

# Paraunitary Oversampled Filter Bank Design for Channel Coding

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**Abstract**—Oversampled filter banks (OSFB) have been considered for channel coding, since their redundancy can be utilised to permit the detection and correction of channel errors. In this paper, we propose an OSFB based channel coder for a correlated additive Gaussian noise channel, of which the noise covariance matrix is assumed to be known. Based on a suitable factorisation of this matrix, we develop a design for the decoder's synthesis filter bank in order to minimise the noise power in the decoded signal, subject to admitting perfect reconstruction through paraunitarity of the filter bank. We demonstrate that this approach can lead to a significant reduction of the noise interference by exploiting both the correlation of the channel and the redundancy of the filter banks. Simulation results providing some insight into these mechanisms are provided.

## I. INTRODUCTION

The redundancy and design freedom afforded by oversampled filter banks (OSFBs) has in the past been exploited for robustness towards quantisation of subband signals [1], [2], [3], reconstruction of erased or erroneously received subband samples [4], [5], or for the design of error correction coders [6], [7]. More recently in [8] a systematic parallelism between block codes and oversampled filter bank systems for channel coding has been drawn, whereby the system design is based on unquantised “soft-input” signals [9].

The channel coding schemes in [2], [3], [6], [7], [8], [9] are based on an encoding stage using a preset analysis filter bank. The design freedom afforded in the decoding stage formed by the oversampled synthesis filter bank is then utilised to find the solution that reconstructs the signal — either perfectly or in the mean square error sense — while ideally projecting away from the noise. The filter banks in [6], [7], [8], [9] are constructed from FFTs, which leads to low cost implementations, have been shown to be very robust towards burst-type errors, and are easily compatible with OFDM-based modulation system.

If the additive channel noise is correlated, the projection in [8] is performed in the direction of the principal components of the noise subspace, which ideally is restricted such that a noise-free signal subspace exists. Also, in [6], [7], [8], [9] the synthesis design is, despite some degrees of freedom (DOFs) due to oversampling, limited by the a-priori choice of the analysis filter bank. In [10], the synthesis filter bank is given more flexibility by aiming the design at the suppression of the channel noise under the constraint of invertibility, such that an analysis filter bank encoder can be derived from the synthesis bank. However, the filter bank design in [10] is based on a crude iterative method that can prove the potential of the approach but is otherwise far

from optimal.

Therefore, in this paper we follow the channel coding scheme in [10] for a correlated additive Gaussian noise channel, but apply a considerably improved constrained synthesis filter bank design method based on the second order sequential best rotation (SBR2) algorithm [11]. By linking the remaining noise variance after decoding to the covariance matrix of the channel noise in dependency of the synthesis filter bank, a suitable broadband eigenvalue decomposition using SBR2 leads to a paraunitary filter bank design that exploits both the correlation of the channel noise as well as the DOFs provided by the OSFBs.

The paper is organised as follows. Based on a brief description of filter banks in Sec. II, the general channel coding structure is presented. With the aim of minimising the impact of additive channel noise on the decoded signal, we derive a noise power term, which can be utilised as a cost function for the channel coder design. The proposed constrained optimisation scheme for the synthesis filter bank is outlined in Sec. III, which aims to minimise the channel noise power at the decoder output subject to the filter bank being paraunitary and therefore perfectly reconstructing. Some insight into the functioning of the channel coder design is provided by simulation in Sec. IV. Conclusions are drawn in Sec. V.

In terms of notation, vector quantities are denoted by either lowercase boldface or underscored variables, such as  $\mathbf{v}$  or  $\underline{V}$ , while matrix quantities are boldface uppercase, such as  $\mathbf{R}$ . Indexed vectors or matrices refer to quantities with polynomial entries, such as  $\mathbf{H}(z)$ . Finally, a transform pair, such as the Fourier or z-transform, is denoted as  $h[n] \circ \bullet H(e^{j\Omega})$  or  $h[n] \circ \bullet H(z)$ , respectively.

## II. SYSTEM MODEL

Based on the description of basic filter bank structures in Sec. II-A and their polyphase description in Sec. II-B, a model of the proposed encoder and decoder together with the transmis-

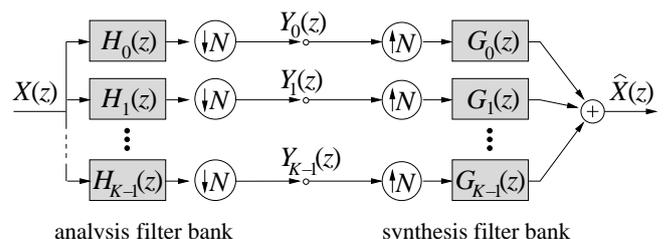


Fig. 1. Subband decomposition of a signal  $X(z)$ .

sion model is discussed in Sec. II-C.

### A. Oversampled Filter Banks

Fig. 1 shows a general filter bank structure comprising of an analysis and a synthesis stage. The analysis filter bank splits a fullband signal  $X(z)$  into  $K$  frequency bands by a series of bandpass filters  $H_k(z)$ ,  $k = 0, 1, \dots, K-1$ , and decimates by a factor  $N \leq K$ , resulting in so-called ‘‘subband’’ signals  $Y_k(z)$ . The dual operation of reconstructing a fullband signal from the  $K$  subband signals is accomplished by a synthesis filter bank, where upsampling by  $N$  is followed by interpolation filters  $G_k(z)$ ,  $k = 0, 1, \dots, K-1$ .

The purpose of oversampling by a ratio  $K/N > 1$  rather than a critical decimation by  $K$  has application specific reasons, and has in the past, for example, enabled subband adaptive filtering techniques for acoustic echo cancellation [12], beamforming [13], [14], [15], or equalisation [16] by permitting independent processing of the subband signals. In these cases, the filters have to be highly frequency selective, and the redundancy introduced through oversampling is located in the spectral overlap region of the filters within the filter bank system.

The redundancy afforded by OSFBs has more recently attracted attention for channel coding [6], [7]. There, a code rate  $N/K < 1$  can ensure robustness against noise interference, with the aim of restoring noise corrupted samples due to the redundant format in which the data is transmitted. The analysis and synthesis filter banks function as encoder and decoder, while the filters  $H_k(z)$  and  $G_k(z)$  are no longer limited to a bandpass design, but will rather be selected according to the characteristics of the interfering noise.

### B. Polyphase Matrices

For implementation and analysis purposes, OSFBs as shown in Fig. 1 are conveniently represented by polyphase analysis and synthesis matrices. The former is based on a type-I polyphase expansion of the analysis filters  $H_k(z)$  [17]

$$H_k(z) = \sum_{n=0}^{N-1} z^{-n} H_{k,n}(z^N) \quad , \quad (1)$$

with polyphase components  $H_{k,n}(z)$ , and a type-II decomposition [17] of the input signal  $X(z)$

$$X(z) = \sum_{n=0}^{N-1} z^{-N+n-1} X_n(z^N) \quad (2)$$

with polyphase components  $X_n(z)$ . This allows us to denote the vector of subband signals,  $\underline{Y}(z)$  as

$$\underbrace{\begin{bmatrix} Y_0(z) \\ \vdots \\ Y_{K-1}(z) \end{bmatrix}}_{\underline{Y}(z)} = \underbrace{\begin{bmatrix} H_{0,0}(z) & \dots & H_{0,N-1}(z) \\ \vdots & \ddots & \vdots \\ H_{K-1,0}(z) & \dots & H_{K-1,N-1}(z) \end{bmatrix}}_{\mathbf{H}(z)} \underbrace{\begin{bmatrix} X_0(z) \\ \vdots \\ X_{N-1}(z) \end{bmatrix}}_{\underline{X}(z)} \quad (3)$$

Therefore, the filter bank can be represented by a demultiplexing of the input signal into  $N$  lines, followed by a multiple-input multiple-output (MIMO) system described by the polyphase analysis matrix  $\mathbf{H}(z)$ . This structure is seen as part of Fig. 2.

Analogously, a polyphase synthesis matrix  $\mathbf{G}(z) \in \mathbb{C}^{N \times K}(z)$  can be defined based on a polyphase expansion of  $G_k(z)$ , yielding the synthesis filter bank representation in Fig. 2 comprising  $\mathbf{G}(z)$  followed by an  $N$  fold multiplexer.

A filter bank system is perfectly reconstructing if

$$\mathbf{G}(z)\mathbf{H}(z) = z^{-\Delta}\mathbf{I}_N \quad . \quad (4)$$

The design of such a system can be demanding in terms of the number of coefficients that need to be optimised. A reduction of the parameter space by, for example, deriving all  $K$  filters from a prototype by modulation [2], [18] or by permitting only symmetric filter impulse responses [18], [4] often makes the problem tractable.

### C. Setup and Channel Coder

The overall model of the considered system is provided in Fig. 2. In the transmitter, the  $N$  polyphase components of  $X(z)$  are encoded by the polyphase analysis matrix  $\mathbf{H}(z)$ . The transmission could either employ  $K$  separate channels as shown in Fig. 2, or multiplex the  $K$  encoder outputs onto a single signal transmitted over a single-input single-output channel. This channel is subject to corruption by additive Gaussian wide-sense-stationary (WSS) noise, and for simplicity is assumed to be non-dispersive.

In the case of a dispersive channel, the model in Fig. 2 can also be applied, if an ideal zero-forcing (ZF) equaliser is employed prior to decoding by the polyphase synthesis matrix  $\mathbf{G}(z)$ . While the channel and the ZF equaliser annihilate each other for the signal path, in the noise path the ZF equaliser can be absorbed into the innovations filter model producing the additive noise components  $W_k(z)$ ,  $k = 0, 1, \dots, K-1$ . This absorption would result in an additional shaping of the channel noise corrupting the  $K$  received signals  $\hat{Y}_k(z)$ , and provide an additional incentive for channel coding that can exploit the spatio-temporal structure of the noise.

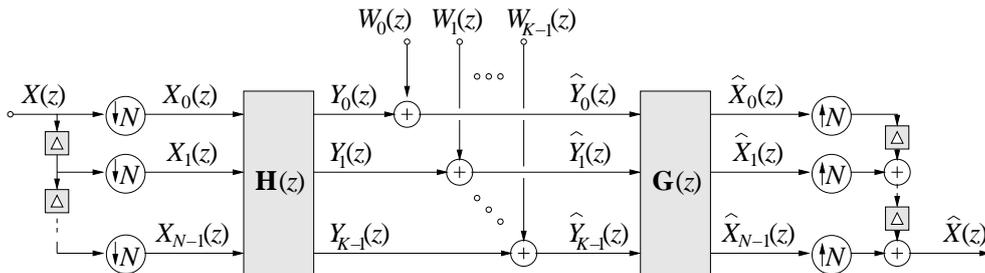


Fig. 2. General setup of a channel coder based on  $K$  channel analysis and synthesis filter banks, arranged around the transmission over  $K$  additive Gaussian noise channels.

In the receiver after decoding, the polyphase components  $\hat{X}_n(z)$  can be collected similar to  $\underline{X}(z)$  in (3) in a vector  $\hat{\underline{X}}(z)$ , which is given by

$$\hat{\underline{X}}(z) = \mathbf{G}(z) (\underline{Y}(z) + \underline{W}(z)) \quad (5)$$

whereby  $\underline{Y}(z) = \mathbf{H}(z)\underline{X}(z) \in \mathbb{C}^K(z)$  and  $\underline{W}(z) \in \mathbb{C}^K(z)$  contain the subband signal components of the transmitted data and the noise, respectively. Selecting perfect reconstruction filter banks  $\mathbf{G}(z)\mathbf{H}(z) = \mathbf{I}_N$ ,

$$\underline{E}(z) = \underline{X}(z) - \hat{\underline{X}}(z) = -\mathbf{G}(z)\underline{W}(z) \quad (6)$$

is obtained.

In order to assess the total received noise variance  $\sigma_e^2$  in  $\hat{X}(z)$ , let the  $N$ -element vector  $\mathbf{e}[m]$  contain the  $N$  time series associated with the  $z$ -domain quantities in  $\underline{E}(z) \bullet \rightarrow \mathbf{e}[m]$ , which depend on the time index  $m$  in the decimated domain. Thus we have

$$\sigma_e^2 = \frac{1}{N} \text{tr}\{\mathcal{E}\{\mathbf{e}[m] \mathbf{e}^H[m]\}\} \quad , \quad (7)$$

where  $\text{tr}\{\cdot\}$  denotes trace and  $\mathcal{E}\{\cdot\}$  is the expectation operator. Defining the auto-correlation matrix

$$\mathbf{R}_{ee}[\tau] = \mathcal{E}\{\mathbf{e}[m] \mathbf{e}^H[m - \tau]\} \quad (8)$$

and its  $z$ -transform  $\mathbf{R}_{ee}(z) \bullet \rightarrow \mathbf{R}_{ee}[\tau]$  denoting the power spectrum of the process  $\mathbf{e}[m]$  [17], the noise variance is given by

$$\begin{aligned} \sigma_e^2 &= \frac{1}{N} \text{tr}\{\mathbf{R}_{ee}[0]\} = \frac{1}{N} \text{tr}\{\mathbf{R}_{ee}(z)\}|_{z=0} \quad (9) \\ &= \frac{1}{N} \text{tr}\{\mathbf{G}(z)\mathbf{R}_{ww}(z)\tilde{\mathbf{G}}(z)\}|_{z=0} \quad . \quad (10) \end{aligned}$$

The notation in (10) uses the parahermitian operator  $\{\tilde{\cdot}\}$ , which applies a complex conjugate transposition and a time reversal [17] to its operand. Note that (6) has been exploited to trace the noise variance back to the power spectrum  $\mathbf{R}_{ww}(z)$ , which is the  $z$ -transform of the covariance matrix of the channel noise,

$$\mathbf{R}_{ww}[\tau] = \mathcal{E}\{\mathbf{w}[m] \mathbf{w}^H[m - \tau]\} \quad (11)$$

with  $\mathbf{w}[m] \bullet \rightarrow \underline{W}(z)$  as defined in Fig. 2.

### III. CHANNEL CODER AND FILTER BANK DESIGN

Based on the idea of the channel coder outlined in Sec. III-A, this section considers a suitable factorisation of the power spectrum at the decoder output in Sec. III-B, admitting a useful coder design in Sec. III-C. An algorithm to construct filter banks achieving this design is reviewed in Sec. III-D.

#### A. Proposed Coding Approach

It is the quantity  $\sigma_e^2$  in (7) which is generally minimised in some sense in channel coding. In [8], for a given  $\mathbf{H}(z)$ , the degrees of freedom (DOFs) in the design of  $\mathbf{G}(z)$  are exploited to minimise  $\sigma_e^2$  in the MSE sense. Note however that this approach limits the DOFs that can be dedicated to fit the synthesis matrix to the spatio-temporal structure of the noise.

Therefore, we proposed to minimise (7) by optimising  $\mathbf{G}(z)$  without restriction by a specific  $\mathbf{H}(z)$ . The only condition placed on  $\mathbf{G}(z)$  is that it admits a right inverse  $\mathbf{G}^\dagger(z)$  such that  $\mathbf{G}(z)\mathbf{G}^\dagger(z) = z^{-\Delta}$ . A stronger restriction than simple invertibility placed on  $\mathbf{G}(z)$  is paraunitarity, which however has two important advantages: (i) the analysis filter banks is immediately given by  $\mathbf{H}(z) = \tilde{\mathbf{G}}(z)$ , and (ii) paraunitarity provides a minimum norm solution such that the transmit power is limited. As a counter example, an invertible  $\mathbf{G}(z)$  might elicit an ill-conditioned  $\mathbf{H}(z)$  which may attempt to transmit highly powered signals over subspaces associated with near rank deficiency.

#### B. Factorisation of the Noise Covariance Matrix

We approach the minimisation of (10) via a factorisation of the power spectrum

$$\mathbf{R}_{ww}(z) = \mathbf{U}(z)\mathbf{\Gamma}(z)\tilde{\mathbf{U}}(z) \quad (12)$$

such that  $\mathbf{U}(z) \in \mathbb{C}^{K \times K}(z)$  is paraunitary and strongly decorrelates  $\mathbf{R}_{ww}(z)$ , i.e.

$$\mathbf{\Gamma}(z) = \text{diag}\{\Gamma_0(z), \Gamma_1(z), \dots, \Gamma_{K-1}(z)\} \quad (13)$$

is a diagonal matrix with polynomial entries  $\Gamma_k(z)$ . This factorisation presents a broadband eigenvalue decomposition, which can be further specified by demanding  $\mathbf{\Gamma}(z)$  to be spectrally majorised [19], [11] such that the power spectral density (PSD) of the  $k$ th noise component  $\Gamma_k(e^{j\Omega}) = \Gamma_k(z)|_{z=e^{j\Omega}}$  evaluated on the unit circle obeys

$$\Gamma_k(e^{j\Omega}) \geq \Gamma_{k+1}(e^{j\Omega}) \quad \forall \Omega \quad \text{and} \quad k = 0, 1, \dots, K-2, \quad (14)$$

similar to the ordering of the singular values in a singular value decomposition. Note that paraunitarity or losslessness of  $\mathbf{U}(z)$  conserves power, i.e.  $\text{tr}\{\mathbf{\Gamma}(z)\}|_{z=0} = \text{tr}\{\mathbf{R}_{ww}(z)\}|_{z=0}$ .

#### C. Channel Coding Design

Using the redundancy  $N < K$  due to oversampling, we can construct  $\mathbf{G}(z)$  from  $\mathbf{U}(z)$  to select the lower (and therefore smallest)  $N$  elements on the main diagonal of  $\mathbf{\Gamma}(z)$ . Let

$$\mathbf{U}(z) = [\underline{U}_0(z) \quad \underline{U}_1(z) \quad \dots \quad \underline{U}_{K-1}(z)] \quad , \quad (15)$$

then

$$\mathbf{G}(z) = \begin{bmatrix} \tilde{\underline{U}}_{K-N}(z) \\ \tilde{\underline{U}}_{K-N+1}(z) \\ \vdots \\ \tilde{\underline{U}}_{K-1}(z) \end{bmatrix} \in \mathbb{C}^{N \times K}(z) \quad , \quad (16)$$

such that  $\mathbf{G}(z)\mathbf{U}(z) = [\mathbf{0}_{N \times K-N} \quad \mathbf{I}_N]$ . If

$$\mathbf{\Gamma}(z) = \begin{bmatrix} \mathbf{\Gamma}_{00}(z) & \mathbf{\Gamma}_{01}(z) \\ \mathbf{\Gamma}_{10}(z) & \mathbf{\Gamma}_{11}(z) \end{bmatrix} \quad (17)$$

with  $\mathbf{\Gamma}_{11}(z) \in \mathbb{C}^{N \times N}$  and the remaining sub-matrices of appropriate dimension, then the noise power at the decoder output becomes

$$\sigma_e^2 = \frac{1}{N} \text{tr}\{\mathbf{\Gamma}_{11}(z)\}|_{z=0} \quad (18)$$

$$= \frac{1}{N} \sum_{i=K-N}^{K-1} \int_0^{2\pi} \Gamma_i(e^{j\Omega}) d\Omega \quad . \quad (19)$$

Therefore, the spectral majorisation in the broadband eigenvalue decomposition (12) is essential to the success of the proposed channel coder design.

#### D. Sequential Best Rotation Algorithm

In order to achieve the factorisation in (12) fulfilling spectral majorisation according to (14), we use the second order sequential best rotation (SBR2) algorithm [11]. In the following, only a brief description of the algorithm is provided, while for an in-depth treatment the reader is referred to [11], [20].

SBR2 is an iterative broadband eigenvalue decomposition technique based on second order statistics only and can be seen as a generalisation of the Jacobi algorithm. The decomposition after  $L$  iterations is based on a paraunitary matrix  $\mathbf{U}_L(z)$ ,

$$\mathbf{U}_L(z) = \prod_{i=0}^L \mathbf{Q}_i \mathbf{\Lambda}_i(z) \quad (20)$$

whereby  $\mathbf{Q}_i$  is a Givens rotation and the matrix  $\mathbf{\Lambda}_i(z)$  a paraunitary matrix of the form

$$\mathbf{\Lambda}_i(z) = \mathbf{I} - \mathbf{v}_i \mathbf{v}_i^H + z^{-\Delta_i} \mathbf{v}_i \mathbf{v}_i^H \quad (21)$$

with  $\mathbf{v}_i = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^H$  containing zeros except for a unit element in the  $\delta_i$ th position. Thus  $\mathbf{\Lambda}_i(z)$  is an identity matrix with the  $\delta_i$ th diagonal element replaced by a delay  $z^{-\Delta_i}$ .

At the  $i$ th step, SBR2 will eliminate the largest off-diagonal element of the matrix  $\mathbf{U}_{i-1}(z) \mathbf{R}_{ww}(z) \tilde{\mathbf{U}}_{i-1}(z)$ , which is defined by the two corresponding sub-channels and by a specific lag index. By delaying the two contributing sub-channels appropriately with respect to each other by selecting the position  $\delta_i$  and the delay  $\Delta_i$ , the lag value is compensated. Thereafter a Givens rotation  $\mathbf{Q}_i$  can eliminate the targeted element such that the resulting two terms on the main diagonal are ordered in size, leading to a diagonalisation and at the same time accomplishing a spectral majorisation.

Hence, each step comprises of optimising the parameter set  $\{\delta_i, \Delta_i, \theta_i\}$ . While the largest off-diagonal element in  $\mathbf{U}_{i-1}(z) \mathbf{R}_{ww}(z) \tilde{\mathbf{U}}_{i-1}(z)$  is eliminated, the remainder of the matrix is also affected. In extensive simulations, SBR2 has proven very robust and stable in achieving both a diagonalisation and spectral majorisation of any given covariance matrix, whereby the algorithm is stopped either after reaching a certain measure for suppressing off-diagonal terms or after exceeding a defined number of iteration [11], [20]. The order  $O_{\text{OSFB}}$  of the filter bank defined by the paraunitary polyphase matrix  $\mathbf{U}_L(z)$  is bounded by  $O_{\text{OSFB}} \leq \sum_{i=0}^L \Delta_i$ . Since the individual delays  $\Delta_i$  are optimised by the algorithm and not known a priori, the filter bank order  $O_{\text{OSFB}}$  cannot be determined or limited priori to applying SBR2 to the power spectral matrix  $\mathbf{R}_{ww}(z)$ .

## IV. SIMULATIONS AND RESULTS

To illustrate the proposed channel coding design, three design examples are demonstrated in the following. The first design assumes an independent transmission across  $K$  sub-channels, while the latter two are based on a time multiplexed transmission leading to correlation between the  $K$  virtual sub-channels.

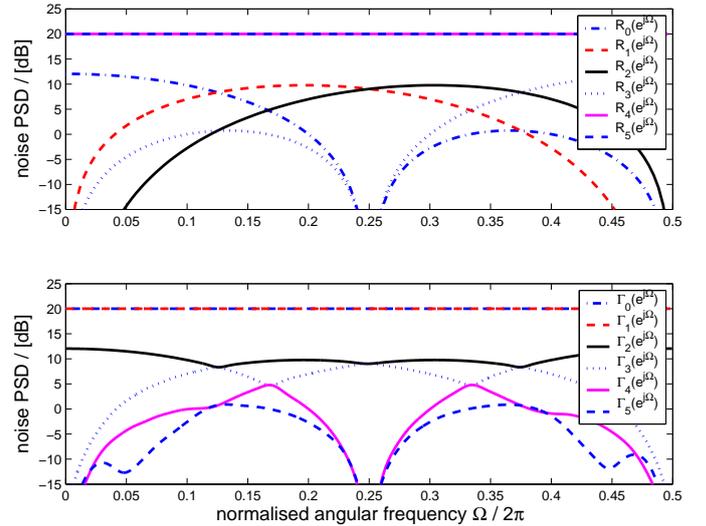


Fig. 3. PSDs on the main diagonals of (top) the power spectrum  $\mathbf{R}_{ww}(z)$  of the channel noise consisting of the  $R_k(e^{j\Omega})$  of (22) and (bottom)  $\mathbf{\Gamma}(z)$  after application of the SBR2 algorithm.

#### A. Multichannel Transmission

We assume the transmission scenario shown in Fig. 2, whereby  $K$  sub-channels are available and are corrupted by Gaussian noise processes  $w_k[m]$ ,  $k = 0, 1, \dots, K-1$ , such that

$$\mathcal{E}\{w_k[m]w_j[m-\tau]\} = \begin{cases} 0 & \text{for } k \neq j; \\ r_k[\tau] \circ \bullet R_k(e^{j\Omega}) & \text{for } k = j. \end{cases} \quad (22)$$

Specifically, for the example below, we assume that  $K = 6$  and that the  $w_k[m]$  are produced by uncorrelated unit variance and zero mean Gaussian processes by passing through innovation filters  $p_k[m] \circ \bullet P_k(z)$  [21],

$$\begin{bmatrix} P_0(z) \\ P_1(z) \\ P_2(z) \\ P_3(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \\ z^{-3} \end{bmatrix} \\ P_4(z) = P_5(z) = 10, \quad (23)$$

such that  $r_k[\tau] = \sum_m p_k[m] p_k^*[m-\tau]$ . The resulting power spectrum  $\mathbf{R}_{ww}(z)$  is a diagonal matrix with PSDs  $R_k(e^{j\Omega})$  as defined in (22) and shown in Fig. 3 (top) on its diagonal.

Prior to running the SBR2 algorithm on  $\mathbf{R}_{ww}(z)$ , its purely diagonal structure must be perturbed through the application of an arbitrary paraunitary matrix. Thereafter, independent of this perturbation, SBR2 achieves a diagonalisation of  $\mathbf{\Gamma}(z)$  after  $L \approx 250$  iterations, whereby a ratio of approximately  $10^{-3}$  between the energy of off-diagonal and on-diagonal terms is reached. However, recall from (17) – (19) that the minimisation of the noise power  $\sigma_e^2$  at the decoder output does not necessitate the diagonality of  $\mathbf{\Gamma}(z)$  but does strongly depend on its spectral majorisation. To examine the latter after convergence of SBR2, the PSDs of the main diagonal elements,  $\Gamma_k(e^{j\Omega})$ , are depicted in Fig. 3 (bottom). Quite clearly, except for a low-power region of the bands  $\Gamma_4(e^{j\Omega})$  and  $\Gamma_5(e^{j\Omega})$  near  $\Omega = \pi$ , spectral

majorisation has been achieved in the sense of (14). Interestingly, the general shape of the PSDs in Fig. 3 (bottom) closely follows those in Fig. 3 (top), but frequency intervals have been re-assigned to different sub-channels and ordered in descending power.

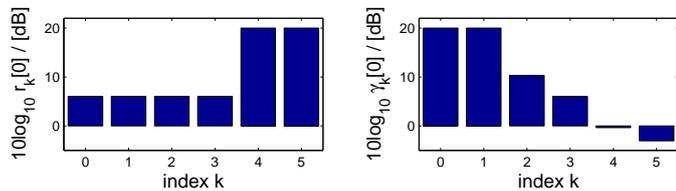


Fig. 4. Variances of (left) uncoded noise,  $r_k[0]$ , and (right) coded sub-channels,  $\gamma_k[0] = \frac{1}{2\pi} \int_0^{2\pi} \Gamma_k(e^{j\Omega}) d\Omega$ .

Integrating over the PSDs in Fig. 3 provides the noise variance of the various sub-channels, which are illustrated in Fig. 4 for  $\mathbf{R}_{ww}(z)$  and  $\mathbf{\Gamma}(z)$  without and with coding, respectively. The coder would then utilise those  $N$  coded sub-channels represented in  $\mathbf{\Gamma}(z)$  that carry the lowest noise power. These  $N$  coded sub-channels convey the  $N$  polyphase components of the transmitted signal  $X(z)$ , which according to Fig. 4 are subject to different levels of noise. Note that the polyphase component transmitted over the lowest sub-channel provides the best protection against noise, while noise introduced on higher sub-channels increases in power. This fact can be exploited for unequal error protection for, for example, high quality high-speed video transmission.

In order to demonstrate how the residual noise power in the decoded sub-channels depends on the order of the filter bank, Fig. 5(top) provides an evolution of the total received noise power in dependence on the number of iterations used for SBR2 and on the number of selected sub-channels  $N$ . If all sub-channels are selected, i.e.  $N = K = 6$ , no redundancy can

be exploited and the total noise power cannot be reduced. For  $N < 4$ , the channel characteristics permit the exploitation of low-noise subspaces, which is achieved through spectral majorisation of the power spectrum due to the filter banks. Note that in Fig. 5 initially a small degradation of the cumulative noise powers for  $N < 6$  with respect to (22) occurs as a result of the random perturbation of the diagonal  $\mathbf{R}_{ww}(z)$  by an arbitrary paraunitary matrix. It is evident from Fig. 5, that the required filter order, and therefore the complexity of the resulting filter bank, depends on the code rate, i.e. the lower  $N$  and hence the higher the oversampling ratio, the more iterations are required to fully exploit the available potential in reducing the output noise power  $\sigma_e^2 = \sum_{k=K-N}^{K-1} \gamma_k[0]$ . The order of the polynomial matrix  $\mathbf{U}_L(z)$ , and therefore the filter bank matrices  $\mathbf{H}(z)$  and  $\mathbf{G}(z)$  after  $L$  iterations is given in Fig. 5(bottom), whereby tails of the filters can be truncated if a lower numerical resolution is sufficient. In the case of channel coding, infinite numerical precision would be wasteful, while quantisation noise is acceptable if its power is well below the level of residual channel noise in Fig. 5(top).

If a decimation factor of  $N = 2$  is chosen for the filter banks, only the two coded sub-channels with the lowest noise variance in Fig. 4 (right) will be utilised. The reduction in noise power results in an SNR enhancement of the coded scheme with respect to a transmission scenario of identical symbol throughput based on maximum ratio combining of the  $K = 6$  channels in Fig. 4 (left) of 7.5 dB. Note that a maximum ratio combiner uses a zero order diagonal  $\mathbf{G}(z)$  and accordingly  $\mathbf{H}(z)$ , with the elements inversely proportional to the standard deviation of the noise in the sub-channels.

Some insight into how the reduction of noise power is gained by the proposed coding method for the case  $N = 2$  is demonstrated in Fig. 6, where the resulting characteristics of a  $K = 6$  channel filter bank decimated by  $N = 2$  are shown. The displayed characteristics refer to the filter bank structure given in Fig. 1, and are plotted against the PSDs of the channel noise after  $N = 2$  fold expansion. Fig. 6 very clearly underlines the functioning of the coder, which effectively excludes the two sub-channels with high noise power from transmission, while in all other sub-channels the transmitted power is concentrated in frequency bands where the noise PSD assumes its lowest values.

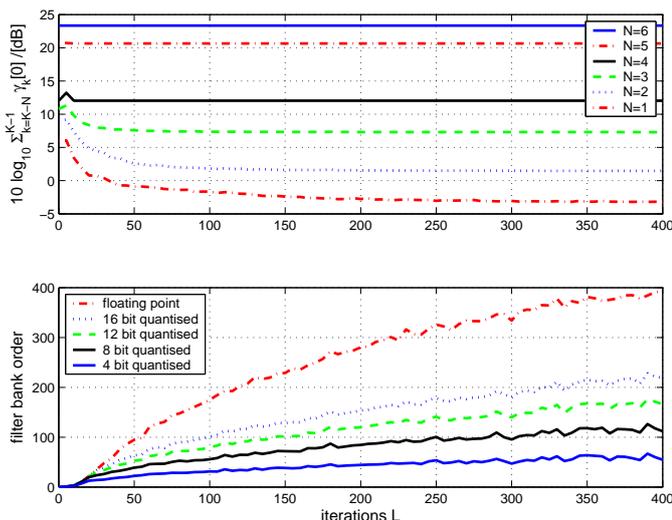


Fig. 5. (top) cumulative variance of  $N$  subchannels containing the lowest noise power after  $L$  iterations of SBR2 and (bottom) filter bank order after  $L$  iterations; the curves are averaged over 50 random trials with different paraunitary matrices perturbing the originally diagonal  $\mathbf{R}_{ww}(z)$ ;  $\gamma_k[0]$  is defined in Fig. 4;

## B. Time-Multiplexed Transmission

In the following we consider the case where the noise in the  $K$  sub-channels in Fig. 2 may be mutually correlated. This can occur through a time multiplexed transmission of the  $K$  encoded symbols over the same channel corrupted by noise  $w[m]$ , which is assumed to be modelled as a unit variance zero mean Gaussian WSS process undergoing an innovation filter  $p[m]$ . Therefore, the auto-correlation function of  $w[m]$  is given by  $r[\tau] = \sum_m p[m]p^*[m - \tau] \circ \bullet R(z)$ . After demultiplexing into  $K$  channels in the receiver, the resulting noise power spectrum  $\mathbf{R}_{ww}(z)$  can be shown to be given by the pseudo-circulant

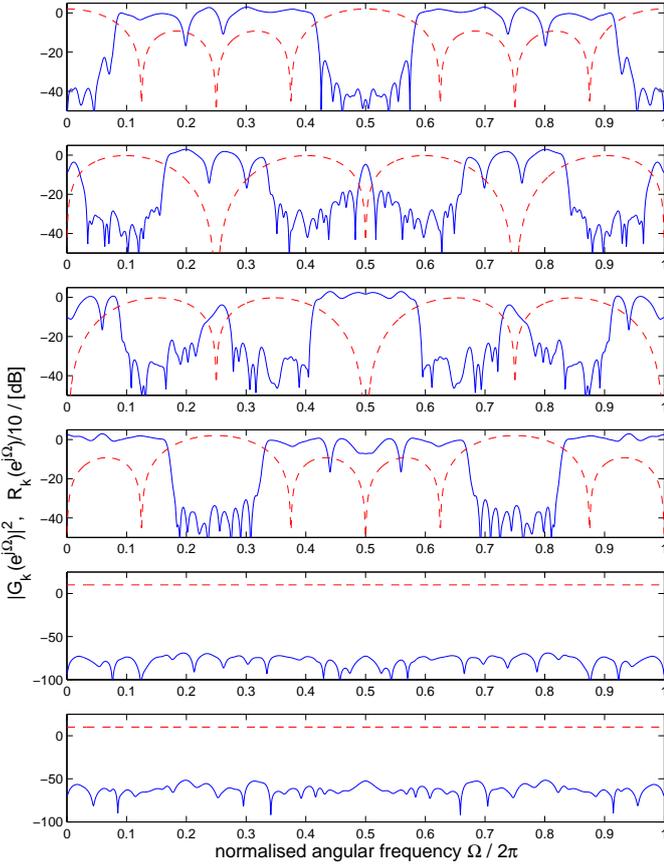


Fig. 6. PSDs of channel noise processes  $w_k[m]$ ,  $k = 0, 1, \dots, K-1$ , decimated by  $N = 2$  (dashed) and magnitude responses of the filters  $|G_k(e^{j\Omega})| = |H_k(e^{j\Omega})|$  (solid).

matrix

$$\mathbf{R}_{ww}(z) = \begin{bmatrix} R_0(z) & R_1(z) & \cdots & R_{K-1}(z) \\ z^{-1}R_{K-1}(z) & R_0(z) & & R_{K-2}(z) \\ \vdots & \ddots & \ddots & \vdots \\ z^{-1}R_1(z) & \cdots & z^{-1}R_{K-1}(z) & R_0(z) \end{bmatrix} \quad (24)$$

containing the  $K$  polyphase components  $R_k(z)$ ,  $k = 0, 1, \dots, K-1$ , of  $R(z)$ ,

$$R(z) = \sum_{k=0}^{K-1} R_k(z^K)z^{-k} \quad (25)$$

*Channel I.* In a first case, the multiplex channel is assumed to be corrupted by an interfering radio signal occupying a quarter of the available bandwidth. The interference is modelled by a zero mean unit variance white Gaussian noise exciting a 49th order bandpass FIR filter, which results in the channel noise PSD shown in Fig. 7. The PSD within each of the sub-channels described by  $\mathbf{R}_{ww}(z)$  for any given  $K$  is identical. Here, different from Sec. IV-A the coder has to additionally exploit the correlation between the  $K$  sub-channels. After application of the SBR2 algorithm, the reduction in noise power — the ratio between the output power of the coder to the power of the channel noise process  $w[m]$  — for various choices of  $K$  and  $N$  is depicted in Fig. 8. In comparison to maximum ratio combining with identi-

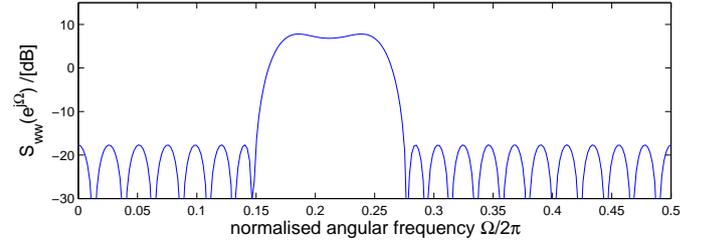


Fig. 7. Channel noise PSD in time multiplex channel II.

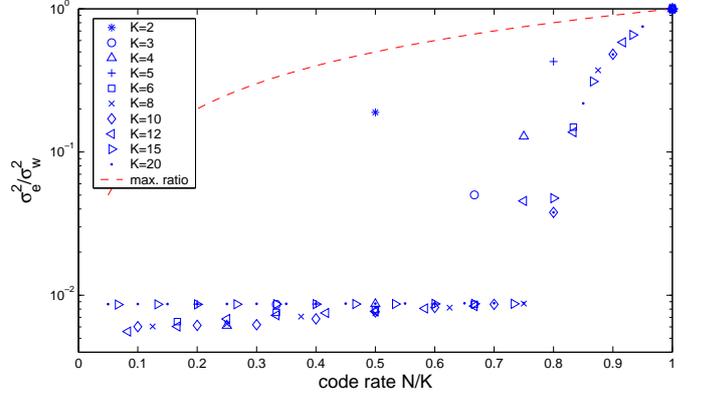


Fig. 8. Noise reduction achieved by the proposed coding scheme over channel II characterised in Fig. 7.

cal symbol throughput, the proposed coder in general performs consistently and considerably better, whereby an increase in  $K$  permits both a finer resolution to exploit spatial correlation as well as the use of more flexible code rates  $N/K$ .

The proposed channel coder can exploit the spectral characteristics of the channel noise well, and, provided a sufficient resolution of the code rate, exhibits an approximately constant output noise power once the code rate reaches the approximately interference-free relative bandwidth of 75% available over the channel.

*Channel II.* We select a power line communication channel (PLC), whose PSD in a worst case scenario can be modelled as [22]

$$S_{\log}(f) = 38.75|f|^{-.72} \text{ dBm/Hz} \quad (26)$$

Sampled at 30 MHz, an iterative least squares fit has been employed to derive an FIR innovation filter with 256 coefficients to produce the PSD characterised in (26) [23]. Applying SBR2 to the resulting noise power spectrum  $\mathbf{R}(z)$ , an example for the resulting spectral majorisation is given in Fig. 9 for a decomposition into  $K = 20$  channels, for which SBR2 yields a 37th order filter bank matrix  $\mathbf{H}(z)$ . The latter is reached with a stopping criterion of  $10^3$  for the ratio between the total power and the power contained in off-diagonal elements in  $\mathbf{U}_L(z)\mathbf{R}_{ww}(z)\mathbf{U}_L^H(z)$ . For this broadband eigenvalue decomposition, a single strong eigenmode of the noise is clearly visible. Therefore, if oversampling is applied and the strongest eigenmodes of the noise subspace can be de-selected from transmission, the noise power in the decoded signal in the receiver can be significantly reduced. The coding gain for the PLC simulation model in (26) is given in Fig. 10 for various selections of channels  $K$ , and compared to maximum ratio combining by

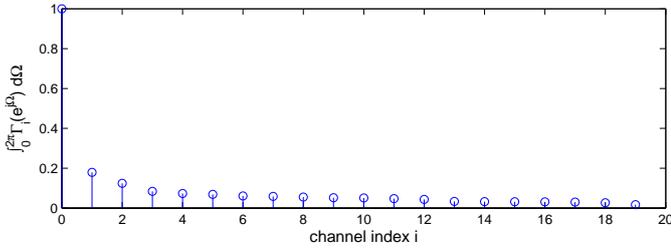


Fig. 9. Spectral majorisation in the decomposition of the noise power spectral matrix  $R(z)$  by SBR2 for  $K = 20$  channels.

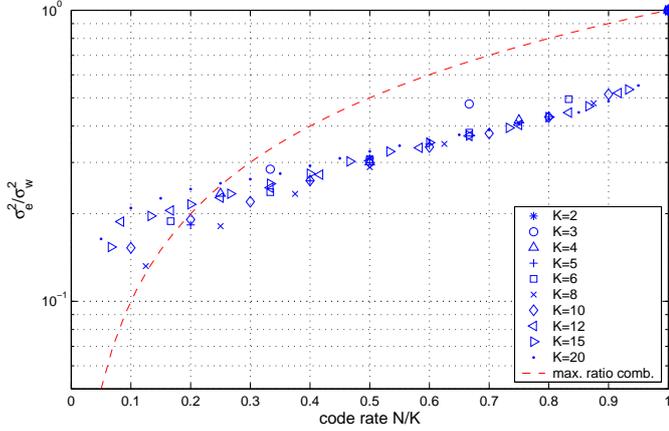


Fig. 10. Coding gain of the OSFB coder applied to the PLC channel defined in (26) for various values of  $K$  and different code rates.

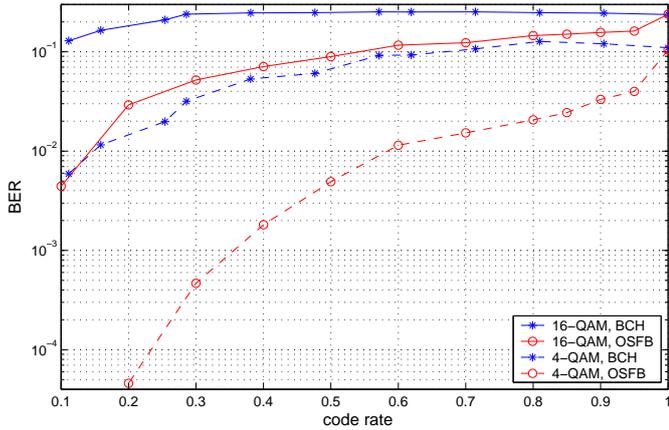


Fig. 11. BER for coding using  $M$ -QAM and OSFB and BCH channel coding, in dependency on the code rate.

repeated transmission of symbols over an otherwise uncoded channel.

Fig. 10 suggests that the OSFB approach can provide considerable coding gain at a high code rate close to unity for the case of highly correlated noise. In order to exploit this,  $K$  has to be chosen sufficiently large in order to offer a high resolution with respect to possible code rates.

In the following we consider transmitting quadrature amplitude modulated (QAM) symbols over the OSFB coded PLC channel. For a channel SNR of 3dB, Fig. 11 presents results for different code rates for a QPSK / 4-QAM and a 16-QAM based transmission.

As a comparison, we also present results for a  $(63, N_{\text{BCH}})$

BCH coded PLC channel, where  $N_{\text{BCH}}$  is varied to achieve various code rates [23]. The BCH encoded bit stream is  $M$ -QAM mapped and transmitted over the PLC channel. In the receiver, after slicing and demapping, a BCH decoder aims to recover the original bit stream. A  $(37,20)$  matrix interleaver, imposing the same processing delay as the OSFB coder, is set to assist in breaking up noise correlation and burst-type errors. Although its computational complexity is higher than the various BCH coders, it is clear that the OSFB coder provides superior protection against correlated channel noise, and almost enables the use of 16-QAM rather than QPSK as opposed to a BCH coder, thus nearly doubling the data throughput without sacrificing error protection.

## V. CONCLUSIONS

In this paper we have proposed a channel coding approach based on OSFBs by first designing a decoder that minimises the influence of correlated channel noise in the receiver, and thereafter obtaining the encoder. By demanding paraunitarity for the decoding OSFB, the latter step is trivial and ensures a strict bound on the transmitted power. An OSFB design method has been proposed, which is based on a broadband eigenvalue decomposition and demonstrates good performance in suppressing the correlated channel noise. Some insight into the effects of the design have been given by considering transmission scenarios over  $K$  independent channels or by time multiplex transmission, where the exploitation of spatial or spectral correlations can bring substantial benefits over a transmission of identical symbol throughput using maximum ratio combining of the sub-channels.

The SNR enhancement gained from the proposed coding architecture can be utilised in conjunction with the transmission of quantised data such as found in binary phase shift keying or multilevel quadrature amplitude modulation symbols, such that the occurrence of symbol or bit errors is reduced. This has been demonstrated by considering a power line communications scenario, whereby the proposed OSFB design can significantly outperform standard channel coding techniques such as BCH, offering a higher data throughput at identical protection against errors.

## VI. ACKNOWLEDGEMENT

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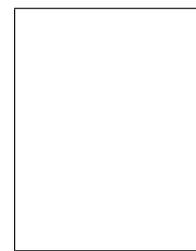
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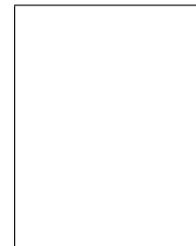
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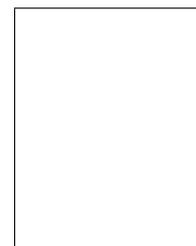


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