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GRAVITATIONAL CAPTURE OPPORTUNITIES FOR ASTEROID RETRIEVAL MISSIONS

J.P. Sanchez

Advanced Space Concepts Laboratory, Mechanical and Aerospace Engineering,
University of Strathclyde, UK, jpau.sanchez@strath.ac.uk

D. García Yáñez,

Advanced Space Concepts Laboratory, Mechanical and Aerospace Engineering,
University of Strathclyde, UK, daniel.garcia-yarnoz@strath.ac.uk

E.M. Alessi,

Department of Mathematics, University of Pisa, Italy, emalessi@adams.dm.unipi.it

C.R. McInnes

Advanced Space Concepts Laboratory, Mechanical and Aerospace Engineering,
University of Strathclyde, UK, colin.mcinnnes@strath.ac.uk

Asteroids and comets are of strategic importance for science in an effort to uncover the formation, evolution and composition of the Solar System. Near-Earth objects (NEOs) are of particular interest because of its accessibility from Earth, but also because of their speculated wealth of resources. The exploitation of these resources has long been discussed as a means to lower the cost of future space endeavours. In this paper, we analyze the possibility of retrieving entire objects from accessible heliocentric orbits and moving them into the Earth's neighbourhood. The asteroid retrieval transfers are sought from the continuum of low energy transfers enabled by the dynamics of invariant manifolds; specifically, the retrieval transfers target planar, vertical Lyapunov and halo orbit families associated with the collinear equilibrium points of the Sun-Earth Circular Restricted Three Body problem. The judicious use of these dynamical features provides the best opportunity to find extremely low energy Earth transfers for asteroidal material. With the objective to minimise transfer costs, a global search of impulsive transfers connecting the unperturbed asteroid's orbit with the stable manifold phase of the transfer is performed. A catalogue of asteroid retrieval opportunities of currently known NEOs is presented here. Despite the highly incomplete census of very small asteroids, the catalogue can already be populated with 12 different objects retrievable with less than 500 m/s of Δv . All, but one, of these objects have an expected size and transfer requirements that can be met by current propulsion technologies. Moreover, the methodology proposed represents a robust search for future retrieval candidates that can be automatically applied to a growing survey of NEOs.

I. INTRODUCTION

Recently, significant interest has been devoted to the understanding of minor bodies of the Solar System, including near-Earth and main belt asteroids and comets. NASA, ESA and JAXA have conceived a series of missions to obtain data from such bodies, having in mind that their characterisation not only provides a deeper insight into the Solar System, but also represents a technological challenge for space exploration. Near Earth Objects (NEO), in particular, have also stepped into prominence because of two important aspects: they are among the easiest celestial bodies to reach from the Earth and they may represent a threat to our planet.

Proposed technologies and methods for deflection of Earth-impacting objects have experienced very significant advances along with increasing knowledge of the asteroid population. While initially devised to mitigate the hazard posed by global threat impacts, the current impact risk is largely posed by the population of small undiscovered objects [1], and thus methods have been discussed to provide subtle orbital changes to

small objects, as opposed to large-scale interventions such as the use of nuclear devices [2]. This latter batch of deflection methods, such as low thrust tugboat [3], gravity tractor [4] or small kinetic impactor [5] are moreover based on currently proven space technologies. They may therefore render the apparently ambitious scenario of manipulating asteroid trajectories a likely option for the near future.

On the other hand, the utilisation of resources in space has long been suggested as the means of lowering the cost of space missions, by means of, for example, providing bulk mass for radiation shielding or distilling rocket propellant for interplanetary transfers [6]. The maturity of technologies for in-situ resource utilisation (ISRU) could become a potentially disruptive innovation for space exploration and utilisation and, for example, enable space concepts that are today considered far-fetched, such as large space solar power satellites or sustaining large communities in space.

Although the concept of asteroid mining dates back to the very first rocketry pioneers [7], evidences of a

renewed interest in the topic can be found in the growing body of literature of recent years [8-10]. As well as in high profile private enterprise announcements such as by Planetary Resources Inc*.

With regards the accessibility of asteroid resources, recent work by Sanchez and McInnes [9, 11] demonstrates that a substantial quantity of resources can indeed be accessed at relatively low energy; on the order of 10^{14} kg of material could potentially be harvested at an energy cost lower than that required to access the resources of the Moon. More importantly, asteroid resources could be accessed across a wide spectrum of energies, and thus, as shown in [11], current technologies could be adapted to return to the Earth's neighbourhood objects from 10 to 30 meters diameter for scientific exploration and resource utilisation purposes.

Hence, advances in both asteroid deflection technologies and dynamical system theory, which allow new and cheaper means of space transportation, are now enabling radically new mission concepts, such as asteroid retrieval missions [12]. These envisage a spacecraft reaching a suitable object, attaching itself to the surface and returning it, or a portion of it, to the Earth's orbital neighbourhood. Moving an entire asteroid into an orbit in the vicinity of Earth entails an obvious engineering challenge, but may also allow a much more flexible mining phase in the Earth's neighbourhood. Not to mention other advantages such as scientific return or possible future space tourism opportunities.

The work presented here aims to provide a feasibility assessment of the latter mission concept by defining a set of preliminary mission opportunities that could be enabled by invariant manifold dynamics. Missions delivering a large quantity of material to the Lagrangian points are of particular interest. The material can be used as test bed for ISRU technology demonstration missions and material processing at affordable costs, ranging from fuel extraction to testing the use of material for radiation shielding. The science return is also greatly improved, with an asteroid permanently, or for a long duration, available for study and accessible to telescopes, probes and even crewed missions to the Lagrangian points. Finally, it sets the stage for other future endeavours, such as the construction of a permanent base around L_2 using the asteroid as the main structure or just as a source of material.

This work assumes the motion of the spacecraft and asteroid under the gravitational influence of Sun and Earth, within the framework of the Circular Restricted Three Body Problem (CR3BP) [13]. The well known equilibrium points of the system are shown in Figure 1.

* <http://www.planetaryresources.com/>

The mass parameter μ considered in the paper is $3.0032080443 \times 10^{-6}$, which neglects the mass of the Moon. Note that the usual normalised units are used when citing Jacobi constant values [13].

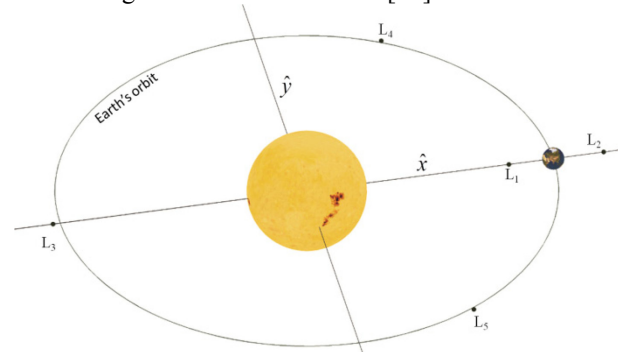


Figure 1: Schematic of the CR3BP and its equilibrium points.

II. LOW ENERGY TRANSPORT CONDUITS

Current interplanetary spacecraft have masses on the order of 10^3 kg, while an asteroidal object of 10 meters diameter will most likely have a mass of the order of 10^6 kg. Hence, already moving such a small object, or an even larger one, with the same ease that a scientific payload is transported today, would demand propulsion systems orders of magnitudes more powerful and efficient; or alternatively, orbital transfers orders of magnitude less demanding than those to reach other planets in the solar system.

Solar system transport phenomena, such as the rapid orbital transitions experienced by comets Oterma and Gehrels 3, from heliocentric orbits with periaapsis outside Jupiter's orbit to apoapsis within Jupiter's orbit, or the Kirkwood gaps in the main asteroid belt, are some manifestations of the sensitivities of multi-body dynamics. The same underlying principles that enable these phenomena allow also excellent opportunities to design surprisingly low energy transfers.

It has for some time been known that the hyperbolic invariant manifold structures associated with periodic orbits around the L_1 and L_2 collinear points of the Three Body Problem provide a general mechanism that controls the aforementioned solar system transport phenomena [14]. In this paper, we seek to benefit from these mathematical constructs in order to find *low-cost* trajectories to retrieve asteroid material to the Earth's vicinity.

II.1 Periodic Orbits and Manifold Structure

In particular, we are interested in the dynamics concerning the Sun-Earth L_1 and L_2 points (see Figure 1), as they are the *gate keepers* for potential ballistic capture of asteroids in the Earth's vicinity.

During the last half a century there has been an intense effort to catalogue all bounded motion near the libration points of the Circular Restricted Three Body

Problem [15]. The principal families of bounded motion that have been discovered are planar and vertical families of Lyapunov periodic orbits, quasi-periodic Lissajous orbits, and periodic and quasi-periodic halo orbits [13, 16]. Some other families of periodic orbits can be found by exploring bifurcations in the aforementioned main families [15].

Theoretically, an asteroid transported into one of these orbits would remain near the libration point for an indefinite time. In practice, however, these orbits are unstable, and an infinitesimal deviation from the periodic orbit will make the asteroid depart asymptotically from the vicinity of the libration point. Nevertheless, small correction manoeuvres can be assumed to be able to keep the asteroid within the periodic orbit [17, 18].

The linear behaviour of the motion near the libration points is of the type *centre x centre x saddle*, which is also a characteristic of all bounded motion near these points [19]. This particular dynamical behaviour ensures that, inherent to any bounded trajectory near the libration points, an infinite number of trajectories exist that asymptotically approach, or depart from, the bounded motion. Each set of trajectories asymptotically approaching, or departing, a periodic or quasi-periodic orbit near the L_1 or L_2 points forms a hyperbolic invariant manifold structure.

There are two classes of invariant manifolds: the central invariant and the hyperbolic invariant. The central invariant manifold is composed of periodic and quasi-periodic orbits near the libration points, while the hyperbolic invariant manifold consists of a stable and an unstable set of trajectories associated with an unstable orbit near an equilibrium point. The unstable manifold is formed by the infinite set of trajectories that exponentially leaves the periodic or quasi-periodic motion to which they are associated. The stable manifold, on the other hand, consists of an infinite number of trajectories exponentially approaching the periodic or quasi-periodic orbit.

It is well known (e.g., [13]) that the phase space near the equilibrium regions can be divided into four broad classes of motion; bound motion near the equilibrium position (i.e., periodic and quasi-periodic orbits), asymptotic trajectories that approach or depart from the latter, transit trajectories, and, non-transit trajectories (see Figure 2). A transit orbit is a trajectory such that its motion undergoes a rapid transition between orbiting regions. In the Sun-Earth case depicted in Figure 2, for example, the transit trajectory approaches Earth following a heliocentric trajectory, transits through the bottle neck delimited by the halo orbit and becomes temporarily captured at Earth. An important observation from dynamical system theory is that the hyperbolic invariant manifold structure defined by the set of

asymptotic trajectories forms a phase space separatrix between transit and non-transit orbits.

It follows from the four categories of motion near the libration points that periodic orbits near the Sun-Earth L_1 and L_2 points can not only be targeted as the final destination of asteroid retrieval missions, but also as natural gateways of low energy trajectories to Earth centred temporarily captured trajectories or transfers to other locations of the cislunar space, such as the Earth Moon Lagrangian points.

In this paper, we will focus on three distinct classes of periodic motion near the Sun-Earth L_1 and L_2 points; Planar and Vertical Lyapunov and Halo Orbits, from now on referred to as a whole as libration point orbits (LPO).

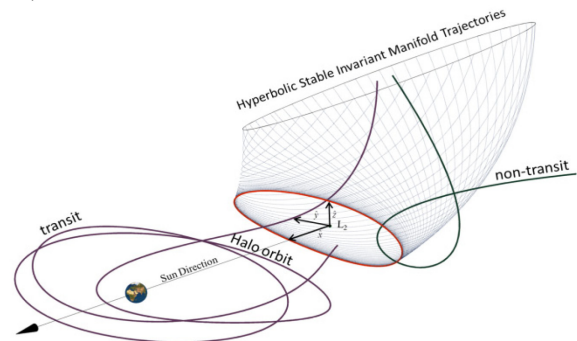


Figure 2: Schematic representation of the four categories of motion near the L_2 point (represented by the set of axes in the figure): periodic motion around L_2 (i.e., halo orbit), hyperbolic invariant manifold structure (i.e., set of stable hyperbolic invariant manifold trajectories), transit trajectory and non transit trajectory.

Lyapunov Orbits

As noted, the linear behaviour of the motion near the L_1 and L_2 points is of the type *centre x centre x saddle*. The *centre x centre* part generates a 4-dimensional central invariant manifold around each collinear equilibrium point when all energy levels are considered. In a given energy level the central invariant manifold is a 3-dimensional set of periodic and quasi-periodic solutions lying on an invariant tori, together with some stochastic regions in between [20].

There exist families of periodic orbits which in a infinitesimally small limit have frequencies related to both centers: ω_p and ω_v . They are known as *planar Lyapunov family* and *vertical Lyapunov family*, see Figure 3, and their existence is ensured by the Lyapunov center theorem. Halo orbits are 3-dimensional periodic orbits that emerge from the first bifurcation of the planar Lyapunov family.

To compute vertical Lyapunov periodic orbits in the neighbourhood of a given libration point, the idea is to search for a fixed point of the CR3BP on the surface of section $z = 0$. Therefore, we start from an initial approximation given by:

$$\begin{aligned} \mathbf{x}_0 &\equiv \mathbf{x}_{t=0} \\ \mathbf{x}_0 &= \mathbf{x}_{L_i} + \epsilon(w_1 \cos(\omega_v t) + w_2 \sin(\omega_v t)) \\ &= \mathbf{x}_{L_i} + \epsilon w_1 \end{aligned} \quad (1)$$

where \mathbf{x}_{L_i} corresponds to the position of the L_i libration point, ω_v is the vertical frequency associated to the L_i equilibrium point, w_1 and w_2 are the corresponding eigenvectors and ϵ is a small parameter, here equal to 10^{-4} .

From the initial condition in Eq. (1) and fixing the value of the Jacobi constant of the periodic orbit, we scan the Poincaré map at section $z = 0$ along a suitable direction. If we do not come back to \mathbf{x}_0 , then we change the initial condition by means of a Newton's iterative procedure. Indeed, we are looking for the zero of $\mathbf{x}^f - \mathbf{x}_0$, where \mathbf{x}^f denotes the vector of position and velocity once reached the desired section. A succession of vertical orbits with increasing energy can be computed by a continuation procedure with decreasing value of the Jacobi constant.

Planar Lyapunov periodic orbits can be computed with a similar procedure but using as reference frequency ω_p .

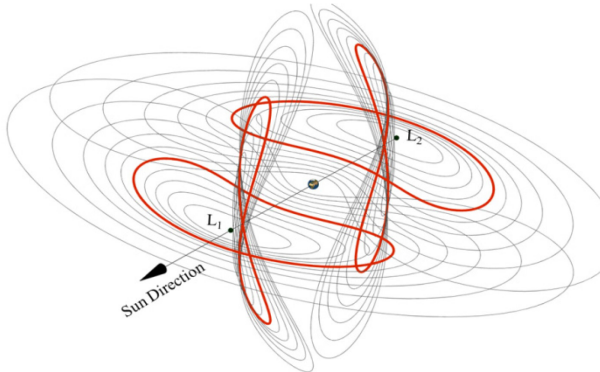


Figure 3: Series of Planar and Vertical Lyapunov orbits associated with the Sun-Earth L_1 and L_2 points. All orbits are plotted ranging from Jacobi constant 3.0007982727 to 3.0000030032. The thicker red line corresponds to a Jacobi constant of 3.0004448196, which corresponds to half distance between the energy at equilibrium in L_2 and L_3 .

Halo Orbits

The term halo orbit refers to the orbit's ring shape and its position relative to the secondary mass, which reminds of the ring of light commonly used in religious iconography to denote holiness. The term was coined by Robert Farquhar, who advocated the use of these orbits near the Earth-Moon L_2 point to obtain a continuous communication relay with the far side of the Moon during the Apollo programme [21].

As previously noted, this type of orbit emerges from a bifurcation in the planar Lyapunov orbits. As the

amplitude of planar Lyapunov orbit increases, eventually a critical amplitude is reached where the planar orbits become vertical critical, as defined by Hénon [22], and new three-dimensional families of periodic orbits bifurcate. Thus, the minimum possible size for Halo orbits in the Sun-Earth system is approximately $(240 \times 660) \cdot 10^3$ km at L_1 and $(250 \times 675) \cdot 10^3$ km at L_2 , sizes denoting the maximum excursion from the libration point in the x and y directions respectively. At the bifurcation point, two symmetric families of halo orbits emerge at each libration point, here referred to as the northern and southern family depending on whether the maximum z displacement is achieved in the northern (i.e., $z > 0$) or southern (i.e., $z < 0$) direction, respectively (see Figure 4).

The set of halo orbits, as shown in Figure 4, was computed by means of the continuation of a predictor-corrector process. The well-known third order approximation by Richardson [23] was used to start this process by providing an analytical approximation of the smallest halo possible (i.e., with z displacement ~ 0). Using the prediction by Richardson's approximation, the corrector phase propagates the analytical approximated solution from the conditions of departure at the x - z plane to the first intersection with the same plane. Since halo orbits are symmetric with respect to the x - z plane, the arrival velocity at the x - z plane must be perpendicular to the plane and so the v_x and v_z components must be zero. This knowledge can be used in a differential corrector procedure to trim Richardson's prediction and obtain the smallest halo possible [13, 24]. We then continue the process by feeding the next iteration with a prediction of a slightly larger displacement in z , but with x and v_y as in the previous step. Note that y , v_x and v_z must be zero since we are departing from the x - z plane on a symmetric orbit. The corrector process iterates the initial x and v_y until a perpendicular velocity to the x - z plane is obtained for the subsequent intersection with the symmetry plane. Repeating this process provides a series of halo orbits with increasing energy, or decreasing Jacobi constant.

The process can however only be continued until a Jacobi constant not far below 3.0004. At this point the direction of the continuation should be changed to the x direction, or a more sophisticated processes of continuation on which the direction is modified at each iteration should be used [25]. In this paper however we chose to stay on the range of halo orbits that can be continued using only the z direction to ensure that each halo orbit is defined by a single Jacobi constant. If halo families are continued beyond that point, they become degenerate in energy since a particular Jacobi constant defines more than one halo orbit.

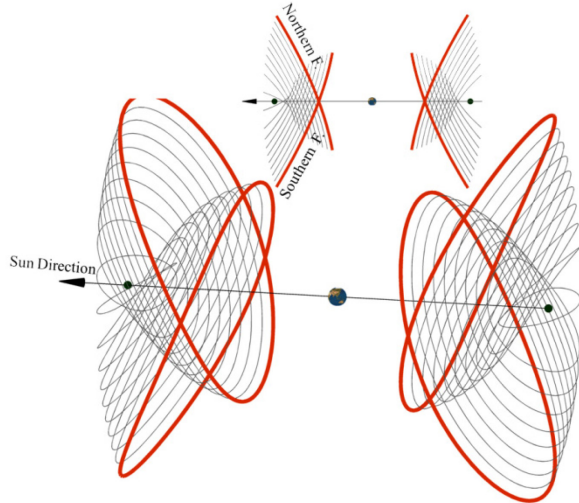


Figure 4: Halo orbits families. The figure represents the northern and southern families of halo orbits at the Sun-Earth L_1 and L_2 points. Halo orbits are plotted ranging from Jacobi constant of 3.0008189806 to 3.0004448196. The thicker red line corresponds to a Jacobi constant of 3.0004448196, as in Figure 3.

III. ASTEROID RETRIEVAL MISSIONS

In the past few years, several space missions have already attempted to return samples from the asteroid population (e.g., Hayabusa [26]) and others are planned for the near future[†]. As shown by Sanchez and McInnes [9, 11], given the low transport cost expected for the most accessible objects, we could also envisage the possibility to return to Earth entire small objects with current or near-term technology. The problem resides on the difficulties inherent in the detection of these small objects. Thus, for example, only 1 out of every million objects with diameter between 5 to 10 meters is currently known and this ratio is unlikely to change significantly in the coming years [27].

In this section then, we will focus our attention to the surveyed population of asteroids in search of the most accessible candidates for near-term asteroid retrieval missions through the invariant hyperbolic stable manifolds.

For this purpose, a systematic search of capture candidates among catalogued NEOs was carried out, selecting the L_1 and L_2 regions as the target destination for the captured material. This gives a grasp and better understanding of the possibilities of capturing entire NEOs or portions of them in a useful orbit, and demonstrates a method that can be applied to newly discovered small bodies in the future when detection technologies improve.

[†] <http://www.nasa.gov/topics/solarsystem/features/osiris-rex.html>
(last accessed 02/05/12)

III.I Invariant Manifold Trajectories to L_1 and L_2

The design of the transfer from the asteroid orbit to the L_1 and L_2 LPO consists of a ballistic arc, with two impulsive burns at the start and end, intersecting a hyperbolic stable invariant manifold asymptotically approaching the desired periodic orbits. This paper only considers the inbound leg of a full capture mission.

Planar Lyapunov, vertical Lyapunov, and Halo orbits around L_1 and L_2 generated with the methods described in previous sections were considered as target orbits. The invariant stable manifold trajectories leading to each of these LPO, computed by perturbing the periodic solutions on the stable eigenvector direction [13], were propagated backwards in the Circular Restricted 3-Body Problem until they reached a fixed section in the Sun-Earth rotating frame. The section was arbitrarily selected as the one forming an angle of $\pm\pi/8$ with the Sun-Earth line ($\pi/8$ for the L_2 orbits, see Figure 5, the symmetrical section at $-\pi/8$ for those targeting L_1). This corresponds roughly to a distance to Earth of the order of 0.4 AU, where the gravitational influence of the planet is considered small. No additional perturbations were considered in the backward propagation.

In this analysis, Earth is assumed to be in a circular orbit 1 AU away from the Sun. This simplification allows the conditions of the manifold trajectories (and in particular in the selected section) to be independent of the insertion time into the final orbit. The only exception is the sum of the right ascension of the ascending node and the argument of perihelion, which varies with the insertion time with respect to a reference time with the following relation:

$$(\Omega + \omega) = (\Omega_{REF} + \omega_{REF}) + \frac{2\pi}{T}(t - t_{REF}) \quad (2)$$

where Ω_{REF} and ω_{REF} are the right ascension of the ascending node and the argument of perihelion at the $\pm\pi/8$ section for an insertion into a target orbit at reference time t_{REF} , and T is the period of the Earth. For orbits with non-zero inclination, the argument of perihelion of the manifolds is also independent of the insertion time and the above equation indicates a variation in Ω . However, in the case of planar Lyapunov with zero inclination, Ω is not defined and an arbitrary value of zero can be selected, resulting in the equation representing a change in argument of perihelion.

The transfer between the NEO orbit and the manifold is then calculated as a heliocentric Lambert arc of a restricted two body problem with two impulsive burns, one to depart from the NEO, the final one for insertion into the manifold, with the insertion constrained to take place before or at the $\pm\pi/8$ section.

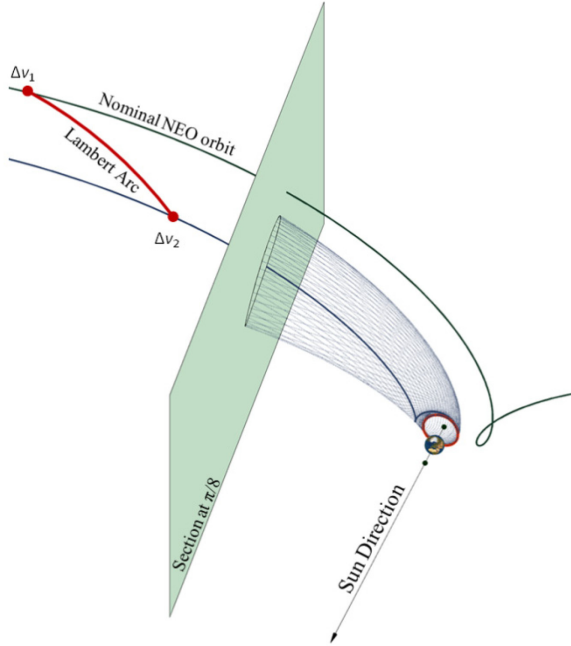


Figure 5: Schematic representation of a transfer to L_2

Thus, the problem can be defined with 5 variables: the Lambert arc transfer time, the manifold transfer time, the insertion date at the target orbit, the energy of the final orbit, and a fifth parameter determining the point in the target orbit where the insertion takes place.

The benefit of such an approach is that the asteroid is asymptotically captured into a bound orbit around a collinear Lagrangian point, with no need for a final insertion burn at arrival. All burns are performed far from Earth, so no large gravity losses need to be taken into account. Furthermore, this provides additional time for corrections, as the dynamics in the manifold are “slow” when compared to a traditional hyperbolic approach. Finally, this type of trajectory is then easily extendable to a low-thrust trajectory if the burns required are small.

The shape of the manifolds in the $r - \dot{r}$ phase space (with r being the radial distance from the Sun) at the intersection with the $\pm\pi/8$ section is shown in Figure 6 for a particular Jacobi constant. For an orbit with exactly the energy of L_1 or L_2 , the intersection is a single point; while for lower Jacobi constants, the shape of the intersection is a closed loop. The intersection corresponding to the bifurcation between planar and halo orbits is also plotted. A few capture candidate asteroids have been included in the plot (+ markers) at their intersection with the $\pi/8$ plane near their next closest approach to Earth. In a planar case, this would already provide a good measure of the distance of the asteroid to the manifolds. However, as we are

considering the 3D problem, information on the z component or the inclination would also be necessary.

Figure 7 and Figure 8 provide a more useful representation of the manifolds in terms of perihelion, aphelion radius and inclination for the two collinear points. The point of bifurcation between the planar Lyapunov and halo orbits, when they start growing in inclination, can easily be identified. Halo orbits extend a smaller range in aphelion and perihelion radius when compared to planar Lyapunov. Vertical Lyapunov orbits have even smaller excursions in radius from a central point, as can already be seen in the smaller loops of vertical Lyapunov in Figure 6, but on the other hand they extend to much lower values of the Jacobi constant and cover a wider range of inclinations.

Several asteroids are also plotted with small markers in the graphs. Their Jacobi constant J is approximated by the Tisserand parameter as defined in Eq. (3).

$$J \approx \frac{1}{a} + 2\sqrt{a(1 - e^2) \cos i} \quad (3)$$

This illustrates the proximity to the manifolds of a number of NEOs. In particular, asteroid 2006 RH120 has been highlighted, due to its proximity to the L_2 manifolds. From these graphs and ignoring any phasing issues, it can already be identified as a good capture candidate, as its perihelion and aphelion radius is close to or within the range of all the three types of manifolds, and its inclination lies also close to the halo orbit manifolds. The manifold orbital elements appear to be a good filter to prune the list of NEOs to be captured.

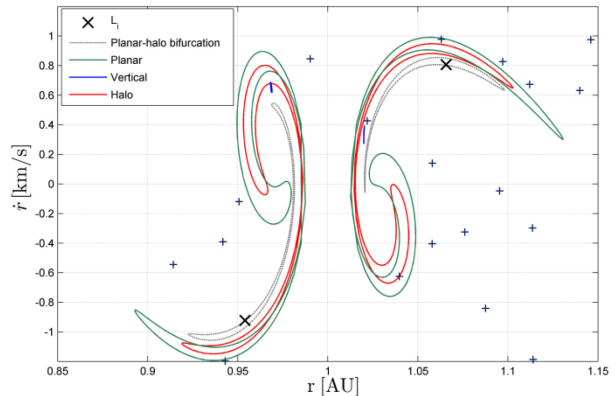


Figure 6: Shape of the manifolds in the $r - \dot{r}$ phase space for a Jacobi constant of 3.0004448196. The manifolds are represented at their intersection with a plane forming a $\pm\pi/8$ angle with the Sun-Earth line in the rotating frame. Manifolds on the left correspond to L_1 , on the right to L_2 . Candidate NEOs are indicated with a + marker.

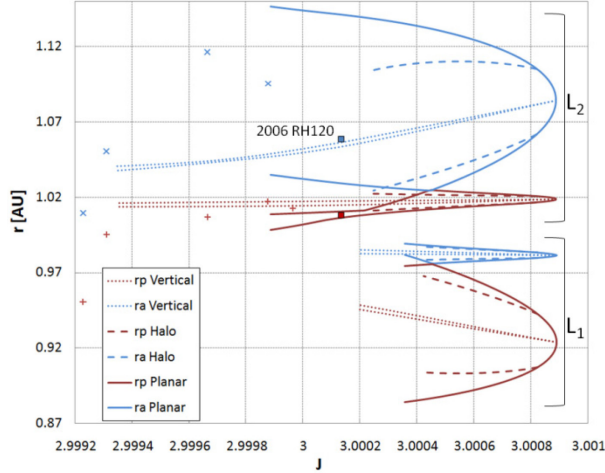


Figure 7: Minimum and maximum perihelion and aphelion radius of the manifolds leading to planar Lyapunov, vertical Lyapunov and halo orbits around L_1 and L_2 .

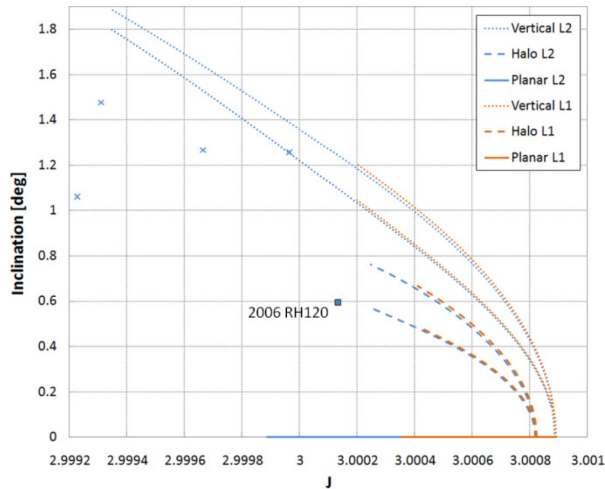


Figure 8: Minimum and maximum inclination of the manifolds leading to planar Lyapunov, vertical Lyapunov and halo orbits around L_1 and L_2 .

III.II Asteroid Catalogue Pruning

For the calculation of capture opportunities, the NEO sample used for the analysis is JPL's Small Bodies Database[‡], downloaded as of 27th of July of 2012. This database represents the catalogued NEOs up to that date, and as such it is a biased population, most importantly in size, as already noted. A large number of asteroids of the most ideal size for capture have not yet been detected, as current detection methods favour larger asteroids. Secondly, there is an additional detection bias related to the type of orbits, with preference for Amors and Apollos in detriment to Atens, as object in Aten orbits spend more time in the exclusion zone due to the Sun.

[‡] <http://ssd.jpl.nasa.gov/sbdb.cgi> (last accessed 27/07/12)

Even with this reduced list, it is a computationally expensive problem and preliminary pruning becomes necessary. Previous work by Sanchez et al. [28] showed that the number of known asteroids that could be captured from a hyperbolic approach with a total Δv less than 400 m/s should be of the order of 10. Although the hyperbolic capture approach in their work and the manifold capture is inherently different, the number of bodies that could be captured in manifold orbits at low cost is expected to be of the same order. Without loss of generality, it is possible to immediately discard NEOs with semi-major axis (and thus energy) far from the Earth's, as well as NEOs in highly inclined orbits. However, more systematic filters needed to be devised.

As a first approximation of the expected total cost in terms of Δv , a bi-impulsive cost prediction with both burns assumed at aphelion and perihelion was implemented. Either of the two burns is also responsible for correcting the inclination. The Δv required to modify the semi-major axis can be expressed as:

$$\Delta v_a = \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_f} \right)} - \sqrt{\mu_s \left(\frac{2}{r} - \frac{1}{a_0} \right)} \quad (4)$$

where μ_s is the Sun's gravitational constant, a_0 and a_f are the initial and final semimajor axis before and after the burn, and r is the distance to the Sun at which the burn is made (perihelion or aphelion distance). On the other hand the Δv required to modify the inclination is given by:

$$\Delta v_i = \sqrt{\frac{\mu_s}{a_0} r^*} \cdot 2 \sin(\Delta i / 2) \quad (5)$$

where Δi is the required inclination change, and r^* corresponds to the ratio of perihelion and aphelion distance if the burn is performed at aphelion, or its inverse if performed at perihelion.

The total cost is then calculated as:

$$\Delta v_t = \sqrt{\Delta v_{a1}^2 + \Delta v_{i1}^2} + \sqrt{\Delta v_{a2}^2 + \Delta v_{i2}^2} \quad (6)$$

with one burn performed at each of the apsis, and one of the two inclination change Δv assumed zero.

The estimated transfer Δv corresponds thus to the minimum of four cases: aphelion burn modifying perihelion and inclination followed by a perihelion burn modifying aphelion, perihelion burn modifying aphelion and inclination followed by an aphelion burn modifying perihelion, and the equivalent ones in which the inclination change is done in the second burn.

For the target manifold final perihelion, aphelion and inclination values, ranges or bands obtained from Figure 7 and Figure 8 were used. For example, for planar Lyapunov orbits at L_2 that corresponds to a range of $\{r_p, r_a, i\} \in \{1.00-1.02, 1.02-1.15, 0\}^c$, or $\{1.01-1.02, 1.025-1.11, 0.59-0.78\}$ for halo orbits at L_2 . Note that the

inclination range for halos was given as the maximum range that corresponds to the highest energy. This is due to the fact that most candidate asteroids have higher energies than the manifolds, and the lowest cost is assumed to take place where the energy difference is minimum.

This approximation provides in general a lower bound Δv estimate for several reasons. Manoeuvres are assumed to take place at aphelion or perihelion, where

the cost of changing semimajor axis is minimum. However, in reality, if the manoeuvre is combined with an inclination change, the burns need to take place at the ascending or descending node of the asteroid plane with the ecliptic, thus resulting in a higher cost for the change of a . Moreover, there is no guarantee, and in fact it is quite unlikely, that a combination of the extremes of the ranges of $\{r_p, r_a, i\}$ used in the filter correspond to proper manifold trajectories.

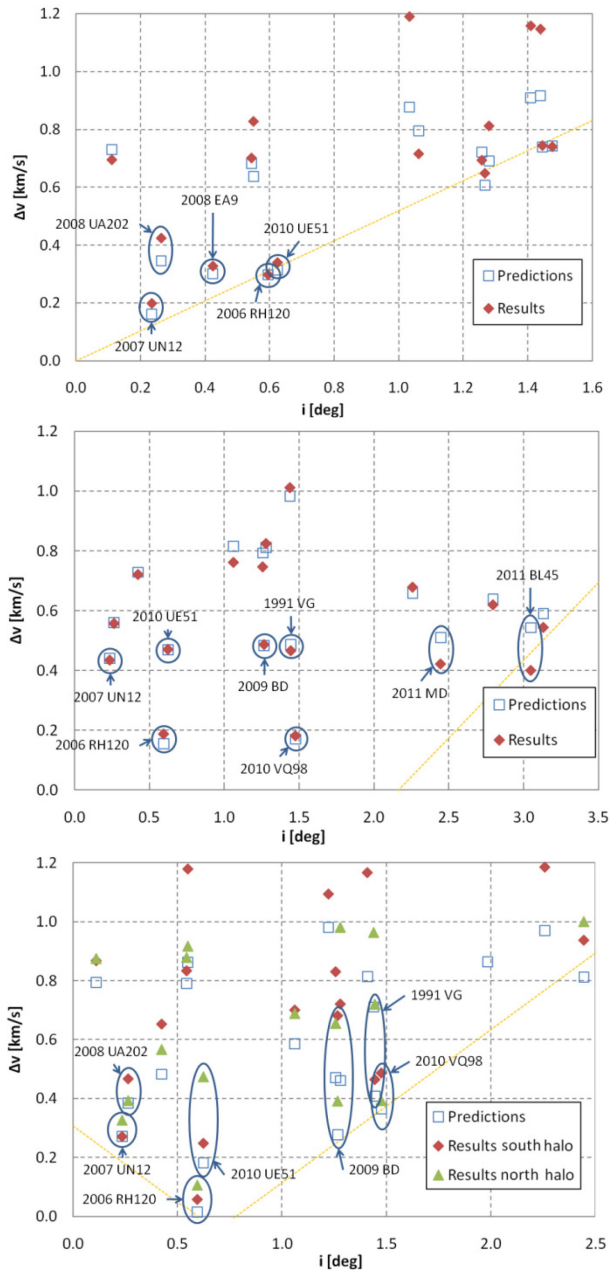


Figure 9: Filter cost estimates and results of the optimisation for planar Lyapunov (top), vertical Lyapunov (middle) and halo orbits (bottom) around L_2 . Yellow lines indicate the cost of changing just the inclination.

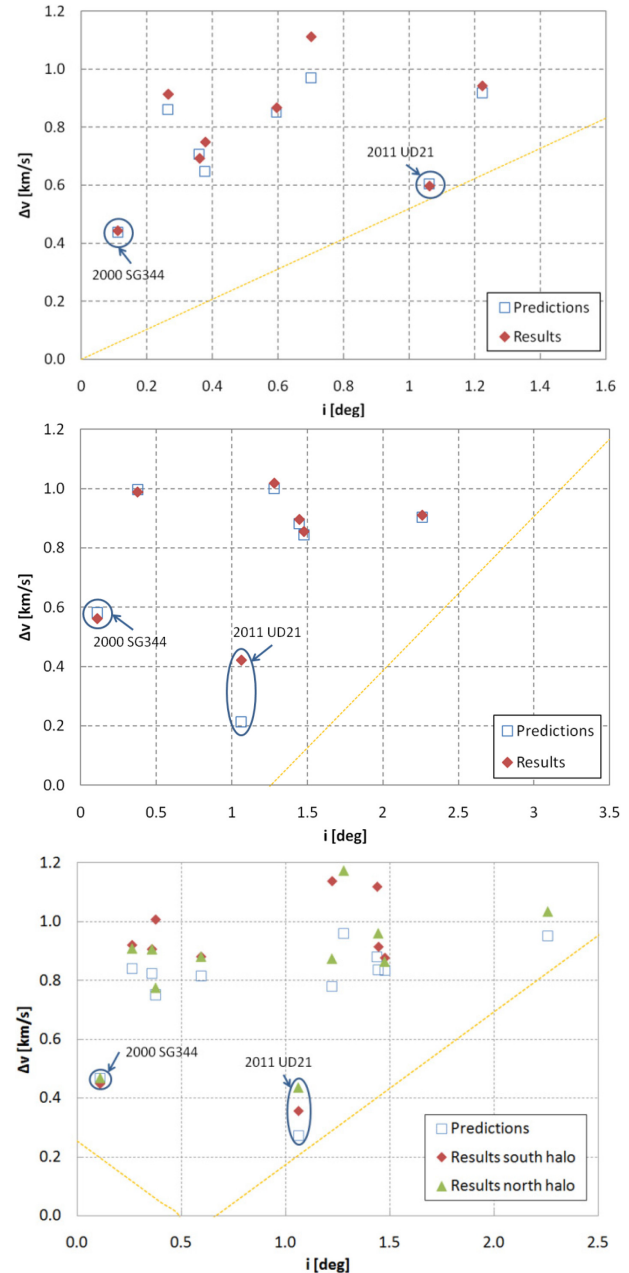


Figure 10: Filter cost estimates and results of the optimisation for planar Lyapunov (top), vertical Lyapunov (middle) and halo orbits (bottom) around L_1 . Yellow lines indicate the cost of changing just the inclination.

Finally, the plane change does not include a modification in right ascension of the ascending node. Although by modifying the phasing any final Ω can be selected, the combination of phasing with the Earth and plane change will also incur in additional costs. North and south halo orbits provide two opportunities with opposite Ω for each transfer, which should result in two different costs for phasing, while the filter provides a single value.

For a few cases, with high initial inclination and associated plane change cost, the filter provides Δv estimates greater than the optimised results. The inclination change is optimal the further from the Sun, namely at aphelion, providing an optimistic estimate in the cases where it is assumed to be performed there. On the other hand, when the burn is assumed at perihelion, the filter can also provide pessimistic results. Another reason for this over-estimate of the Δv comes from the possibility of splitting the large plane change into the two burns, which can potentially result in a lower cost.

Figure 9 and Figure 10 plot the results of the filter estimates plotted together with the results of the optimisation for L_2 and L_1 . It can be observed that the filter provides in general a good estimate of the total cost to be expected. It is a useful tool to select candidates and prioritise lists of asteroids for optimisation, and to quickly predict if any newly discovered asteroid is expected to have low capture costs. Dotted yellow lines have been added to the plot as indicators of the ideal cost of just the inclination change at a circular orbit at 1 AU. Predicted and optimised results are expected to fall above or close to these lines, which already limits the maximum inclination for each type of transfer for a given Δv . For example, in the case of L_2 , the maximum inclination that an asteroid should have to be captured with 500 m/s or less is for planar, vertical Lyapunov and halo approximately 1.0, 3.2 and 1.8 degrees respectively. The filter does however provide a quick and much more accurate estimate of the costs taking into consideration the shape of the original orbit as well as the inclination.

III.III Capture Transfers

For each of the filtered NEOs with estimated Δv below 1 km/s, feasible capture transfers with arrival date in the interval 2016-2100 were obtained. The Lambert transfers between the asteroid initial orbit and the manifolds were optimised using EPIC, a global optimisation method that uses a stochastic search blended with an automatic solution space decomposition technique [29]. Single objective optimisations with total transfer Δv as the cost function were carried out. Trajectories obtained with EPIC were locally optimised with MATLAB's built-in function `fmincon`. This process was repeated in a smaller domain around the optimum insertion date. Lambert arcs with up to 3

complete revolutions before insertion into the manifold were considered. For cases with at least one complete revolution, the two possible solutions of the Lambert problem were optimised. This implies that 7 full problem optimisations needed to be run for each NEO.

Table 1 shows the asteroids with costs lower than 500 m/s. Twelve asteroids of the whole NEO catalogue can be captured at this cost, ten of them around L_2 plus two Atens around L_1 . The table provides the orbital elements, minimum orbit intersection distance according to the JPL Small Bodies Database, and an estimate of the size of the object. This estimate is calculated with the following relation [30]:

$$D = 1329 \text{ km} \times 10^{-H/5} p^{-1/2} \quad (7)$$

where the absolute magnitude H is provided in the JPL database, and the albedo p is assumed to range from 0.05 (dark) to 0.50 (very bright icy object).

As expected, planar Lyapunov orbits are optimal for lower inclination NEOs, while NEOs with higher inclination favour transfers to vertical Lyapunov.

| | a | e | i | MOID | Diameter | Type | Δv |
|------------|-------|-------|-------|---------------------|-----------|------------|--------------|
| | [AU] | | [31] | [AU] | [m] | | [km/s] |
| | | | | | | 2Hs | 0.058 |
| 2006 RH120 | 1.033 | 0.024 | 0.595 | 0.0171 | 2.3- 7.4 | 2Hn | 0.107 |
| | | | | | | 2V | 0.187 |
| | | | | | | 2P | 0.298 |
| | | | | | | 2V | 0.181 |
| 2010 VQ98 | 1.023 | 0.027 | 1.476 | 0.0048 | 4.3-13.6 | 2Hn | 0.393 |
| | | | | | | 2Hs | 0.487 |
| | | | | | | 2P | 0.199 |
| 2007 UN12 | 1.054 | 0.060 | 0.235 | 0.0011 | 3.4-10.6 | 2Hs | 0.271 |
| | | | | | | 2Hn | 0.327 |
| | | | | | | 2V | 0.434 |
| | | | | | | 2Hs | 0.249 |
| 2010 UE51 | 1.055 | 0.060 | 0.624 | 0.0084 | 4.1-12.9 | 2P | 0.340 |
| | | | | | | 2V | 0.470 |
| | | | | | | 2Hn | 0.474 |
| 2008 EA9 | 1.059 | 0.080 | 0.424 | 0.0014 | 5.6-16.9 | 2P | 0.328 |
| | | | | | | 1Hs | 0.356 |
| 2011 UD21 | 0.980 | 0.030 | 1.062 | 0.0043 | 3.8-12.0 | 1V | 0.421 |
| | | | | | | 1Hn | 0.436 |
| 2009 BD | 1.062 | 0.052 | 1.267 | 0.0053 | 4.2-13.4 | 2Hn | 0.392 |
| | | | | | | 2V | 0.487 |
| | | | | | | 2Hn | 0.393 |
| 2008 UA202 | 1.033 | 0.069 | 0.264 | $2.5 \cdot 10^{-4}$ | 2.4- 7.7 | 2P | 0.425 |
| | | | | | | 2Hs | 0.467 |
| 2011 BL45 | 1.033 | 0.069 | 3.049 | 0.0040 | 6.9-22.0 | 2V | 0.400 |
| 2011 MD | 1.056 | 0.037 | 2.446 | 0.0018 | 4.6-14.4 | 2V | 0.422 |
| | | | | | | 1P | 0.443 |
| 2000 SG344 | 0.978 | 0.067 | 0.111 | $8.3 \cdot 10^{-4}$ | 20.7-65.5 | 1Hs | 0.449 |
| | | | | | | 1Hn | 0.468 |
| 1991 VG | 1.027 | 0.049 | 1.445 | 0.0037 | 3.9-12.5 | 2Hs | 0.465 |
| | | | | | | 2V | 0.466 |

Table 1: NEO characteristics for transfer trajectories with Δv below 500 m/s. The type of transfer is indicated by a 1 or 2 indicating L_1 or L_2 plus the letter P for planar Lyapunov, V for vertical Lyapunov, and Hn or Hs for north and south halo.

| | | Date [yyyy/mm/dd] | | | J manifold | Total Duration [yr] | Δv [m/s] | | Isp = 300s | | Isp = 3000s | |
|------------|-----|-----------------------|-----------------------|------------------|---------------|---------------------------|------------------|-----|--------------|--------------------|--------------|--------------------|
| | | Asteroid departure | Manifold insertion | L_1 arrival | | | Dep | Ins | Mass [kg] | \emptyset [m] | Mass [kg] | \emptyset [m] |
| 2006 RH120 | 2Hs | 2021/02/01 | 2024/03/30 | 2028/08/05 | 3.000421 | 7.51 | 58 | 0 | 398144 | 6.64 | 4067256 | 14.40 |
| 2006 RH120 | 2Hn | 2023/05/11 | 2024/02/20 | 2028/08/31 | 3.000548 | 5.31 | 52 | 55 | 213657 | 5.39 | 2222273 | 11.77 |
| 2010 VQ98 | 2V | 2035/02/14 | 2035/09/01 | 2039/11/15 | 3.000016 | 4.75 | 177 | 4 | 121879 | 4.47 | 1304330 | 9.86 |
| 2007 UN12 | 2P | 2013/10/22 | 2016/04/28 | 2021/02/19 | 3.000069 | 7.33 | 199 | 0 | 110313 | 4.33 | 1188630 | 9.56 |
| 2011 UD21 | 1Hs | 2037/11/20 | 2038/07/03 | 2042/07/19 | 3.000411 | 4.66 | 149 | 207 | 57441 | 3.48 | 659549 | 7.85 |
| 2011 UD21 | 1V | 2036/07/20 | 2038/11/16 | 2041/06/21 | 3.000667 | 4.92 | 226 | 196 | 47017 | 3.26 | 555160 | 7.42 |
| 2011 UD21 | 1Hn | 2039/10/24 | 2040/06/15 | 2043/08/30 | 3.000504 | 3.85 | 210 | 226 | 45236 | 3.21 | 537325 | 7.34 |
| 2000 SG344 | 1P | 2024/02/11 | 2025/03/11 | 2027/06/18 | 3.000357 | 3.35 | 195 | 248 | 44380 | 3.19 | 528741 | 7.30 |

Table 2: Capture trajectories and mass estimates for the best trajectory of each type.

Table 2 presents the best trajectory for each type of transfer for L_2 and L_1 (highlighted in bold in Table 1). The cheapest transfer, below 60 m/s, corresponds to a trajectory inserting asteroid 2006 RH120 into a halo orbit. Solutions to planar and vertical Lyapunov were also found for 2006 RH120 at higher costs. This agrees well with the interpretation of Figure 7. The pruning method was also predicting that this transfer would be the cheapest, with a minimum estimated Δv of 15 m/s. It is important to emphasise that the total Δv comprises both burns at departure from the asteroid and insertion into the manifold, but it does not include any navigation costs or corrections. The NEO orbit may intersect the manifold directly, and in that case the transfer to the target orbit can be done with a single burn, as in this particular case.

The total duration of the transfers range from 3 to 7.5 years. For the longer transfers it is possible to find faster solutions with less revolutions in the Lambert arc at a small Δv penalty.

Retrieved mass estimates

In order to obtain an estimate of the mass and size of the asteroid that could be retrieved, we can consider a basic system mass budget exercise. Assuming a spacecraft of 5500 kg dry mass and 8100 kg of propellant at the NEO (as proposed in the Keck study report for asteroid retrieval [12]), it is possible to estimate the total asteroid mass that can be transferred. A full system budget would require a larger fuel mass to deliver the spacecraft to the target, and thus an analysis of the outbound leg, but that is beyond the scope of this paper.

Results are appended for each trajectory on Table 2 for two different engine configurations. The total mass for a high thrust engine of specific impulse 300s ranges from 44 to about 400 tons, which represents 3 to 30 times the wet mass of the spacecraft at arrival to the NEO. The trajectories presented assume impulsive burns, so in principle they are not suitable for low-thrust transfers. However, due to their low Δv and long time of flight, transformation of these trajectories to low-thrust is in principle feasible, and will be considered in future work. If an equivalent

trajectory could be flown with a low-thrust engine of higher specific impulse (3000s) the asteroid retrieved mass would be over ten times that of the high-thrust case, up to an impressive 4000 tons in the case of a hypothetical transfer from the orbit of 2006 RH120 to a halo orbit.

For an average NEO density of 2.6 gr/cm³ [30], and assuming spherical bodies, the equivalent diameter of the asteroid that can be captured is also included in the table. This shows that reasonably sized boulders of 3-7 m diameter, or small asteroids of that size, could be captured with this method. Capture of entire bodies of larger size is still challenging, but the derived size of a few of the candidates fall actually within this range. With the higher specific impulse, the capturable diameter increases by a factor of 2. Asteroid 2000 SG344, with a derived size in the range of 20 to 65 meters, is the only object that fails to meet the capturable range shown in Table 2.

III.IV Overview of the Selected Candidates

The capture candidates are all of small size (perhaps with the exception of 2000 SG344), which is ideal for a technology demonstrator retrieval mission. In fact, seven of them fit the small-Earth approachers (SEA) definition by Brassier and Wiegert [32]. They showed, focusing on object 1991 VG, that the orbit evolution of these type of objects is dominated by close encounters with Earth, with a chaotic variation in the semi-major axis over long periods of time. A direct consequence of this is that reliable capture transfers can only be designed with accuracy over one synodic period, before the next encounter with Earth changes the orbital elements significantly. One could argue that finely tuning these encounters could also be used to shepherd these objects into trajectories that have a lower cost to be inserted into a manifold [33].

The candidates NEOs in Table 1 are well-known, and there has been speculation about the origin of a few of them, including the possibility that they were man-made objects, lunar ejecta after an impact [32, 34-36], or even an alien probe [37]. In particular

object 2006 RH120 has been thoroughly studied [35, 38], as it was a temporarily captured object that was considered the “second moon of the Earth” until it finally escaped the Earth in July 2007. Granvik shows that the orbital elements of 2006 RH120 changed from being an asteroid of the Atens family pre-capture, to an Apollo post-capture, having followed what we refer in this paper a transit orbit inside Earth’s Hill sphere. An additional object in the list, 2007 UN12, is also pointed out by Granvik as a possible candidate to become a TCO (Temporarily Captured Object).

Regarding their accessibility, a recent series of papers [39-41] considered up to 7 of the above objects as possible destinations for the first manned mission to a NEO (and the other 5 were not discovered at the time). They proposed human missions during the same close approaches as the capture opportunities calculated. However, the arrival dates at the asteroids are later than the required departure date for the capture, so their outbound legs could not apply to our proposed capture trajectories. An additional study by Landau and Strange [42] presents crewed mission trajectories to over 50 asteroids. It shows that a mission to 6 of the considered asteroids is possible with a low-thrust Δv budget between 1.7 and 4.3 km/s. The costs presented are for a return mission of a spacecraft with a dry mass of 36 tons (including habitat) in less than 270 days. A longer robotic mission with a final mass at the NEO of 13,600 kg and a manifold capture as the one proposed would result in much lower fuel costs as the thrust-to-mass ratio increases. NASA also publishes the Near-Earth Object Human Space Flight Accessible Target Study (NHATS) list [43], which will be continuously updated and identifies potential candidate objects for human missions to asteroids. The NEOs are ranked according to the number of feasible return trajectories to that object found by an automated search within certain constraints. Eleven of our 12 capturable objects appear in the top 25 of NASA’s NHATS list as of September 2012, seven of them in the top 10. This seems to indicate that the objects found by our pruning and optimisation are indeed easily accessible, even if the outbound part of the trajectory was not considered in our calculation.

III.V Method Limitations

One of the first objections that can be raised to the approach presented involves some of the simplifications in the model. The main simplifying assumptions are placing the Earth in a circular orbit, assuming Keplerian propagation for the NEOs orbital elements, and not including other types of perturbations, in particular the Moon third body perturbation. While the influence of the first two assumptions should be relatively small, and the

trajectories obtained can be used as first guesses for a local optimisation with a more complex model with full Earth and NEOs ephemerides, not including the Moon as a perturbing body can have a much greater influence. Granvik [38] shows that the Moon plays an important role in the capture of TCO, and so the trajectories of the manifolds would be also affected by it. However, the general behaviour and the type of NEOs that can be captured are not expected to change. Other perturbations, such as the changes in the orbit of small bodies affected by solar radiation pressure are of little importance within the timescales considered.

III.VI Earth to LPO Transfer

The paper has so far investigated the costs of transporting asteroids to weakly-bound orbits in Earth’s vicinity and has identified several plausible targets for asteroid retrieval missions. The ultimate purpose of such a mission would be to exploit the advantages of more flexible space operations at the neighbourhood of the Earth and the accessibility that would allow the transport of larger payloads to the L_1/L_2 libration point orbits.

Thus, this section presents a brief overview of the inherent costs to access LPOs from Earth. The transports costs, in terms of Δv -change, are computed assuming a departure from a dedicated geostationary transfer orbit (GTO). To obtain an estimate of the cost needed to reach a L_1/L_2 LPO from Earth, we propagate backward in time the stable invariant manifold associated with the given LPO (either halo or vertical Lyapunov orbit) for a maximum time interval of about 300 days up to fulfilling the condition:

$$\mathbf{r}^{sid} \cdot \mathbf{v}^{sid} = 0, \quad (8)$$

where \mathbf{r}^{sid} and \mathbf{v}^{sid} refer to the vector of position and velocity, respectively, in the inertial reference frame centred at the Earth. More details on this procedure can be found in [44].

It can be seen that stable hyperbolic invariant manifold trajectories associated with halo orbits satisfy Eq. (8) within the Low Earth Orbit (LEO) region. Hence, given a GTO such that $r_{perigee} = h + r_{Earth}$ and $r_{apogee} = 42164$ km with $r_{Earth} = 6378.14$ km, we can apply at the perigee of the GTO a tangential manoeuvre to insert directly into the manifold. In Figure 11 we show the cost associated with this manoeuvre as a function of perigee altitude h and Jacobi constant J of the LPO.

As Figure 11 shows, a wide range of halo orbits can be reached with a manoeuvre at a typical periapsis altitude GTO orbit. At 250 km, for example, the cost of insertion into a stable manifold leading to a halo orbit ranges from 740 m/s to 745 m/s. The variation in cost appears to be more sensitive to the

altitude of the periapsis, and thus the lower the periapsis the cheaper the manoeuvre. The time of flight of the transfer, although not shown in the figure, ranged from 185 to 210 days. The inclination of the GTO required for each specific halo orbit also varies largely, between 15° and 90° with respect to the ecliptic plane, increasing with decreasing Jacobi constant. Finally, the costs of reaching planar Lyapunov are similar to those of the halo orbits.

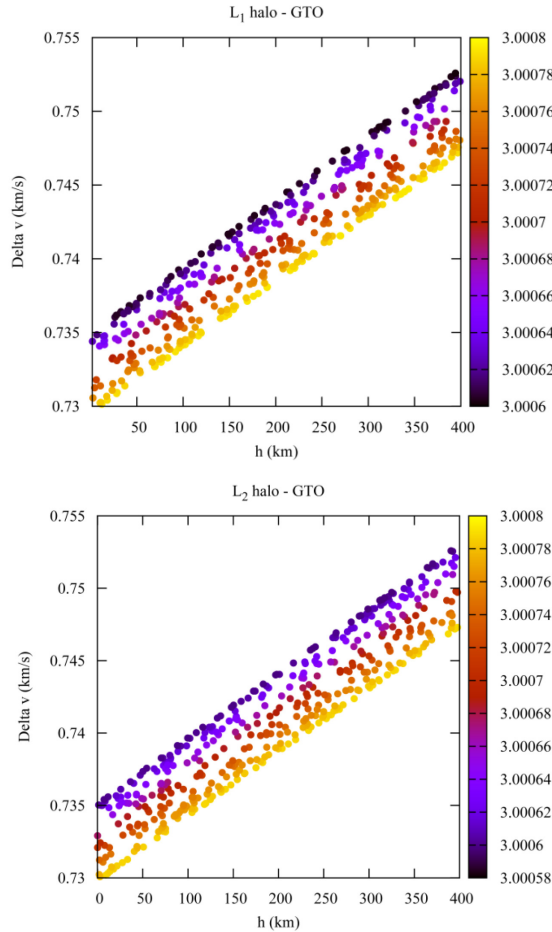


Figure 11: Cost of the manoeuvre needed to depart from a GTO by following the stable invariant manifold of a halo of a given energy near L_1 (top) or L_2 (bottom) in the Sun-Earth system as a function of h and J (colour bar).

For vertical Lyapunov orbits, the procedure to compute the transfer costs is slightly different since Eq. (8) is not satisfied within the distance reached by the GTO orbit. Thus, the transfer to a vertical Lyapunov comprises of two different manoeuvres: The first one is applied at the perigee of the GTO, now fixed at 250 km from the surface of the Earth, in order to raise the apogee to $r_{\text{apogee}} = h + r_{\text{Earth}}$. Then a second manoeuvre is applied at the new apogee to reach the stable invariant manifold. In Figure 12, we show the cost of these manoeuvres as a function of h and J , suggesting that for vertical Lyapunov is always

more convenient to be inserted into the hyperbolic stable invariant manifold as far as possible from Earth.

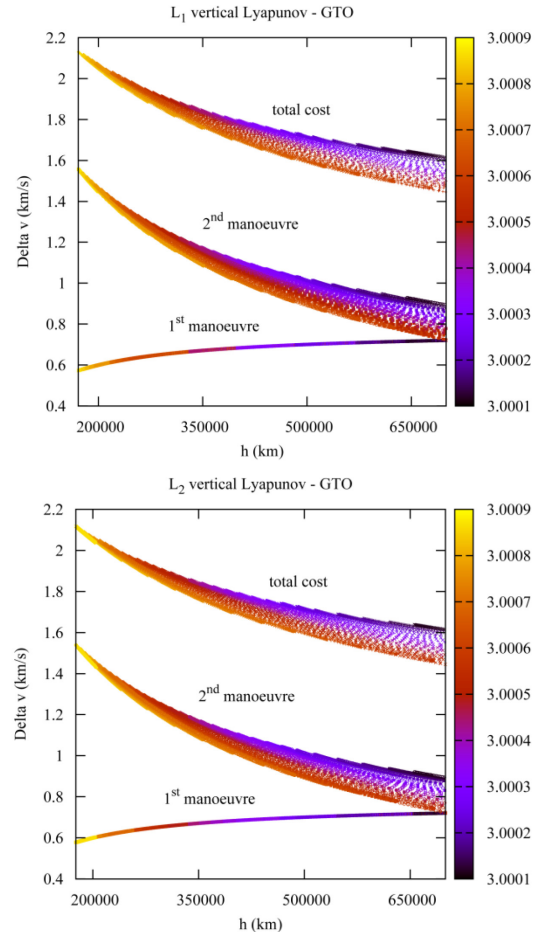


Figure 12: Cost of the manoeuvres needed to reach the stable invariant manifold of a given vertical Lyapunov orbit around L_1 (top) and L_2 (bottom) in the Sun-Earth system from a GTO at 250 km of altitude as a function of h and J (colour bar).

IV. CONCLUSIONS AND FUTURE WORK

The possibility of capturing a small NEO or a segment from a larger object would be of great scientific and technological interest in the coming decades. It is a logical stepping stone towards more ambitious scenarios of asteroid exploration and exploitation, and possibly the easiest feasible attempt for humans to modify the Solar System environment outside of Earth, or attempting any large scale macro-engineering project.

This paper has shown that the retrieval of a full asteroid is well within today's technological capabilities. Taking advantage of this, the utilisation of asteroid resources may be a viable means of providing substantial mass in Earth orbit for future space ventures. Despite the largely incomplete survey of very small objects, the current known population

of asteroids provides a good starting platform to begin with the search for easily capturable objects. With this goal, a robust methodology for systematic pruning of a NEO database and optimisation of capture trajectories through the hyperbolic invariant stable manifold into different types of LPO around L_1 and L_2 has been implemented and tested. Twelve possible candidates for affordable full asteroid retrieval missions have been identified among known NEOs with capture opportunities during the next 30 years. Transfers to the libration points region have been calculated for all these targets. These transfers enable the capture of bodies within 3-7 meters diameter with low propellant costs.

The proposed method can be easily automated to prune the NEO database on a regular basis, as the number of objects in orbits of interest is expected to grow asymptotically with the new efforts in asteroid detection. Any new occurrence of a low-cost candidate asteroid can be optimised to obtain the next available phasing and transfer opportunities and the optimal target LPO.

Moreover Sun-Earth LPOs can also be considered as natural gateways to the Earth system. Thus, the problem to transfer an asteroid to an Earth or Moon centred orbit can be decoupled into the initial phase

of inserting the asteroid into a stable invariant manifold and then providing the very small correction manoeuvres required to continue the transit into the Earth system. While a method to find optimal LPO capture trajectories and possible targets has been defined in this paper, the transit trajectories can potentially allow the asteroid to move to the Earth-Moon L_1/L_2 or other locations within cislunar space taking advantage of heteroclinic connections between collinear points.

This paper has also shown the costs of accessing the capture material at the Sun-Earth collinear equilibrium points. Given the costs associated with reaching the Sun-Earth LPOs, one can imagine the scientific, mining and even touristic advantages of bringing asteroids close to Earth, as oppose to reaching them on their unperturbed heliocentric orbits.

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