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NEAR-EARTH ASTEROID RESOURCE ACCESSIBILITY AND FUTURE CAPTURE MISSION OPPORTUNITIES

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In-Situ Resource Utilization (ISRU) has always been suggested for ambitious space endeavours; and asteroids and comets in particular are generally agreed to be ideal sources, both in terms of its accessibility and wealth. The future utilisation of asteroid resources is here revisited by, firstly, providing an estimate of the total amount of accessible resources in the Earth’s neighbourhood and, secondly, by envisaging a series of missions in order to retrieve resources from the most accessible objects known today. An analytical multi-impulsive transfer model is proposed in order to define the region in Keplerian space from which resources are accessible, and mapped subsequently into a near-Earth asteroid model, to understand the availability of material. This estimate shows a substantial amount of resources can be accessible at relatively low energy-cost; on the order of $10^{14}$ kg of material could potentially be accessed at an energy cost lower than that required to access the resources in the Moon. Most of this material is currently undiscovered, but the current surveyed population of near-Earth asteroid provides a good starting point for a search for future capture opportunities. The possibility of capturing, i.e., placing the asteroid into an orbit in permanent close proximity to Earth, a small-size NEO or a segment from a larger object would be of great scientific and technological interest in the coming decades. A systematic search of capture candidates among catalogued NEOs is presented, which targets the L$_2$ region as the destination for the captured material. A robust methodology for systematic pruning of candidates and optimisation of capture trajectories through the stable manifold of planar Lyapunov orbits around L$_2$ has been implemented and tested. Five possible candidates for affordable asteroid retrieval missions have been identified among known NEOs, and the transfers to the L$_2$ region calculated. These transfers enable the capture of bodies with 2-8 meters diameter with modest propellant requirements. Because of the optimal departure dates, two of them have been identified as attractive targets for capture missions in the 2020-2030 time frame.

1. INTRODUCTION

Asteroids and comets have long been the target of speculative thinking on resources for future space exploitation [1] (and even more recently*). The utilisation of their resources has often been suggested as necessary for future ambitious space endeavours [2]. This concept is here revised by, firstly, providing an estimate of the total amount of accessible resources in the Earth’s neighbourhood and, secondly, by envisaging a series of missions in order to retrieve resources from the most accessible known objects today.

An estimate of the total accessible resources can be obtained by analysing the volume of Keplerian orbital element space from which the Earth can be reached under a certain specific energy threshold and then mapping this onto an existing statistical near Earth object model. The specific energy required to transport resources back to the Earth can then be defined by a multi-impulsive transfer, requiring an impulse to both phase the asteroid with the Earth and reduce the Minimum Orbital Intersection Distance (MOID) below a minimum critical distance that allows a final second insertion burn at the periapsis of the hyperbolic encounter with Earth.

A resource map can then be devised estimating the amount of material available as a function of energy investment to access different types of resources. The map then shows that considerable resources can be accessed at relatively low energy levels. More importantly, asteroid resources can be accessed with an entire spectrum of energy levels, unlike other more massive bodies such as the Earth or Moon, which require a minimum energy threshold implicit in their gravity well.

As of 23 of April of 2012, 8933 NEOs are known. The smallest object among the surveyed asteroids is estimated to be of only a few meters diameter, while the larger is of 32 km diameter. The surveyed portion of the NEO population is only a small fraction of the total existing population, especially at very small sizes, on

* http://www.planetaryresources.com/ (last accessed 02/05/12)
the order of a few meters diameter, for which the surveyed fraction well below 1% [3]. Thus, the statistical approach provides a good framework for the future development of the concept. Nevertheless, plausible near-term missions to asteroids need consider the existing population of catalogued asteroids. Hence, the paper also reviews all these surveyed objects in search of possible candidates for relatively near-term scenarios for asteroid exploitation (or technology demonstrators).

The paper envisages moving a whole asteroid or a large segment of raw material to a bound Earth orbit for a later utilisation, which would allow more flexible mining operations or utilisation in the Earth’s neighbourhood. Under this premise, a preliminary set of capture missions can already be conceived for the best set of capture opportunities. A set of capture opportunities can be estimated by optimising Lambert transfers to stable invariant manifolds associated with planar Lyapunov orbits. This process is computationally slow and requires an initial pruning for candidate asteroids, which can be done by means of the Jacobi constant, approximated as Tisserand’s parameter, or Minimum Orbit Intersection Distance (MOID) related filters.

As an outcome of the paper, a series of realistic examples of asteroid utilisation targets, suitable for missions ranging from small technology demonstrators to missions delivering material resources to support future human space ventures, are presented.

II. ASTEROID RESOURCE TRANSPORT

This section will now describe the methodology followed to estimate the requirements for transporting asteroid material to the vicinity of the Earth.

Two different scenarios are envisaged; the transport of mined material and the transport of the entire asteroid. The first scenario, the transport of mined material, requires less energy to transport resources, since less mass is transported to Earth orbit, while it requires that the mining operations occur in-situ. The latter requirement results in very long duration crewed missions, with the complexity that this entails, or, if the mining is performed robotically, the need for advanced autonomous systems due to both the communication delay between asteroid and Earth and the complexity of mining operations. The second scenario, on the other hand, requires moving a large mass, with the difficulty that this involves, but allows more flexible mining operations in the Earth’s vicinity. The ultimate optimality of these two scenarios would depend on each particular asteroid (i.e., structure and composition) and the future development of the key technologies required for such missions.

The analysis presented in the paper focuses on the use of $\Delta v$ as a figure of merit (FoM) for the transport cost. This is also a measure of the specific energy, i.e., energy per unit of mass, required to transport material to Earth and, therefore the two envisaged transportation scenarios can benefit from the same FoM to draw conclusions about the feasibility of a mission.

Ideally, one would like to compute the cost of transporting resources from an initial asteroid orbit to Earth by optimising an impulsive or low-thrust trajectory. This, of course is a highly complex numerical process, to which more complexity can be added by considering gravity assists and manifold dynamics.

The paper seeks, at first, a statistical order of magnitude estimate of the total amount of asteroid resources likely to exist in the Earth’s neighbourhood. As will be clearer later, an analytical multi-impulsive transfer model is necessary in order to define the region in phase space from which resources are accessible, and map this into a near-Earth asteroid model, to understand the availability of material. After this initial analysis, the paper will follow to analyse the existing population of asteroids and discuss the results of a series of asteroid retrieval mission examples, for which much more complex transfers were sought.

III. Analytical transport formulae

We thus seek a simple conservative estimate of the transport cost of asteroid resources. A two-impulse analytical transfer is envisaged for this purpose. This is composed of an initial interception manoeuvre, which is necessary in order to insert the asteroid into a trajectory that will come in close proximity to the Earth. At the Earth encounter, a secondary manoeuvre provides the final insertion to an Earth captured orbit. The two-impulse transfer model is now briefly described in the following subsections. For a more detailed discussion the interested reader should refer to previous work by the authors [4].

Insertion manoeuvre

At the Earth’s encounter, we envisage a final impulse that modifies the hyperbolic motion of the asteroid into a capture orbit around the Earth. Since a parabolic orbit is the threshold between a permanent orbit around a planet and a single passage, the cost of capturing an asteroid can be estimated as the difference between the hyperbolic and parabolic velocities at the pericentre of the Earth’s fly-by, thus:

$$\Delta v_{\text{cap}} = \sqrt{\frac{2\mu_e}{r_p}} + v_\infty^2 - \sqrt{\frac{2\mu_e}{r_p}}$$

where $v_\infty$ is the hyperbolic excess velocity of the asteroid, $r_p$ is the pericentre altitude and $\mu_e$ the Earth gravitational constant. The asteroid’s hyperbolic excess velocity at the Earth encounter can be conveniently expressed as a function of the semi-major axis $a$, the semi-minor axis $b$, the eccentricity $e$, and the longitude of ascending node $\Omega$ as:
eccentricity $e$ and inclination $i$ of the asteroid by means of Opik’s encounter theory [5] as:

$$v_e = \sqrt{\mu_s \left(3 - \frac{1}{a} - 2\sqrt{a\left(1 - e^2\right)} \cdot \cos i\right)}$$  \(2\)

where $\mu_s$ is the Sun’s gravitational constant and, together with the asteroid’s semi-major axis, must be expressed in AU units of length. Eq.(2) assumes a circular Earth orbit with a semi-major axis of 1 AU.

### Interception manoeuvre

Previous to the final insertion manoeuvre described by Eq.(1), an interception manoeuvre is generally required in order to allow the asteroid to come in close proximity to the Earth. Note that even Earth-crossing asteroids, which by definition have periapsis and apoapsis radius smaller and larger than 1 AU respectively, do not generally cross the orbit of the Earth, unless coplanar with Earth (see Figure 1).

Figure 1: Representation of all possible orientations of an orbit as a function of argument of the periapsis $\omega$. The two crosses mark the Earth orbital crossing points which are possible only for four different values of the argument of the periapsis $\omega$. Any orbit with other values of $\omega$, represented by dashed lines, does not intersect the orbit of the Earth.

Thus, as shown by Figure 1, unless the elliptical Earth-crossing orbit has a very particular orientation within the asteroid’s orbital plane (or is coplanar with the Earth’s motion) a manoeuvre is necessary to transfer asteroid resources to Earth. The transfer model implemented here assumes a manoeuvre such that the asteroid’s orbital orientation is modified to allow the asteroid to come close enough to the Earth to undergo a hyperbolic fly-by. The change of orientation is modelled as an instantaneous change of argument of periapsis $\omega$ of the orbit, thus a rotation of the orbit within its orbital plane, or a change of plane manoeuvre, thus a rotation of angle $i$ of the orbital plane.

In order to compute the required change of the argument of periapsis $\Delta \omega$, we first require to identify the four different periapsis orientations $\omega_{enc}$, as depicted in Figure 1, that allow an asteroid with Keplerian elements $\{a, e, i\}$ to intersect the Earth’s orbit:

$$\omega_{enc} = \left\{\pi - \theta_{enc} - \theta_{enc} - \theta_{enc} - \theta_{enc}\right\}$$  \(3\)

where $\theta_{enc}$ is the true anomaly of the asteroid’s orbit such that the Sun is at 1 AU distance, thus:

$$\theta_{enc} = \pm \cos^{-1}\left(\frac{p - 1}{e}\right)$$  \(4\)

where $p$ is the asteroid’s semilatus rectum.

Hence, for an initial argument of periapsis $\omega_i$, the orbit requires to be changed by $\Delta \omega = |\omega_{enc} - \omega_i|$, where $\omega_i$ is either the closest $\omega_{enc}$ value to $\omega_i$, or a close value that allows a more adjusted fly-by with a given specific MOID, or minimum orbital intersection distance, that ensures a fly-by with a given perigee altitude. See more details on this in other work by the authors [4]. Note that the four values of $\omega_{enc}$ enforce a MOID equal to zero, which is not necessary to ensure a fly-by.

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Figure 2: Representation of a manoeuvre providing a change of orientation $\Delta \omega$ of an orbit.

The cost of rotating an orbit’s argument of periapsis by a given angle $\Delta \omega$, as described by Figure 2, can be computed by noticing that at the intersection between the orbits with arguments $\omega_i$ and $\omega_i$, both the radial distance to the Sun and angular position within the asteroid’s orbital plane must be the same. This defines two intersection points between the two orbits occurring at true anomalies $\{\Delta \omega/2, -\Delta \omega/2\}$ and $\{\Delta \omega/2 + \pi, \pi - \Delta \omega/2\}$. It is then straightforward to see that at these intersection points the change of velocity required to jump from orbit with $\omega_i$ to orbit with $\omega_i$ is equal to twice the radial velocity of the intersecting orbits, which is the same at the two intersections. Hence, the final $\Delta v$ manoeuvre to change $\omega_i$ to $\omega_i$, without changing $\{a, e, i\}$, can be defined as:

$$\Delta v_{\Delta \omega} = 2\sqrt{\frac{\mu_s}{p}} e \sin\left(\frac{\Delta \omega}{2}\right).$$  \(5\)

In general, Eq.(5) provides a conservative estimate of the transfer cost, that is, a Lambert arc optimisation will always provide a lower value for the same transfer.
In some occasions, for low inclinations and high eccentricities, the manoeuvre described by Eq.(5) becomes a very poor option, and a simple change of plane such that the asteroid is inserted into an orbit with coplanar motion with the Earth becomes a better option. For this case a manoeuvre such as:

\[ \Delta v_{\text{nc}} = 2v_{\text{LoN}} \cdot \sin \left( \frac{i}{2} \right) \]  

(6)

where \( v_{\text{LoN}} \) corresponds to the asteroid’s velocity at the line of nodes, is computed instead [4].

**\( \Delta v \)-leveraging manoeuvre**

Hitherto we have only considered Earth-crossing asteroids. Non-Earth crossing asteroids, those with \( a<1 \) and \( r_p<1 \) or \( a>1 \) and \( r_p>1 \), can also be transferred to Earth by applying a manoeuvre at one of the apsidal points, such that the opposite apses changes in such a way that an intersection with the Earth’s orbit occurs (see Figure 3).

**Figure 3: Manoeuvre representation for a non-Earth crossing asteroid.**

Given a non-Earth crosser asteroid \( \{a,e,i,\omega\} \), the apsidal manoeuvre necessary to obtain an Earth intersecting trajectory can be computed as:

\[ \Delta v = \sqrt{\mu} \left( 2 \frac{1}{a_f} - \frac{2}{r_m} - \frac{1}{a} \right) \]  

(7)

where \( r_m \) is the apsidal point at which the impulsive manoeuvre is performed, thus the periapsis for asteroid with \( a<1 \) and the apoapsis when \( a>1 \), and \( a_f \) is the semimajor axis of the transfer trajectory, which can be computed as:

\[ a_f = a \left( 1 \pm \frac{e}{e_f} \right) \]  

(8)

where \( e_f \) is the eccentricity of the transfer trajectory, which is defined as:

\[ e_f = \frac{r_e - 1}{\cos \theta \pm r_e} \]  

(9)

where \( \theta \) is the true anomaly of the closest node to the periapsis when \( a>1 \) (i.e., \( \theta=\pi-\omega \)) or to the apoapsis when \( a<1 \) (i.e., \( \theta=\pi+\omega \)). The upper sign and lower signs in Eq. (8) and (9) also correspond to the \( a>1 \) and \( a<1 \) case respectively.

**Accessibility in the Earth’s orbital neighbourhood**

Eq. (1) to (9) describe a set of manoeuvres that allows a quick estimate of the transport costs of asteroidal resources. These formulae can also be expressed as an explicit function of the \( \Delta v \) costs, which can be used to define the 4-D limits, in \( \{a,e,i,\omega\} \)-space, for which transfers are ensured to be below a given \( \Delta v \) limit. Estimating the fraction of the NEO population that is within this 4-D volume will allow us to estimate the total amount of available resources accessible given a \( \Delta v \) threshold.

**Figure 4:** For example, shows the \( \{a,e,i\} \)-Keplerian element space defined by a \( \Delta v \) threshold of 2.37 km/s, which corresponds to Moon’s escape velocity. This \( \Delta v \) threshold defines an accessible Keplerian element space that can be exploited at an equivalent energy to that necessary to exploit resources at the Moon.

**Figure 4: Accessible Keplerian space at a \( \Delta v \) threshold of 2.37 km/s as defined by the analytical transport model presented.**

### III. AVAILABLE RESOURCES

By 9th April 2012, there are almost 9000 known near-Earth objects. By convention, an object is considered a Near Earth Object (NEO) if its perihelion is smaller than 1.3 AU and its aphelion is larger than 0.983 AU. This is a very broad definition which includes predominantly asteroids, but also a small fraction of comets. NEOs are then the closest objects to the Earth and therefore the obvious first targets for any resource exploitation mission (excluding the Moon).

A quick analysis of the transportation costs for all the surveyed asteroids, by means of the model described above, already shows a series of interesting target objects for future exploitation missions that require a \( \Delta v \) in the order of a few hundred meters per second. Figure 5 summarises the number of objects that could be exploited for a given level of \( \Delta v \). The figure also shows different colour bands for each different type of transfer considered as part of the two-impulse transfer model (i.e., change of \( \omega \) versus change of \( i \), Earth-crosser type of asteroid or non-Earth crosser).
Figure 5: Number of known objects whose resources are accessible for a given $\Delta v$ threshold. The set of manoeuvres are briefly described in this paper, except the Earth-crosser with no interception manoeuvre, which can be found at Sanchez & McInnes [4] under the one-impulse manoeuvre description.

The purpose of the transfer models however is not to compute the transport cost of existing objects, for which more complex and less conservative transfers should be considered, but to delimit the Keplerian element space regions from which asteroids could be transported to Earth, as shown by Figure 4. Once the accessible Keplerian element space is known, an estimate of the total amount of resources can be made by means of a NEO model capable of providing the number and size of objects expected to be found within these regions.

III.I NEO Orbital and Size Distribution

In order to estimate the amount of available resources we thus require a sound statistical NEO population model. The NEO model used here is composed by two separate parts: an orbital and a size distribution. The NEO size model is based on Harris’s population estimates [6], whereas the NEO orbital distribution is based on the theoretical distribution model published in Bottke et al. [7].

The size distribution model allows us to estimate the probability of an object to have a given size. As noted before, it is based on the latest estimates of the NEO population [6], which indicate a slight deviation from the commonly used single slope power law distribution [8]. The paper uses a multi-slope power law distribution, which approximates the results shown by Harris [6]. This is a drop, with respect to previous estimates, of a factor of 2/3 on the cumulative number of objects with diameter larger than 100 m.

The NEO orbital distribution used is based on an interpolation from the theoretical distribution model by Bottke et al. [7]. The data used was very kindly provided by W.F. Bottke (personal communication, 2009). Bottke et al. [7] built an orbital distribution of NEOs by propagating in time thousands of test bodies initially located at all the main source regions of asteroids (i.e., the $\nu_6$ resonance, intermediate source Mars-crossers, the $3:1$ resonance, the outer main belt, and the trans-Neptunian disk). By using the set of asteroids discovered by Spacewatch at that time, the relative importance of the different asteroid (or comets) sources could be best-fitted. This procedure yielded a steady state population of near Earth objects from which an orbital distribution as a function of semi-major axis $a$, eccentricity $e$ and inclination $i$ can be interpolated numerically. Figure 6 shows a representation of Bottke’s NEO density function $\rho(a,e,i)$. The remaining three Keplerian elements, not shown in Figure 6, the right ascension of the ascending node $\Omega$, the argument of periapsis $\omega$ and the mean anomaly $M$, can be assumed to be uniformly distributed random variables [9].

III.II Accessible Resource Mass

The total mass of the population of near Earth objects, larger than 1 meter diameter, is estimated to be approximately $5 \times 10^{16}$ kg [4]. The question that arises now is how much of this mass can be easily accessed and exploited. Figure 7 shows the portion of the aforementioned total asteroid mass that can be accessed with a given $\Delta v$ threshold. The same figure also shows, with a thinner dashed line, the fraction of the accessible mass that has been currently surveyed. This is only a rough estimate of the total mass of surveyed objects computed from the visual magnitude data [10].
Figure 7: Rough estimate of the total and surveyed amount of accessible asteroid resources as a function of $\Delta v$ threshold.

Figure 7 provides a tool to measure and compare the availability of resources, defined here as a total asteroid mass (without distinguishing between different materials), and the difficulty to access that mass. Thus for example, the figure shows that at the same cost of accessing lunar resources (i.e., 2.37 km/s) the total pool of asteroid mass available is of the order of $1 \times 10^{14}$ kg. This of course is orders of magnitude lower than the total pool of resources at the Moon (i.e., the mass of the Moon, ~7.36x10^{22} kg), although it is still a very reasonable amount. The main advantage of asteroid resources however is that asteroid material can be exploited at a whole spectrum of $\Delta v$, rather than at a minimum threshold, as is the case for lunar material (i.e., 2.37 km/s). Thus, for example, with 100 m/s of $\Delta v$ budget, approximately $8.5 \times 10^9$ kg of asteroid resource could potentially be exploited. Note however, from Figure 7, that while for higher $\Delta v$ thresholds the fraction of surveyed asteroid is largely complete, for very low $\Delta v$ exploitable targets still need to be discovered.

III.2. Accessibility of Asteroids

One of the important issues not yet resolved by the results shown in Figure 7 is the number of missions that are required to exploit all or part of the accessible pool of asteroid resources. This issue is of key importance, since if a given resource is spread in a large number of small objects, gathering all of them may become a cumbersome task, and therefore not economically worthwhile.

Figure 8 casts some light on this issue by computing the median asteroid size expected to be found within the Keplerian element space that ensures transport at a given $\Delta v$ cost. For a given $\Delta v$ budget we shall expect a complete population of asteroids, whose sizes follow Harris’s population estimates [6]. Thus, the figure shows the largest object, but also the tenth, hundredth and thousandth largest asteroid accessible with the same $\Delta v$ budget. The overlapped grey regions represent the 90% confidence region of each one of these objects. Note that the 90% confidence region accounts only for the statistical variance of a population of objects perfectly described by Harris’s population estimates, not by epistemic uncertainties on the assumed distributions. For details on the calculations of the results shown in Figure 8 the reader should refer to [4].

The information in the figure can be read as follows: let us set, for example, the $\Delta v$ threshold at 100 m/s, the largest accessible object has a 50% probability to be equal to or larger than 33 meters in diameter, while we can say with 90% confidence that its size should be between 83 meters and 20 meters. The following set of data in the decreasing ordinate axis is the group referring to the 10th largest object found within the region of feasible capture given by a $\Delta v$ threshold of 100 m/s, whose median diameter is at 13 meters diameter. The 100th largest object is foreseen to have a diameter of 5 meters and 1000th largest of 2 meters. This provides an overview of all the objects that could be exploited with a $\Delta v$ of 100 m/s or lower.

Figure 8: Expected size of the accessible asteroid.
planned for the near future\(^\dagger\). Given the low transport cost expected for the most accessible objects, we could also envisage the possibility to return to Earth entire small objects with current or near-term technology. The problem resides on the difficulties inherent to the detection of these small objects. Thus, for example, only 1 out of every million objects with diameter between 5 to 10 meters is currently known and this ratio is unlikely to change significantly in the coming years [12].

As indicated in Figure 5 and Figure 7, there are already some known asteroids that are relatively accessible. In this section then, we will shift our attention to the surveyed population of asteroids in search of the most accessible candidates for near term asteroid retrieval missions.

The transfers presented previously were only intended to provide an approximate estimate of the transport cost, as with a Hohmann transfer analysis. In order to minimise the transfer costs, it is necessary now to use judiciously the Earth’s gravitational perturbation. Exploiting the dynamics of invariant manifolds, associated with periodic orbits around the Sun-Earth Lagrangian points, provides good opportunities for low-energy transfers.

For this purpose, a systematic search of capture candidates among catalogued NEOs was carried out, selecting the L\(_2\) region as the target destination for the captured material. This gives a grasp and better understanding of the possibilities of capturing entire NEOs or portions of them in a useful orbit, and demonstrates a method that can be applied to newly discovered smaller bodies in the coming future when detection technologies improve.

Missions delivering a large quantity of material to the Lagrangian points are of particular interest. The material can be used as test bed for ISRU technology demonstration missions and material processing at affordable costs, ranging from fuel extraction to testing the use of material for radiation shielding. The science return is also greatly improved, with an asteroid permanently, or for a long duration, available for study and accessible to telescopes, probes and even crewed missions to L\(_2\). Finally, it sets the stage for other future endeavours, such as the construction of a permanent base around L\(_2\) using the asteroid as the main structure or just as a source of material.

IV. 4 Invariant Manifold Trajectories to L\(_2\)

The design of the transfer from the asteroid orbit to the L\(_2\) region consists of a Lambert arc that intersects an invariant stable manifold leading to periodic orbits around L\(_2\). Only the inbound leg of a full capture mission was considered. Planar Lyapunov orbits around L\(_2\) were selected as the target final parking orbit for the asteroid. Future work should consider both outbound and inbound trajectories, as well as other non-planar orbits such as vertical Lyapunov and halo orbits.

Planar Lyapunov orbits around L\(_2\) with Jacobi constants ranging from 3.00089 (~L\(_2\)) to 2.999388 were considered and generated through a process of differential correction and numerical continuation [13]. The invariant stable manifold trajectories leading to each of these planar Lyapunov orbits were propagated backwards in the circular restricted 3-body problem until they reached a fixed section in the Sun-Earth rotating frame. The section was arbitrarily selected as the one forming an angle of \(\pi/8\) with the Sun-Earth line (see Figure 9). This corresponds roughly to a distance of order 0.4 AU to the Earth, where the gravitational influence of the planet is considered small. Only Sun and Earth point-mass gravitational fields were considered in the backward propagation.

The shape of the manifolds in the \(r - \dot{r}\) phase space (with \(r\) the radial distance from the Sun) at the intersection with the \(\pi/8\) section is shown in Figure 10. For an orbit with exactly the energy of L\(_2\), the intersection is a single point; while for lower Jacobi constants, the shape of the intersection is a closed loop. The lower the Jacobi constant, the larger the loop becomes and the more the manifolds curve around themselves.

\[\text{Figure 9: Schematic representation of the transfer}\]

In the analysis, Earth is assumed to be in a circular orbit 1 AU from the Sun. This simplification allows for the orbital elements of the manifold trajectories (and in particular in the selected section) to be independent of the insertion time into the Lyapunov orbit. The only exception is the argument of perihelion, which varies

\[^\dagger\text{http://www.nasa.gov/topics/solarsystem/features/osiris-rex.html (last accessed 02/05/12)}\]
with the insertion time with respect to a reference time with the following relation:

$$\omega = \omega_{REF} + \frac{2\pi}{T}(t - t_{REF})$$  \hfill (10)

where $\omega_{REF}$ is the argument of perihelium at the $\pi/8$ section for an insertion into a Lyapunov orbit at reference time $t_{REF}$, and $T$ is the period of the Earth.

The transfer between the NEO orbit and the manifold is calculated as a heliocentric Lambert arc with two impulsive burns, one to depart from the NEO, the final one for insertion into the manifold, with the insertion constrained to take place before or right after the $\pi/8$ section. For the Lambert arc, no Earth, Moon or other planet third body perturbation is included in the computations.

Thus, the problem is defined with 5 variables: the Lambert arc transfer time, the manifold transfer time, the insertion date at the Lyapunov orbit, the energy of the final orbit, and a fifth parameter determining the point in the Lyapunov orbit where the insertion takes place.

The benefit of such an approach is that the asteroid is asymptotically captured into a “stable” bound orbit around $L_2$, with no need of a final insertion burn at arrival. All burns are performed far from Earth, so no large gravity losses need to be taken into account. Furthermore, this provides additional time for corrections, as the dynamics in the manifold are “slow” when compared to a traditional hyperbolic approach. Finally, this type of trajectory is then easily extendible to a low-thrust trajectory if the burns obtained are small. A further advantage is that the NEO ends up in an orbit with the possibility of low energy transfers through heteroclinic connection to other destinations of interest, such as $L_1$ or the Moon [14].

### IV. II Asteroid Catalogue Pruning

The NEO sample for the study is the Minor Planet Centre (MPC) database [15] of minor bodies as of 3rd of February 2012. This database represents the catalogued NEOs up to that date, and as such it is a biased population, most importantly in size, as already noted. A large number of asteroids of the most ideal size for capture have not yet been detected, as current detection methods favour larger asteroids. Secondly, there is an additional detection bias related to the type of orbits, with preference for Amors and Apollos in detriment to Aten, as object in Aten orbits spend more time in the exclusion zone due to the Sun.

Even with this reduced list, it is a computationally expensive problem and preliminary pruning becomes necessary. Figure 5 showed that the number of asteroids that could be captured with impulsive burns lower than 400 m/s should be of the order of 10. Although the approach in the former section and the manifold capture is different, the number of bodies that could be captured in Lyapunov orbits at low costs is expected to be of the same order. Without loss of generality, it is possible to immediately discard NEOs with semi-major axis (and thus energy) far from the Earth’s, as well as NEOs in highly inclined orbits. However, more systematic filters needed to be devised. Three filters were tested against the MPC population:

- **Jacobi constant**: the Jacobi constant $J$ is approximated by the Tisserand parameter as defined in Eq (11). Objects with Jacobi constant close to $L_2$ are filtered.

  $$J \approx \frac{1}{a} + 2\sqrt{a(1 - e^2)\cos(i)}$$ \hfill (11)

- **Percentage of orbit close to Earth**: selecting an arbitrary distance threshold of 0.05 AU, objects with the largest percentage of their orbits at a distance lower than the threshold are filtered out.

- **MOID detection filtering**: the number of local minima for the distance between two orbits can at most be 4 [16], but in most cases there will only be two such points, one close to each of the nodes. These points are referred to as MOID points. However, for a circular Earth orbit, an object which lies in a circular orbit slightly larger or smaller than Earth would have by definition infinite MOID points. A geometrical method for finding these points [17] calculates additional “fake” MOID points for orbits that are close to this situation. We have found out that filtering NEOs in such orbits, though unconventional, provides good candidates for the capture method presented.

![Figure 10: Shape of the manifolds in the $r - \dot{r}$ phase space for different Jacobi constant levels. The manifolds are represented at their intersection with a plane forming a $\pi/8$ radians angle with the Sun-Earth line in the rotating frame.](image-url)
Candicates were additionally filtered by semi-major axis, constraining it in the range 0.85-1.15 AU to allow for a relatively short low energy transport to the Earth’s neighbourhood [18]. This constraint was later relaxed at the end of the study but no better capture candidates were found.

Table 1 lists the best candidates according to the different filters tested. Most of the NEOs ranking best in each filter appear in more than one list. The bodies that are uniquely appearing in a particular filter are shaded in grey in the table. A combined list with the most promising 36 candidates was used for the capture trajectory search and optimisation.

<table>
<thead>
<tr>
<th>Jacobi Filter</th>
<th>Distance Filter</th>
<th>MOID Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 MD</td>
<td>2006 RH120</td>
<td>2000 SG344</td>
</tr>
<tr>
<td>2011 BL45</td>
<td>2010 VQ98</td>
<td>2006 RH120</td>
</tr>
<tr>
<td>2009 BD</td>
<td>2010 JW34</td>
<td>2010 VQ98</td>
</tr>
<tr>
<td>2008 HU4</td>
<td>1991 VG</td>
<td>2007 UN12</td>
</tr>
<tr>
<td>2007 TF15</td>
<td>2000 SG344</td>
<td>2008 EA9</td>
</tr>
<tr>
<td>2006 RH120</td>
<td>2006 QG56</td>
<td>2010 UE51</td>
</tr>
<tr>
<td>2005 LC</td>
<td>2008 UA202</td>
<td>2009 BD</td>
</tr>
<tr>
<td>2003 SM84</td>
<td>2010 UJ</td>
<td>1991 VG</td>
</tr>
<tr>
<td>2010 JW34</td>
<td>2007 UN12</td>
<td>2010 JW34</td>
</tr>
<tr>
<td>2006 QG56</td>
<td>2010 UE51</td>
<td>2011 MD</td>
</tr>
<tr>
<td>2000 SG344</td>
<td>2008 EA9</td>
<td>2008 HU4</td>
</tr>
<tr>
<td>2010 VQ98</td>
<td>2009 BD</td>
<td>2006 QG56</td>
</tr>
<tr>
<td>2010 UE51</td>
<td>2003 YN107</td>
<td>2008 UA202</td>
</tr>
<tr>
<td>2008 UA202</td>
<td>2001 GP2</td>
<td>2011 BQ50</td>
</tr>
<tr>
<td>2008 EA9</td>
<td>2010 UY7</td>
<td>2009 OS5</td>
</tr>
<tr>
<td>2007 UN12</td>
<td>2009 YR</td>
<td>1999 AO10</td>
</tr>
<tr>
<td>2008 JL24</td>
<td>2005 KA</td>
<td>2011 BQ50</td>
</tr>
<tr>
<td>2011 BQ50</td>
<td>2011 BL45</td>
<td>2011 BP40</td>
</tr>
<tr>
<td>2011 CL50</td>
<td>2011 CE22</td>
<td>2011 MD</td>
</tr>
<tr>
<td>2010 VQ</td>
<td>1999 AO10</td>
<td>2006 JY26</td>
</tr>
<tr>
<td>2006 KT</td>
<td>2010 TE55</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Candidate NEOs for the different filters. Results are shown for an approximate Jacobi filter larger than 2.995, a percentage of the orbit close to Earth larger than 33%, and NEOs with additional MOID points.

IV. III Capture Transfers and Mass Estimates

For each of the selected NEOs, feasible capture transfers were obtained in the date interval of 2016-2100. The Lambert transfers between the asteroid initial orbit and the manifolds were optimised using EPIC, a global optimisation method that uses a stochastic search blended with an automatic solution space decomposition technique [19]. Single objective optimisations with total transfer \( \Delta N \) as cost function were carried out. Trajectories obtained with EPIC were locally optimised with MATLAB’s built-in function fmincon. This process was repeated in a smaller domain around the optimum insertion date. Lambert arcs with up to 3 complete revolutions before insertion into the manifold were considered. For cases with at least one complete revolution, the two possible solutions of the Lambert problem were optimised. This implies that 7 full problem optimisations needed to be run for each NEO.

Table 2 presents the optimisation variables and the search domain where they were allowed to vary. The first two variables \( J_{\text{Lyapunov}} \) and \( \gamma_{\text{Lyapunov}} \) define the size of the target planar Lyapunov orbit and the point of insertion into this orbit. The final conditions are completely defined once the final time of insertion into the Lyapunov \( E_{\text{Earth}} \) is fixed. The Lambert transfer time \( \Delta t_{\text{Lambert}} \) is defined as a function of the number of complete revolutions \( nco \) as indicated in the table.

One of the main outputs of the optimisation is that the cost in terms of \( \Delta N \) for the Lambert solutions with more than one complete revolution is of the same order as those with zero revolutions. This favors then the later shorter transfers. All solutions take place during a close approach of the asteroid to the Earth, as expected. The departure from the asteroid and insertion into the manifold occur in general before the time of closest approach, while the final insertion into the Lyapunov orbit usually happens after. When there were several close encounters with Earth in the date range, only results for the first approach were stored and are presented here. Later encounters may provide lower \( \Delta N \) but the orbital elements of the NEO would have changed significantly after an Earth flyby and the estimated costs are less reliable.

Table 3 shows the best 5 opportunities that resulted from the search. The five NEOs appeared in the candidates obtained with all 3 filters (they are highlighted in bold in Table 1), which indicates that a combination of filters may be the best option for pruning. All five have a total \( \Delta N \) of less than 500 m/s.
Table 3: Capture trajectories to L2 and mass estimates.

<table>
<thead>
<tr>
<th>Asteroid</th>
<th>Date [yyyy/mm/dd]</th>
<th>Duration</th>
<th>Δv</th>
<th>Isp = 300s</th>
<th>Isp = 3000s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asteroid departure</td>
<td>Manifold insertion</td>
<td>L2 arrival</td>
<td>[yr]</td>
<td>[m/s]</td>
</tr>
<tr>
<td>2007 UN12</td>
<td>2019/05/02</td>
<td>2020/04/06</td>
<td>2021/12/03</td>
<td>2.59</td>
<td>195</td>
</tr>
<tr>
<td>2006 RH120</td>
<td>2026/03/31</td>
<td>2027/02/11</td>
<td>2029/11/30</td>
<td>3.67</td>
<td>302</td>
</tr>
<tr>
<td>2008 EA9</td>
<td>2018/10/30</td>
<td>2019/07/19</td>
<td>2022/01/29</td>
<td>2.53</td>
<td>325</td>
</tr>
<tr>
<td>2008 UA202</td>
<td>2027/03/02</td>
<td>2027/12/18</td>
<td>2030/07/29</td>
<td>3.41</td>
<td>416</td>
</tr>
</tbody>
</table>

The cheapest transfer, below 200 m/s, corresponds to 2007 UN12. It is important to emphasise that this Δv comprises both burns at departure from the asteroid and insertion into the manifold (but it does not include any navigation costs and corrections). The NEO orbit may intersect the manifold directly, and in that case the transfer to the Lyapunov orbit can be done with a single burn. This is noted in the table by shading in gray the dates of the burns with zero Δv.

The total duration of the transfers is in general of the order of 3 years. The only exception is 2010 UE51, with a total duration close to 5 years. For 2008 EA9 the duration is calculated as the time from manifold insertion until arrival to the Lyapunov orbit, as the first burn of departure is zero.

Finally, Figure 13 plots the manifold shape on the phase space at its intersection with the π/8 plane. The NEO orbit intersection with the plane is represented with a red circle, while the intersections of the orbit after the first and second burn are plotted with a right cyan triangle and a cross. In this particular case both points lie on the manifold and at the same position as the second burn is basically zero.

Figure 12: Distance to Earth for 2010 UE51. The stable manifold trajectories are plotted in light grey.

Figure 13: Manifold representation at the phase space intersection with π/8 section for 2010 UE51.
In order to obtain an estimate of the mass and size of the asteroid that could be retrieved to the L₂ region with this type of trajectory, we can consider a basic system mass budget exercise. Assuming a spacecraft of 6 tons can be delivered to the asteroid before the required departure date, consisting of 2000 kg of dry mass and 4000 kg of propellant it is possible to estimate the total asteroid mass that can be transferred. That mass fraction is similar to Cassini at launch⁴ with 2150 dry mass excluding Huygens, and 5574 wet mass. A full system budget would require a larger fuel mass to deliver the spacecraft to the target, and thus an analysis of the outbound leg, but that is beyond the scope of this paper.

Results are appended for each trajectory on Table 3. The total mass for a high thrust engine of specific impulse 300s ranges from 25 to 57 tons, which is 5 to 11 times the wet mass of the spacecraft at arrival to the NEO. The trajectories presented assume impulsive burns, so in principle they are not suitable for low-thrust transfers. Due to their low Δv and long time of flight, transformation of these trajectories to low-thrust is possible, and will be considered in future work. If an equivalent trajectory could be flown with a low-thrust engine of higher specific impulse (3000s) the asteroid retrieved mass would be ten times that of the high-thrust case, up to an impressive 600 tons in the case of 2007 UN12.

For an average NEO density of 2.6 gr/cm³ [20], and assuming spherical bodies, the equivalent diameter of the asteroid that can be captured is also included in the table. This shows that reasonable sized boulders of 3m diameter, or small asteroids of that size, could be captured with this method. Capture of entire bodies of larger size is still challenging even for these low level Δv/s. With the higher specific impulse the largest asteroid that can be retrieved is still under 8 meter diameter.

IV. IV Overview of the Selected Candidates

Table 4 presents the orbital elements of the NEOs that resulted from our search. The table also includes the MOID published by the Minor Planet Centre [15] and an estimate of the size of the object. This estimate is calculated with the following relation [20]

\[ D = 1329 \text{ km} \times 10^{-H/5} p^{-1/2} \]  

(12)

where the absolute magnitude \( H \) is obtained from the MPC database, and the albedo \( p \) is assumed to range from 0.05 to 0.50.

All NEOs in Table 4 are well-known, and there has been speculation about the origin of some of them, including the possibility that they were man-made objects or lunar ejecta after an impact. In particular object 2006 RH120 has been thoroughly studied [21, 22], as it was a temporarily captured orbit and was considered the “second moon of the Earth” until it finally escaped the Earth in July 2007. Granvik shows that the orbital elements of 2006 RH120 changed from being an asteroid of the Atens family pre-capture, to an Amor post-capture. The first object in our list, 2007 UN12, is also pointed out by Granvik as a possible candidate to become a TCO (Temporarily Captured Object).

Several papers in 2010, [23-25], some of them presented in the second SBAG (Small Bodies Assesment Group) workshop, considered three of the above objects 2007 UN12, 2006 RH120, and 2008 EA9 as possible destinations for the first manned mission to a NEO. They proposed human missions during the same close approaches as the capture opportunities calculated. However, the arrival dates at the asteroids are later than the required departure date to capture material around L₂, so their outbound legs could not apply to our proposed capture trajectories.

An additional study by Landau [26] presents crewed mission trajectories to over 50 asteroids. It shows that a mission to each of the 5 above considered asteroids is possible with a low-thrust Δv budget between 2 and 4.3 km/s. The costs presented are for a return mission of a spacecraft with a dry mass of 36 tons (including habitat) in less than 270 days. A longer robotic mission with a final mass at the NEO of 5000 kg and a manifold capture as the one proposed would result in much lower costs as the thrust-to-mass ratio increases.

<table>
<thead>
<tr>
<th>a</th>
<th>e</th>
<th>i</th>
<th>MOID</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>[AU]</td>
<td>[deg]</td>
<td>[AU]</td>
<td>[m]</td>
<td></td>
</tr>
<tr>
<td>2007 UN12</td>
<td>1.054</td>
<td>0.060</td>
<td>0.201</td>
<td>0.0011</td>
</tr>
<tr>
<td>2006 RH120</td>
<td>1.033</td>
<td>0.024</td>
<td>0.602</td>
<td>0.0165</td>
</tr>
<tr>
<td>2008 EA9</td>
<td>1.059</td>
<td>0.080</td>
<td>0.401</td>
<td>0.0014</td>
</tr>
<tr>
<td>2010 UE51</td>
<td>1.055</td>
<td>0.060</td>
<td>0.602</td>
<td>0.0084</td>
</tr>
<tr>
<td>2008 UA202</td>
<td>1.033</td>
<td>0.068</td>
<td>0.298</td>
<td>2.6e-4</td>
</tr>
</tbody>
</table>

Table 4: Orbital elements for the NEOs with lower capture Δv.

The capture candidates are all of small size, which is optimal for a retrieval mission. It is important to point out that this is not a constraint in any of the filters, and the search included all sizes of NEOs. It shows that the retrieval of a full asteroid is well within today’s capabilities for low-thrust missions or even more ambitious high-thrust engine missions with a larger wet mass at the asteroid.

‡ http://saturn.jpl.nasa.gov/multimedia/products/pdfs/cassini_msn_overview.pdf (last accessed 02/05/12)
NASA also publishes the Near-Earth Object Human Space Flight Accessible Target Study (NHATS) list [27], which will be continuously updated and identifies potential candidate objects for human missions to asteroids. The NEOs are ranked according to the number of feasible return trajectories to that object found by an automated search with certain constraints. All five capturable objects appear in NASA’s NHATS list, as of March 2012, ranking 7th, 8th, 5th, 9th and 12th respectively. This indicates that the objects found by our filtering and optimisation are indeed easily accessible, even if the outbound part of the trajectory was not considered in our calculation.

As interesting as the asteroids included in the results of our search is the analysis of some notable absences. For example, asteroid 2000 SG344 appeared in all three of our pruned lists, ranking first with the MOID filter. It also ranks first in NASA’s NHATS list, it is the cheapest accessible asteroid in Ladau’s study (under 500 m/s in total), it is one of the candidates in all human missions studies by Adamo, Barbee and Hopkins, and it was proposed for a capture mission to the Lagrangian points [28]. However, in our systematic search, the best solutions that could be found for asteroid 2000 SG344 had a Δv cost of 700 m/s. This is still low, but one would expect to see it among the top 5 capturable objects. The reason for this may be better understood in the next subsection, where the limitations of our approach are analysed.

IV. V Method Limitations

One of the first objections that can be raised to the approach presented involves some of the simplifications in the model. The main simplifying assumptions are placing the Earth in a circular orbit, assuming Keplerian propagation for the NEOs orbital elements, and not including other types of perturbations, in particular the Moon third body perturbation. While the first two assumptions influence should be relatively small, and the trajectories obtained can be used as first guesses for a local optimisation with a more complex model with full Earth and NEOs ephemerides, not including the Moon as a perturbing body can have a much greater influence. Granvik [21] shows that the Moon plays an important role in the capture of TCO, and so the trajectories of the manifolds would be also affected by it. However, the general behaviour and the type of NEOs that can be captured are not expected to change. Other perturbations, such as the changes in the orbit of small bodies affected by solar radiation pressure are of less importance.

The main drawback of the proposed approach, which can explain the absence of 2000 SG344 in our top five list, is that the method is not well suited for asteroids of the Atens family, with semi-major axis lower than 1. If the L2 stable manifold is targeted, the NEO should be travelling clockwise in the co-rotating frame. Atens move counterclockwise in such a frame and even if feasible trajectories can be found, these trajectories pass the Earth and require a larger burn after the time of closest approach to change direction and become captured. This is inefficient in terms of Δv, and the orbital elements of the asteroid after the first approach are not considered reliable, if only Keplerian propagation is considered. Figure 14 plots the total capture Δv as a function of the inclination. Atens asteroids, marked with a blue diamond, show significant Δv cost when compared to other asteroids of the same inclination. NEO 2000 SG344 is the bottom left blue diamond with a Δv of 700 m/s, still low. As a possible solution, the same method can be implemented for the stable manifold leading to L1, and asteroids in the Aten and Atira families are expected to be favoured in that type of trajectories.

Another point to note is the high cost of changing inclination. Figure 14 shows that this is in fact the main driver for Δv. The single burn impulsive change of inclination in an Earth like orbit can be estimated as:

\[
\Delta v_i \approx 2V_E \sin (i/2)
\]  

with \(V_E\) being the orbital velocity of the Earth. This estimate is plotted as a red line in Figure 14, and it can be clearly seen that, apart from the Atens, it matches quite well the results for other NEOs. The main reason for this is the use of planar Lyapunov orbits for simplicity, whose manifold trajectories lie in the ecliptic plane with zero inclination. Using other target orbits, namely vertical Lyapunov or halo orbits should reduce the required inclination change and
lower the size of the burns. The results presented are thus conservative in terms of $\Delta v$.

IV. VI. Simplified $\Delta v$ Estimates

It was observed that the main effects of the two impulsive burns were to reduce the inclination to 0, to place the perihelion of the asteroid orbit close to $L_2$ distance, and finally to reduce the aphelion in the cases where it was much larger than 1.15 AU.

On view of the results, a new quick filter which only takes into account the orbital elements of the asteroid can be devised, consisting in the sum of two theoretical $\Delta v$. The first is a hypothetical burn performed at aphelion of the asteroid, which at the same time changes the inclination to zero and adjusts the perihelion. A second burn assumed to be performed at the new perihelion reduces the aphelion to 1.15 AU if necessary. In reality the burns may need to take place at different positions in the orbit, if the aphelion is not at one of the nodes, which would increase the size of the burn. On the other hand, the inclination change cost, which is usually the largest, could be split between the two burns, reducing the total $\Delta v$. These two effects, together with phasing adjustments, may incline the $\Delta v$ estimate in one direction or another, but it is believed that the result can still be used as an effective filter.

Table 5 presents the $\Delta v$ estimates with this new filter, compared to the results of the optimisation for all Amor/Apollo asteroids with $\Delta v$ smaller than 1 km/s. The estimates match quite accurately the optimisation $\Delta v$. A similar symmetric filter can be devised for the Aten family NEOs, in which the aphelion is adjusted to $L_1$ distance and the perihelion raised to 0.85 AU if needed. The table also presents the estimates for all Atens with $\Delta v$ under 1 km/s, providing with a list of candidates for a similar optimisation as the one presented but leading to Lyapunov orbits around $L_1$.

Table 5: Estimated $\Delta v$ for different NEO families. NEOs in bold appeared in three of the filters used in section IV.II, while NEOs in lighter shades of gray appear in just one filter, or none if in italics.

<table>
<thead>
<tr>
<th>Amors / Apollos</th>
<th>$\Delta v_{opt}$ [m/s]</th>
<th>$\Delta v_{atm}$ [m/s]</th>
<th>Atens</th>
<th>$\Delta v_{atm}$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007 UN12</td>
<td>195</td>
<td>169</td>
<td>2010 UJ</td>
<td>392</td>
</tr>
<tr>
<td>2006 RH120</td>
<td>302</td>
<td>301</td>
<td>2000 SG344</td>
<td>402</td>
</tr>
<tr>
<td>2008 E9</td>
<td>325</td>
<td>319</td>
<td>2011 BQ50</td>
<td>452</td>
</tr>
<tr>
<td>2010 UES1</td>
<td>355</td>
<td>316</td>
<td>2011 UD21</td>
<td>579</td>
</tr>
<tr>
<td>2008 UA202</td>
<td>416</td>
<td>376</td>
<td>2009 YR</td>
<td>647</td>
</tr>
<tr>
<td>2009 BD</td>
<td>663</td>
<td>624</td>
<td>2007 YL6</td>
<td>799</td>
</tr>
<tr>
<td>2008 HU4</td>
<td>729</td>
<td>741</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1991 VG</td>
<td>739</td>
<td>726</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010 VQ98</td>
<td>754</td>
<td>760</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001 GP2</td>
<td>828</td>
<td>713</td>
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<td></td>
</tr>
<tr>
<td>2008 JL24</td>
<td>880</td>
<td>678</td>
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</tr>
</tbody>
</table>

V. CONCLUSIONS AND FUTURE WORK

The possibility of capturing a small NEO or a segment from a larger object would be of great scientific and technological interest in the coming decades. It is a logical stepping stone towards more ambitious asteroid exploration and exploitation missions.

This paper has initially shown that the utilisation of asteroid resources may be a viable mean of providing substantial mass in Earth orbit for future space ventures. A statistical population of near Earth asteroids allows us to determine the approximate amount of accessible asteroid resources within a given specific transfer energy. These results devise an energy-cost framework, or resource map, that provides some hints for the future utilisation of asteroids. For example, it shows that there is a reasonable mass of accessible asteroid resources with transfer energies lower than those required to exploit the Moon. Moreover, these resources can be accessed with an incremental level of energy, while lunar resources would require a minimum threshold equal to the Moon’s escape velocity. Exploitation of higher energy transfers may only be justifiable if the required resource is not available on the Moon. The size distribution of near-Earth objects guarantees that most of the exploitable mass could be successfully harvested by only a few mining or capture missions. Small objects with a diameter of order tens of meters to a few hundred meters could potentially be the first targets for strategic resources.

Despite the largely incomplete survey of very small objects, the current known population of asteroids provides a good starting platform to begin with the search for easily capturable objects. With this goal, a robust methodology for systematic pruning of a NEO database and optimisation of capture trajectories through the stable manifold into planar Lyapunov orbits around $L_2$ has been implemented and tested. Five possible candidates for affordable full asteroid retrieval missions have been identified among known NEOs, and the transfers to the $L_2$ region calculated. These transfers enable the capture of bodies with 2-8 meters diameter with low propellant costs. Because of the departure dates, two of them can be attractive targets for capture missions in the 2020-2030 time frame. A new effective filter that at the same time provides an estimate of the required $\Delta v$ has also been introduced.

The proposed method can be easily extended to additional libration point orbits, such as vertical Lyapunov or halo orbits, and to trajectories to the $L_1$ region through the stable manifolds.

The optimisation has been automated to search for transfers of newly detected NEOs once they are included in the MPC database. The same approach
could also be automatically applied to a NEO orbital size and distribution model such as the one described in section III.I, to obtain a statistical estimate of the amount of material that can be captured in such orbits, and to define the optimal region in orbital elements space that could be accessed under a certain \( \Delta v \) threshold.

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