *at most*, 2 steps away. The following quantum keys between Alice and Bob can therefore be established by utilising the bit-transport technique outlined above;

Quantum Key	QKD Channel
$QK_{AR_1R_2R_3B}$	$AR_1R_2R_3B$
$QK_{AR_2R_3B}$	$A - R_2R_3B$
$QK_{AR_1R_2B}$	$AR_1R_2 - B$
$QK_{AR_1R_3B}$	$AR_1 - R_3B$
QK_{AR_2B}	$A - R_2 - B$

We can see that there is at least one channel in which a given relay does not participate. So Alice and Bob establish these separate quantum keys and simply XOR them together to obtain their final key. None of the relays now possess this final key, and in order to obtain the key an attacker must compromise all of the relays in the channel. If only one is not compromised, and therefore trusted, the attacker cannot obtain the final key. For this particular example Alice and Bob could establish either of the following final quantum keys⁴:

⁴Of course there are other final quantum keys that can be established. It is important, however, that any relay has not participated in the establishment of all of the keys used in the XOR. Thus, for example, if we tried to establish a key $QK_{AR_2R_3B} \oplus QK_{AR_1R_2B}$ then relay 2 knows the final key and the attacker would only have to compromise this relay in order to obtain the final key.

$$QK_{AB} = QK_{AR_1R_2R_3B} \oplus QK_{AR_2R_3B} \oplus QK_{AR_1R_2B} \oplus QK_{AR_1R_3B} \oplus QK_{AR_2B}$$

$$QK'_{AB} = QK_{AR_2R_3B} \oplus QK_{AR_1R_2B} \oplus QK_{AR_1R_3B} \oplus QK_{AR_2B}$$

The second key here would seem intuitively more preferable since relays 1 and 3 only participate in 2 out of the 4 channels. It is obvious how to extend this to any general network configuration. Indeed, on multi-path networks we could employ a combination of this single-channel secret sharing and the multi-path secret sharing developed in [11]. Furthermore, on multi-path networks we could choose any path, at random, for each key bit we wish to establish using the bit-transport technique. The attacker would then be in the position of having to compromise *all* of the relays on the multi-path network, or collect *all* the data exchanged on all possible paths.

4 Discussion and Conclusions

The notion of bit-transport that we developed for single channels that use intercept-resend relays (IR relays) allowed those channels to establish a key between the endpoints such that the same fraction of received bits could be used in the key-establishment process as for the standard QKD channel without relays. Elsewhere, we have shown how bit-transport can be used on a duplex QKD channel to establish a key between 2 users (without relays) in such a way that public bit comparison is not required to determine the error-rate on the channel [12]. In this paper we have shown how bit-transport can be used to provide another useful function for QKD networks or channels. This feature is the ability to establish a key in an *asynchronous* fashion. In other words, bit-transport allows us to establish a quantum key using data that does not have to have been exchanged in the *same* transmission.

This ability gives us a great deal of flexibility not only in the data we use for key-establishment, but also in the technique we use to establish the key. Some of these various methods have been briefly alluded to above. In essence we use the relays to establish correlations between disparate data sets and it is this new feature for QKD channels that gives us the possibility of considering different operating modes for QKD networks. Bit transport is a classical processing methodology that can be used after the quantum transmissions and measurements on a channel, but coupled with a QKD protocol, such as that of BB84 we have considered in this paper, it gives us a powerful new way of approaching QKD network design and functionality.

The technique we have presented is quite general and can be applied to a variety of specific network designs. In this sense it should be considered not as a specific network, but more of a design element that can be employed when building a practical network. It is also clear that both the random dropout method using relays operated in pass-through mode (PT mode) and the

bit transport technique can be employed on a network employing link-by-link relays (LL relays). Indeed, it is not difficult to design a suitable relay that can be operated in any of the 3 modes, LL, PT or IR. Once again, this gives us a great deal of flexibility in the operation and control of any QKD network that employs these elements.

One early goal of QKD was to establish a kind of cryptographer's 'Holy Grail' where quantum keys were used in a one-time pad (OTP). This desirable goal couples a provably secure key transmission (on the assumption that the laws of physics are correct and the correct technological and procedural operation of the devices used) with a provably secure encryption algorithm. For such schemes questions of transmission efficiency become critical. If final quantum keys can only be established at a rate of a few bits per second then the operation of the OTP is somewhat limited. With the possibility of using asynchronous quantum key establishment we are not reliant on the efficiency of a single transmission, but can establish quantum keys from data that has been collected previously. The transmission limit, for something like the OTP, is then determined by questions of storage, the processing required to establish the correlation between the disparate data sets, and the database search/look-up times. Indeed, we can establish a large amount of quantum key data in advance using the asynchronous technique should it be so desired. In general, however, we should not be too obsessed with the use of the OTP for anything but messages of the highest security. With the ability to establish many quantum keys for use in algorithms employing 128 or 256 bit keys we can consider more imaginative key management and refresh policies above the standard master/session key methodology.

In our previous work [8,9] we considered single channels and how to secure the IR relays used. In this paper we have discussed how, in principle, this can be extended to networks with multiple paths between two end users. By using asynchronous quantum key establishment on a multi-path network with random drop-outs using relays in PT mode we make the job of an eavesdropper significantly more difficult. Whilst conceivable, even a limited attack on such a multi-path network would require the eavesdropper to have monitored all possible paths and to have collected all the quantum key data that had been exchanged. The overhead associated with keeping track of the various correlations that have to be established on the different paths is clearly not something that can be entirely ignored and for many paths and relays this overhead will likely be prohibitive for generating long quantum keys that are required instantaneously, such as keys that might be used in a OTP application between two users that have not previously established a shared quantum key.

In conclusion, therefore, we have shown how, with a suitable adaptation of our previous bit-transport and secret-sharing techniques [8,9], an asynchronous quantum key can be established between any two users on a network in such a way that an attacker has to compromise all of the intermediate network elements to obtain the final key. Indeed, in order to obtain even a limited amount of information about the key the attacker must collect the data between all network elements, even on different paths. The possibility of asynchronous quantum key establishment is another application of the notion of bit transport that can be

employed when designing a specific QKD network. Of course, it should be noted that the standard operating assumptions of a normal single link QKD channel must be observed. So, for example, the public communications between the various elements must be authenticated and any side-channel information must be protected.

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References

1. Bennett, C.H.; Brassard, G. Quantum Cryptography: Public Key Distribution and Coin Tossing. *Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing*, Bangalore, India, p.175, 1984
2. Gisin, N.; Ribordy, G.; Tittel, W.; Zbinden, H. Quantum Cryptography, *Rev. Mod. Phys.*, **2002**, *74*, 145–195
3. See, for example, Katz, J.; Lindell, Y. *Introduction to Modern Cryptography*; Chapman & Hall: Florida, 2008
4. Zbinden, H.; Gisin, N.; Huttner, B.; Muller, A.; Tittel, W. Practical aspects of quantum key distribution. *J. Cryptology*, **1998**, *11*, 1-14
5. Sasaki, M.; *et.al.* Field test of quantum key distribution in the Tokyo QKD Network. *Optics Express*, **2011**, *19* (11), 10387-10409
6. Shor, P.W. Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer. *SIAM J. Comput.*, **1997**, *26* (5), 1484–1509
7. Barnett, S.M. *Quantum Information*; Oxford University Press: Oxford, 2009
8. Barnett, S.M.; Phoenix, S.J.D. Extending the Reach of QKD Using Relays. *Proceedings of the IEEE GCC Conference*, Dubai, UAE, pp.140-142, 2011
9. Barnett, S.M.; Phoenix, S.J.D. Securing a Quantum Key Distribution Relay Network Using Secret Sharing, *Proceedings of the IEEE GCC Conference*, Dubai, UAE, pp.143-145, 2011
10. Bechmann-Pasquinucci, H.; Pasquinucci, A. Quantum Key Distribution with Trusted Quantum Relay. **2005**, <http://arxiv.org/abs/quant-ph/0505089> (accessed Mar 19, 2012)

11. Beals, T.R.; Sanders, B.C. Distributed relay protocol for probabilistic information theoretic security in a randomly-compromised network. *Third International Conference on Information Theoretic Security (ICITS)*, pp.29–39, 2008
12. Barnett, S.M.; Phoenix, S.J.D. Detecting an Eavesdropper in QKD Without Public Bit Comparison. 2012, <http://arxiv.org/abs/1203.0633> (accessed May 11, 2012)