

COMPUTATIONALLY LIGHT ATTITUDE CONTROLS FOR RESOURCE LIMITED NANO-SPACECRAFT

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Nano-spacecraft have emerged as practical alternatives to large conventional spacecraft for specific missions (e.g. as technology demonstrators) due to their low cost and short time to launch. However these spacecraft have a number of limitations compared to larger spacecraft: a tendency to tumble post-launch; lower computational power in relation to larger satellites and limited propulsion systems due to small payload capacity. As a result new methodologies for attitude control are required to meet the challenges associated with nano-spacecraft. This paper presents two novel attitude control methods to tackle two phases of a mission using zero-propellant (i) the detumbling post-launch and (ii) the repointing of nano-spacecraft. The first method consists of a time-delayed feedback control law which is applied to a magnetically actuated spacecraft and used for autonomous detumbling. The second uses geometric mechanics to construct zero propellant reference manoeuvres which are then tracked using quaternion feedback control. The problem of detumbling a magnetically actuated spacecraft in the first phase of a mission is conventionally tackled using BDOT control. This involves applying controls which are proportional to the rate of change of the magnetic field. However, real systems contain sensor noise which can lead to discontinuities in the signal and problems with computing the numerical derivative. This means that a noise filter must be used and this increases the computational overhead of the system. It is shown that a time-delayed feedback control law is advantageous as the use of a delayed signal rather than a derivative negates the need for such a filter, thus reducing computational overhead. The second phase of the mission is the repointing of the spacecraft to a desired target. Exploiting the analytic solutions of the angular velocities of a symmetric spacecraft and further using Lax pair integration it is possible to derive exact equations of the natural motions including the time evolution of the quaternions. It is shown that parametric optimisation of these solutions can be used to generate low torque reference motions that match prescribed boundary conditions on the initial and final configurations. Through numerical simulation it is shown that these references can be tracked using nano-spacecraft reaction wheels while eigenaxis rotations, used for comparison, are more torque intensive. As the method requires parameter optimisation as opposed to optimisation methods that require numerical integration, the computational effort is reduced.

Key words: nano-spacecraft, time delay, geometric control, repointing

I. INTRODUCTION

Due to their low cost and short time to launch, CubeSat style “nano” spacecraft have become increasingly common for earth observation missions and trialling new technologies in the space environment. However, these spacecraft have a number of drawbacks: a post launch tumbling motion; lower computational power in relation to larger satellites and limited hardware (e.g. actuators and sensors). As a result, classical control methods are often ill-suited to the control of nano-spacecraft and novel control laws are required. In this paper two novel attitude control methods are presented to tackle two mission phases using zero-propellant and with low

computational overhead. These are (i) the detumbling post-launch and (ii) the repointing of nano-spacecraft. The first method consists of a time-delayed feedback control (TDFC) law which is applied to a magnetically actuated spacecraft and used for autonomous detumbling. The second uses tools from geometric mechanics to construct an analytic description of the natural angular velocities and their corresponding quaternion rotational representation. These are exploited to design zero propellant reference manoeuvres (e.g. for repointing) by optimising the initial angular velocities to match a prescribed final pointing position. The generated motions are then tracked using quaternion feedback control.

On ejection from the launcher small spacecraft can experience a “tumbling” motion (rotating about all 3 axes) with rates of up to 5rpm. This post separation tumbling motion is potentially chaotic and if not stabilised will likely result in failure of the spacecraft: a tumbling motion makes effective communication with ground stations or payload pointing impossible. As such detumbling can be considered the most vital manoeuvre in a spacecraft’s lifetime. Conventional proportional controllers [1], where the control applied is proportional to the spacecraft body rates, are often applied in the detumbling of spacecraft [2], [3]. These laws are simple and effective, however, their implementation relies on accurate data from rate gyroscopes concerning the spacecraft body rates. Rate gyroscopes are susceptible to errors due to noise and gyroscopic drift [4] introducing potentially large errors into the control signals. Where magnetic actuators (magnetorquers) are employed the convention is to use a BDOT control law: that is, a control which is proportional to the rate of change of the magnetic field [5], [6], [7]. This control has been implemented on many real spacecraft, including Compass-1 and RADARSAT, and negates the need for rate gyro inputs in the detumbling control law. However the measurement of the magnetic field is susceptible to noise which can introduce errors in taking the derivative of the magnetic field.

A time-delayed feedback control (TDFC) law is presented here as a computationally light means of stabilising the tumbling motion. Control laws resembling TDFC have previously been used to detumble magnetically actuated spacecraft [7]. However the control appears as a simple approximation of the rate of change of the magnetic field and to date the benefits of TDFC over conventional control laws have not been reported in the wider engineering literature.

The TDFC is based on applying a control that is proportional to the difference between the current value of a variable and its state at a specified time in the past, viz:

$$\bar{u}(t) = -K[\bar{x}(t) - \bar{x}(t - \tau)] \quad (1)$$

Where $\bar{u}(t)$ is the control, $\bar{x}(t)$ is the value of \bar{x} at time t , $\bar{x}(t - \tau)$ is the variable at some time τ in the past and K is a gain matrix. An advantage of this approach is that the delayed trajectory can evolve continuously over time which negates the need for a reference trajectory to be generated. The TDFC method was first proposed by Pyragas [8] and has been successfully applied to a number of applications, including control of chaos in electronic oscillators [9], mechanical pendulums [10] and control of heart conductivity [11]. In addition, TDFC has also been applied to astrodynamics applications [12] and has been shown to tolerate noisy signals well [13], [14], [15].

Re-pointing exploiting natural motions

When the spacecraft has been detumbled, the next

stage in most missions is repointing to a specific configuration e.g. to enable payload pointing and communications. However the limited hardware and software of nano-spacecraft poses significant problems to their repointing. Micro reaction wheels commonly used on nano-spacecraft are typically capable of providing at most 0.01Nm of torque [16]. Meanwhile the low on-board processing power means that any control law must be of low computational intensity in order to be implemented. Therefore in order to design propellant free manoeuvres for nano-spacecraft the references must be trackable with low torque and the generation method must be computationally light.

The repointing of spacecraft has been addressed via classical control methods such as quaternion feedback control and eigenaxis manoeuvres [1], [17]. While these methods are effective they can be torque intensive which makes them unsuitable for nano-spacecraft.

This paper presents a novel way of repointing an axisymmetric spacecraft by revisiting the integrable symmetric Euler equations and exploiting their analytical solution for use in an optimisation procedure. A Lax pair integration is also undertaken on the Special Unitary Group $SU(2)$ in order to analytically define the corresponding quaternion kinematic representation of the motion in matrix form. The resulting analytic formulas are then optimised to match prescribed boundary conditions imposed on the end points. The method consists of two stages: (i) generation of natural motions using tools from geometric control theory [19] and numerical parameter optimisation and (ii) tracking of these reference motions using an augmented quaternion feedback controller [20].

The design of the reference motions takes the form of a constrained optimal attitude control problem. As the method exploits the kinematics and dynamics of the system to create a “natural (or free) motion”, the torque demands are low. Also since parametric optimisation is utilised rather than the computationally heavy pseudo-spectral methods recently employed on-board the International Space Station [21], the method has low computational overhead and so is suitable for use on-board a nano-spacecraft.

Recent advances in geometric control theory have enabled the application of co-ordinate free, global approaches to attitude control. Moreover, geometric control theory has been used to derive co-ordinate free necessary and sufficient conditions for controllability using gas jet actuators and momentum exchange devices for the attitude control problem. These results were used to design co-ordinate free control laws that stabilise the system around an equilibrium state [22]. However the actual motion planning algorithms required for attitude control generally require the introduction of Euler angles and thus only small re-orientations can be planned at any one time, see for example [23]. A co-ordinate free formulation of an optimal control problem was proposed for the attitude motion planning of a spacecraft in Spindler [24]. For a trivial case of constant optimal angular velocities exact

solutions for the corresponding rotations were derived. However, for the more general, time-dependent optimal angular velocities the method reverted to numerical methods involving numerical integration to solve the corresponding motions. The disadvantage of using numerical methods in this setting, in addition to the increased demand on the on-board processor, is that the trajectory is not guaranteed to match the prescribed final orientation and shooting methods have to be employed. Furthermore, this kinematic control problem did not take torque requirement into consideration when planning motions.

In this paper we exploit the natural dynamics of the system to design feasible motions for nano-spacecraft. Exact expressions for the natural angular velocities (free motion) and the corresponding quaternions are derived in closed form using Lax pair integration. As the natural motions can be expressed in closed-form in terms of the initial angular velocities, they can be optimised to match the end-points of prescribed orientations. Theoretically once the initial and final angular velocities can be matched then the torque required during the motion will be zero. In practice it is shown that these motions significantly decrease the torque requirement when compared to traditional eigenaxis rotations. Moreover, the augmented quaternion feedback then illustrates that the designed reference motions are feasible for nano-spacecraft in the near term.

Paper Layout

This paper is presented as follows: first the kinematic and dynamic models used are introduced. The time-delayed feedback control law is then described and a comparison with the conventional BDOT control made under ideal and perturbed conditions. Next the motion planning method is explained, beginning with the alternative kinematic expressions used. The kinematics are expressed as quaternions and equivalently in the less conventional matrix form on the Special Unitary Group $SU(2)$. The quaternion differential equations are better suited for numerical integration as they do not include any imaginary component or singularities and are conventionally used on-board spacecraft. However, the equivalent kinematic formulation on $SU(2)$ enables an elegant formulation of the natural quaternion motions. The exact analytic form of the natural motions for a symmetric spacecraft including their quaternion representations are exploited to generate low-torque reference motions. Moreover, a numerical parameter optimisation method is used to optimise the available parameters of the analytic solution to match the prescribed final pointing direction. This allows the rapid generation of reference motions that theoretically will require zero torque to track provided the boundary conditions on the angular velocities are matched. The effectiveness of the reference motions in terms of torque requirement is demonstrated in simulation through comparison with

conventional eigenaxis manoeuvres.

II. MODELS

The general equations describing the attitude control problem are that of an asymmetric rigid body with external forces describing the effect of the actuators and perturbations. The dynamic equations describe the evolution of the angular velocities while the kinematic equations relate these angular velocities to the angular position of the spacecraft.

Dynamic Model

Euler's rotational equations of motion of a rigid body are defined as:

$$J \cdot \dot{\bar{\omega}} + \bar{\omega} \times J \cdot \bar{\omega} = \bar{N} \quad (2)$$

Where J denotes the moment of inertia matrix of the spacecraft, $\bar{\omega}$ and $\dot{\bar{\omega}}$ the angular velocities and angular accelerations in each axis respectively and \bar{N} the external torques. Defining a body frame originating from the spacecraft centre of mass and coincident with the principal axis of the spacecraft, Euler's Equations reduce to:

$$\begin{aligned} \dot{\omega}_1 &= \frac{N_1 + (J_2 - J_3)\omega_2\omega_3}{J_1} \\ \dot{\omega}_2 &= \frac{N_2 + (J_3 - J_1)\omega_3\omega_1}{J_2} \\ \dot{\omega}_3 &= \frac{N_3 + (J_1 - J_2)\omega_1\omega_2}{J_3} \end{aligned} \quad (3)$$

Where J_1, J_2 and J_3 are the principal moments of inertia of the spacecraft.

Kinematic Model

The attitude kinematics of the spacecraft can be parameterised using quaternions:

$$\frac{d\bar{q}}{dt} = \frac{1}{2}\Omega\bar{q} \quad (4)$$

Where $\bar{q} = [q_1, q_2, q_3, q_4]^T$ denotes the quaternions and $\frac{d\bar{q}}{dt}$ their rate of change and:

$$\Omega = \begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix} \quad (5)$$

The quaternions must satisfy the constraint $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$. The quaternion differential equations are used as they do not suffer from problems with singularities or imaginary numbers.

Magnetic Model

A simple magnetic dipole model from [25] was used, with a rotation and translation applied to model the

offset between the geomagnetic and earth centred reference frames. The components of the Earth's magnetic field vector in the geomagnetic reference frame are given as:

$$\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = -\frac{\mu}{r^3} \begin{bmatrix} 3 \sin \lambda_m \cos \lambda_m \cos \eta_m \\ 3 \sin \lambda_m \cos \lambda_m \sin \eta_m \\ 3 \sin^2 \lambda_m - 1 \end{bmatrix} \quad (6)$$

Where $[\bar{b}]$ is the magnetic field vector in the geomagnetic reference frame, μ represents the Earth's magnetic dipole strength, r is the orbit radius, λ_m is the latitude with respect to the geomagnetic equatorial plane and η_m is the longitude along the magnetic equator. The magnetic field in the body frame, $[\bar{B}]$, can be found via the product of a quaternion rotation matrix $[A]$ [4] and the magnetic field vector in the geomagnetic reference frame, $[\bar{b}]$:

$$[\bar{B}] = [A][\bar{b}] \quad (7)$$

where:

$$[A] = \begin{bmatrix} 1 - 2(q_2^2 + q_3^2) & 2(q_1 q_2 + q_3 q_4) & 2(q_1 q_3 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & 1 - 2(q_1^2 + q_3^2) & 2(q_2 q_3 + q_1 q_4) \\ 2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & 1 - 2(q_1^2 + q_2^2) \end{bmatrix} \quad (8)$$

III. TIME DELAYED FEEDBACK CONTROL LAW

Magnetic actuators (magnetorquers) operate via the interaction between current driven coils and the geomagnetic field. This produces a control torque that is orthogonal to the magnetic field and the magnetic moment of the actuators. A consequence of this is that the control is time-varying and that at any time instant only 2 independent control torques can be applied: the spacecraft is fundamentally under actuated. However over the course of an orbit the variation of the magnetic field enables control in each axis. The control torques, \bar{N} , on a spacecraft with magnetic actuators is given by:

$$\bar{N} = \bar{M} \times \bar{B} \quad (9)$$

Where \bar{M} is the magnetic dipole and \bar{B} the magnetic field in the body frame. The controllable parameter is the magnetic moment \bar{M} . Therefore the TDFC law was designed such that the requested moment \bar{M} would provide control torques to detumble the spacecraft i.e.:

$$\bar{M} = \bar{u}(t) \quad (10)$$

Where $\bar{u}(t)$ is the control signal. The time delay was applied to the magnetic field vector in the body frame, giving the control law:-

$$\bar{u}(t) = -K[\bar{B}(t) - \bar{B}(t - \tau)] \quad (11)$$

Where K is a gain and τ is the delay parameter. This differs from BDOT control as BDOT utilises the rate of change of the magnetic field rather than a delayed signal, giving:

$$\bar{u}(t) = -K[\dot{\bar{B}}(t)] \quad (12)$$

As a delayed signal is tracked with TDFC rather than the derivative of the magnetic field, the control should be more robust to fluctuations in the magnetic field due to noise or perturbations. This was tested via simulation.

Simulations

In order to assess the suitability of TDFC for spacecraft detumbling, a number of simulations were carried out. The spacecraft model was based on that of the UKube-1 spacecraft, with the simplifying assumption of axisymmetry. This gave principal moments of inertia of $J = [0.0109, 0.05, 0.05]^T \text{kgm}^2$. A worst case scenario tip off rate of 0.52 rad/s (5rpm) was applied in each axis. The orbit was a 600km altitude circular orbit giving an orbital rate of $1.083 \times 10^{-3} \text{ rad/s}$ and a period of 5801.06 seconds . The inclination of the orbit was 97.79° and perturbations including magnetic dipole, solar radiation pressure, air drag and gravity gradient were modelled. The maximum requested magnetic dipole was restricted to 0.1Am^2 as in the UKube-1 specifications, and a time delay of 2 seconds was used.

Initially an ideal case without any perturbations or sensor noise was considered in order to assess the basic suitability of the control law. The results are shown in Figures 1 and 2:

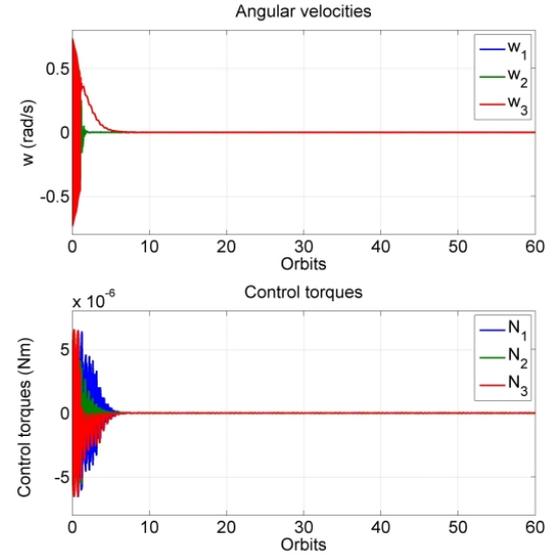


Figure 1: Angular velocities and control torques with TDFC.

With TDFC the spacecraft detumbles in 46500 seconds ($\approx 8 \text{ orbits}$) while with BDOT control it detumbles in around 35000 seconds ($\approx 6 \text{ orbits}$.) Therefore in the ideal case BDOT control outperforms the time-delayed feedback control law.

Sensor noise and perturbations were then introduced to create a more realistic environment in which to test the control laws. Sensor noise was modelled as random Gaussian noise of the order of 10^{-5} Teslas and com-

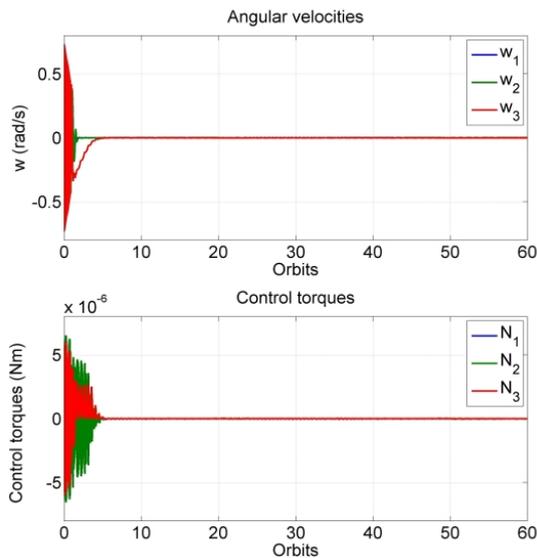


Figure 2: Angular velocities and control torques with BDOT control.

bined with the magnetic field model output. The TDFC law was again compared with the results of the BDOT control as shown in Figures 3 and 4:

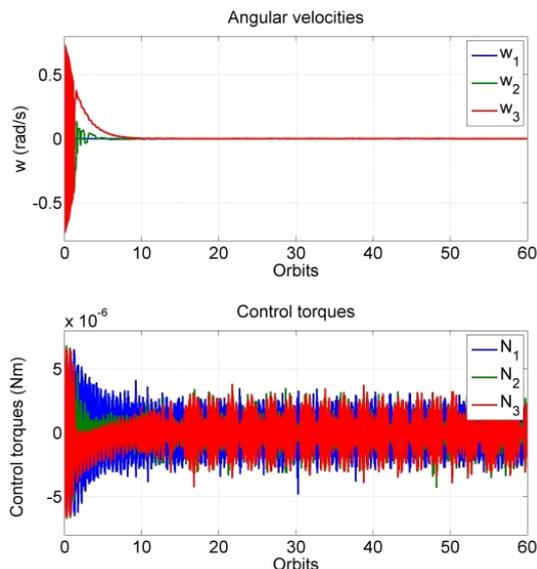


Figure 3: Angular velocities and control torques with TDFC under effect of noise and perturbations.

With perturbations and noise the TDFC law detumbles the spacecraft in 58000 seconds (≈ 10 orbits) while the BDOT control law detumbles in 290000 seconds (≈ 50 orbits.) It is clear that the TDFC law copes better with the inclusion of noise and perturbations. This is because the noisy signal can contain step-like sections which when differentiated (as in BDOT) return either zero or infinite rate of change, introducing large errors in the control signal. Since signal noise is generated by inaccuracies in the spacecraft sensors and is an unavoidable part of real

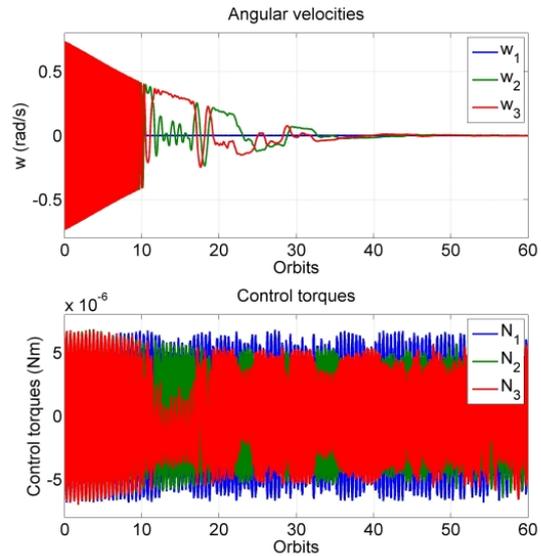


Figure 4: Angular velocities and control torques with BDOT control under effect of noise and perturbations.

system operation, a filter (for example a Kalman filter) must be employed to reduce the effect of the noise. This places further demands on the limited on-board processor and increases the computational overhead. Since the TDFC method negates the need to calculate the derivative of the field and therefore the need to apply a filter, it is shown here that it is a potentially viable method for reducing the computational overhead of nano-spacecraft detumbling controls. The TDFC law also ensures that the spacecraft could be stabilised effectively should the noise filter fail, provided the magnetometer remained functional.

IV. NATURAL MOTIONS

Alternative kinematics

To obtain a global description of the problem the angular position is usually denoted by a rotation matrix in the Special Orthogonal Group $SO(3)$. In addition the angular position may be described locally by parameterising the rotation matrix using Euler angles [22]. However, in this paper we will use both a quaternion representation and an equivalent matrix formulation on the Special Unitary Group $SU(2)$. $SU(2)$ is isomorphic to the unit quaternions [19]. In addition the formulation on $SU(2)$ is used as it is the most natural and convenient for the analytical calculations. The quaternion differential previously described (4) is equivalent to the following kinematic matrix representation on $SU(2)$:

$$\frac{dR(t)}{dt} = R(t)(\omega_1 A_1 + \omega_2 A_2 + \omega_3 A_3) \quad (13)$$

Where $R(t) \in SU(2)$ represents the spacecraft's orientation. A_1, A_2 and A_3 form a basis for the Lie algebra

$\mathfrak{su}(2)$ of the Lie group $SU(2)$:

$$\begin{aligned} A_1 &= \frac{1}{2} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \\ A_2 &= \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ A_3 &= \frac{1}{2} \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \end{aligned} \quad (14)$$

with its commutator defined by $[X, Y] = XY - YX$ called the Lie bracket with $X, Y \in \mathfrak{su}(2)$ such that $[A_1, A_2] = -A_3$, $[A_2, A_3] = -A_1$ and $[A_1, A_3] = A_2$. $R(t) \in SU(2)$ is of the form:

$$R(t) = \begin{pmatrix} z_1 & z_2 \\ \bar{z}_1 & \bar{z}_2 \end{pmatrix} \quad (15)$$

with $z_1, z_2 \in \mathbb{C}$ and \bar{z}_1, \bar{z}_2 their complex conjugates such that $|z_1|^2 + |z_2|^2 = 1$. Physically the basis vectors A_1, A_2, A_3 describe the infinitesimal motion of the spacecraft in the roll, pitch and yaw directions respectively. Furthermore, the two sets of kinematic equations (4) and (13) are equivalent with the isomorphism $F : SU(2) \leftrightarrow \mathbb{H}$:

$$F : \begin{pmatrix} z_1 & z_2 \\ \bar{z}_1 & \bar{z}_2 \end{pmatrix} \leftrightarrow z_1 + z_2j = q_1i + q_2j + q_3k + q_4e \quad (16)$$

Defining the co-ordinate change.

Analytic derivation of the reference motion

The first stage in the derivation of the reference motion is to derive the exact analytical equations describing the free motion of the spacecraft. In this case these are the Euler Equations in the symmetric case i.e. with $J_2 = J_3$. The angular velocities $\omega_1, \omega_2, \omega_3$ for a symmetric spacecraft where the moments of inertia $J_2 = J_3 = J$ are described by:

$$\begin{aligned} \dot{\omega}_1 &= 0 \\ \dot{\omega}_2 &= \beta\omega_1\omega_3 \\ \dot{\omega}_3 &= -\beta\omega_1\omega_2 \end{aligned} \quad (17)$$

where $\beta = \frac{J_1 - J}{J}$. As $\omega_1 = \omega_1(0)$ is constant denote it c and let $\lambda = c\beta$. Then the symmetric equations in their most simple form are:

$$\begin{aligned} \dot{\omega}_1 &= 0 \\ \dot{\omega}_2 &= \lambda\omega_3 \\ \dot{\omega}_3 &= -\lambda\omega_2 \end{aligned} \quad (18)$$

Note by differentiating with respect to time the energy and magnitude of angular momentum are constant:

$$\begin{aligned} KE &= \frac{1}{2} (J_1c^2 + J\omega_2^2 + J\omega_3^2) \\ H^2 &= J_1^2c^2 + J^2\omega_2^2 + J^2\omega_3^2 \end{aligned} \quad (19)$$

Using the total kinetic energy KE we can write:

$$r^2 = \omega_2^2 + \omega_3^2 \quad (20)$$

where $r^2 = \frac{2KE - J_1c^2}{J}$. This suggests using polar coordinates to solve for ω_2, ω_3 :

$$\begin{aligned} \omega_2 &= r \sin \theta \\ \omega_3 &= r \cos \theta \end{aligned} \quad (21)$$

Substituting (21) into (18) gives:

$$\dot{\theta} = \lambda t + \delta \quad (22)$$

At $t = 0$ we have:

$$\begin{aligned} \delta &= \sin^{-1} \left(\frac{\omega_2(0)}{r} \right) \\ \delta &= \cos^{-1} \left(\frac{\omega_3(0)}{r} \right) \end{aligned} \quad (23)$$

or $\delta = \tan^{-1} \left(\frac{\omega_2(0)}{\omega_3(0)} \right)$. Therefore we can write:

$$\begin{aligned} \omega_2 &= \text{sgn}[\omega_3(0)]r \sin(\lambda t + \tan^{-1} \left(\frac{\omega_2(0)}{\omega_3(0)} \right)) \\ \omega_3 &= \text{sgn}[\omega_3(0)]r \cos(\lambda t + \tan^{-1} \left(\frac{\omega_2(0)}{\omega_3(0)} \right)) \end{aligned} \quad (24)$$

The $\text{sgn}[\omega_3(0)]$ term is added to ensure that the arctangent function is in the right quadrant. Note that the original Euler equations (2) can be written in Lax pair form:

$$\frac{dL(t)}{dt} = [L(t), \omega] \quad (25)$$

where:

$$\begin{aligned} L(t) &= J_1cA_1 + J\omega_2A_2 + J\omega_3A_3 \\ \omega &= cA_1 + \omega_2A_2 + \omega_3A_3 \end{aligned} \quad (26)$$

It is well know that the Lax Pair equation and the kinematic equations are connected through the relation:

$$L(t) = R(t)^{-1}L(0)R(t) \quad (27)$$

and it is this relation we use to solve for the corresponding rotations. Equivalently we can write:

$$R(t)L(t)R(t)^{-1} = L(0) \quad (28)$$

where $L(0)$ is a matrix of constant entries and $R(t)L(t)R(t)^{-1}$ describes the conjugacy class of $L(t)$. An initial $R(0)$ can be chosen such that $L(0) = HA_1$. Therefore, it suffices to integrate the particular solution:

$$\bar{R}(t)L(t)\bar{R}(t)^{-1} = HA_1 \quad (29)$$

giving:

$$L(t) = H\bar{R}(t)^{-1}A_1\bar{R}(t) \quad (30)$$

As $\exp(\varphi_1A_1)$ is the stabilizer of A_1 it is convenient to introduce the coordinate form:

$$\bar{R}(t) = \exp(\varphi_1A_1) \exp(\varphi_2A_2) \exp(\varphi_3A_3) \quad (31)$$

and substituting into (30) yields:

$$L(t) = \frac{iH}{2} \begin{pmatrix} \cos \varphi_2 & e^{-i\varphi_3} \sin \varphi_2 \\ e^{i\varphi_3} \sin \varphi_2 & -\cos \varphi_2 \end{pmatrix} \quad (32)$$

Equating (32) with $L(t)$ in (26) yields:

$$\begin{aligned} J_1 c &= H \cos \varphi_2 \\ J \omega_2 + i J \omega_3 &= H e^{-i \varphi_3} \sin \varphi_2 \\ -J \omega_2 + i J \omega_3 &= H e^{i \varphi_3} \sin \varphi_2 \end{aligned} \quad (33)$$

Immediately providing an expression for φ_2 . These expressions can also be simplified to obtain:

$$\varphi_3 = \frac{c(J - J_1)}{J} t + \tan^{-1} \left(\frac{\omega_2(0)}{\omega_3(0)} \right) \quad (34)$$

To obtain an expression for φ_1 substitute (31) into (13):

$$R(t)^{-1} \frac{dR(t)}{dt} = (\omega_1 A_1 + \omega_2 A_2 + \omega_3 A_3) \quad (35)$$

Using the previously derived expressions for the angular velocities and euler angles yields:

$$\varphi_1 = (H(2KE - J_1 c^2) / (H^2 - (J_1 c)^2)) t \quad (36)$$

Substituting φ_2 and φ_3 in from equations (33) and (34) into (31) and pulling the solution back to the identity via:

$$R(t) = R_{int} \bar{R}(0)^{-1} \bar{R}(t) \quad (37)$$

where R_{int} is the initial orientation and $\bar{R}(0)^{-1}$ is the inverse of $\bar{R}(t)$ at $t = 0$ gives the solution on $SU(2)$. Finally, using the isomorphism (16) and comparing the real and imaginary parts yields the quaternion equations in terms of φ :

$$\begin{aligned} q_1 &= -\cos\left[\frac{\varphi_2(0) + \varphi_2(t)}{2}\right] \cos\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] \sin\left[\frac{\varphi_1(0) - \varphi_1(t)}{2}\right] \\ &\quad - \frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) + \varphi_2(0) - \varphi_2(t)}{2}\right] \sin\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] - \\ &\quad \frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t)}{2}\right] \sin\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] \\ q_2 &= \text{sgn}(\omega_3(0)) \left[-\cos\left[\frac{\varphi_1(0) - \varphi_1(t)}{2}\right] \cos\left[\frac{\varphi_3(0) + \varphi_3(t)}{2}\right] \right. \\ &\quad \left. \sin\left[\frac{\varphi_2(0) - \varphi_2(t)}{2}\right] - \frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) - \varphi_2(0) - \varphi_2(t)}{2}\right] \right. \\ &\quad \left. \sin\left[\frac{\varphi_3(0) + \varphi_3(t)}{2}\right] + \frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) + \varphi_2(0) + \varphi_2(t)}{2}\right] \right. \\ &\quad \left. \sin\left[\frac{\varphi_3(0) + \varphi_3(t)}{2}\right] \right] \\ q_3 &= \text{sgn}(\omega_3(0)) \left[-\frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) - \varphi_2(0) - \varphi_2(t)}{2}\right] \right. \\ &\quad \left. \cos\left[\frac{\varphi_3(0) + \varphi_3(t)}{2}\right] + \frac{1}{2} \cos\left[\frac{\varphi_1(0) + \varphi_1(t) + \varphi_2(0) + \varphi_2(t)}{2}\right] \right. \\ &\quad \left. \cos\left[\frac{\varphi_3(0) + \varphi_3(t)}{2}\right] + \cos\left[\frac{\varphi_1(0) - \varphi_1(t)}{2}\right] \sin\left[\frac{\varphi_2(0) - \varphi_2(t)}{2}\right] \right. \\ &\quad \left. \sin\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] \right] \\ q_4 &= -\frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) + \varphi_2(0) - \varphi_2(t)}{2}\right] \cos\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] \\ &\quad + \frac{1}{2} \cos\left[\frac{\varphi_1(0) - \varphi_1(t) - \varphi_2(0) + \varphi_2(t)}{2}\right] \cos\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] - \\ &\quad \cos\left[\frac{\varphi_2(0) - \varphi_2(t)}{2}\right] \sin\left[\frac{\varphi_1(0) - \varphi_1(t)}{2}\right] \sin\left[\frac{\varphi_3(0) - \varphi_3(t)}{2}\right] \end{aligned} \quad (38)$$

Where the $\text{sgn}(\omega_3(0))$ term is added to ensure that the quaternions take on the correct initial sign. With analytical expressions for the quaternions and angular velocities, a parametric optimisation can now be undertaken to find values of the initial angular velocities

such that the spacecraft will follow a natural motion to the target.

Simulations

The feasibility of the references generated were tested through simulation. The motion planning method described in this paper was compared with an eigenaxis rotation between the same start and end points. The eigenaxis manoeuvre takes the form [1] :-

$$\bar{u}(t) = -K \bar{q}_e - C \bar{\omega}_e + \bar{\omega} \times J \bar{\omega} \quad (39)$$

Where \bar{q}_e is the quaternion error, $\bar{\omega}_e$ the angular velocity error and K and C are gains. The spacecraft modelled was again based on the UKube-1 configuration with moments of inertia $J_1 = 0.0109 \text{kgm}^2$, $J_2 = J_3 = 0.05 \text{kgm}^2$. The slew manoeuvre time was set at 110 seconds. In the final 10 seconds the angular velocity reference track was switched to $\bar{\omega} = [0, 0, 0]^T$ in order to complete the rest to rest manoeuvre. Reaction wheels were implemented as actuators and an augmented quaternion feedback controller was used for tracking the reference motion [20]:-

$$\bar{u}(t) = \dot{\bar{h}} - \bar{\omega} \times \bar{h} = -K \bar{q}_e - C \bar{\omega}_e + \bar{\omega} \times (J \bar{\omega} + \bar{h}) \quad (40)$$

This controller contains gyroscopic compensation terms which take into account the dynamics of the reaction wheels while the quaternion feedback control terms enable motion tracking. The gains were tuned until the minimum torque manoeuvre which satisfied the boundary conditions was found.

The results of large slew manoeuvres using the free motion tracking and eigenaxis rotation methods are presented here. The selected manoeuvre was from $\bar{q}_i = [0, 0, 0, 1]^T$ to $\bar{q}_f = [0.5, 0.5, 0.5, 0.5]^T$. The results of the free motion reorienting manoeuvre are presented in Figures 5, 6 and 7:

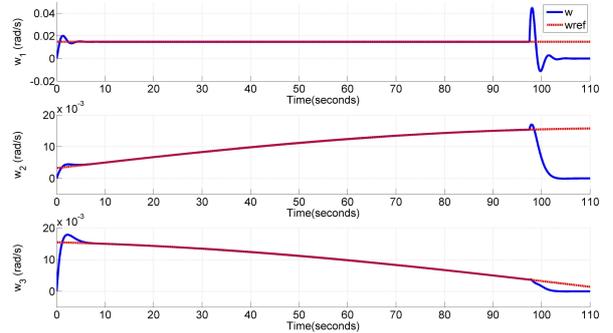


Figure 5: Angular velocity tracks using free motion planner.

It is evident that once the angular velocities match that of the natural motion minimal torque is required to maintain the reference track. The maximum torques occur at the beginning of the manoeuvre to drive the spacecraft onto the reference track and at the end of the

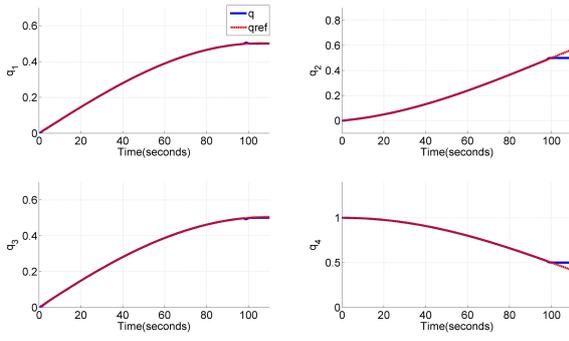


Figure 6: Quaternion tracks using free motion planner.

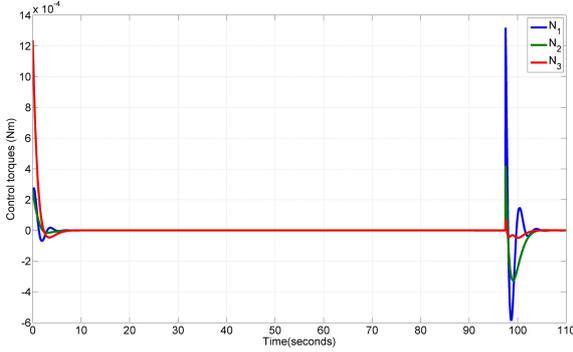


Figure 7: Control torques using free motion planner.

manoeuvre to bring the spacecraft to rest. The maximum torque required is of the order of $10^{-3} Nm$, well within the limits of existing reaction wheel technology. The absolute value of the torque in each axis was found to be $[0.0195, 0.0152, 0.0243]^T Nm$. The results of the eigenaxis manoeuvre are shown below in 8, 9 and 10:

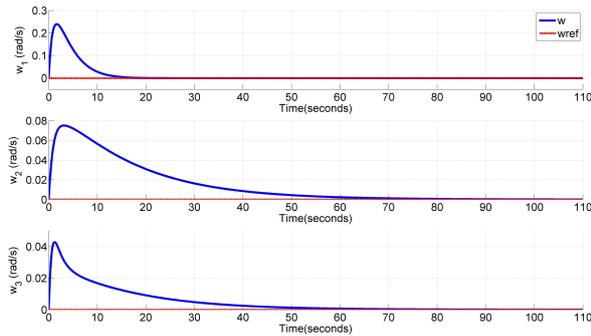


Figure 8: Angular velocities during eigenaxis manoeuvre.

In the case of the eigenaxis rotation the control torques are an order of magnitude larger. Summing the torques gives absolute values of $[0.1013, 0.1252, 0.0973]^T Nm$. The natural motion method therefore provides a considerable reduction (82%) in required torque. Since the control torques are only required at the start and end of the manoeuvre, fewer control signals are sent to the actuators. This

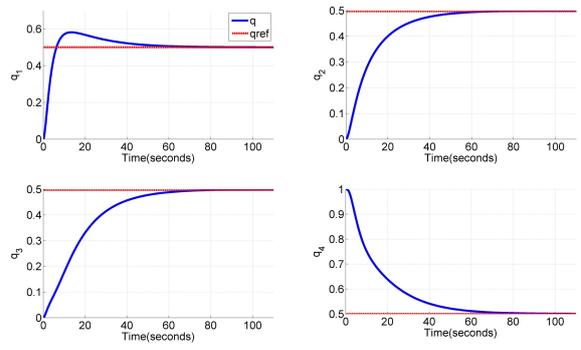


Figure 9: Quaternions during eigenaxis manoeuvre.

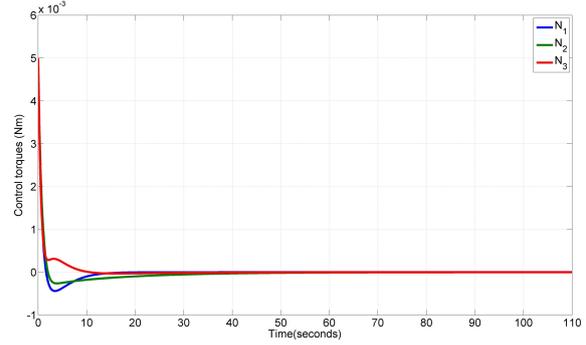


Figure 10: Control torques during eigenaxis manoeuvre.

combined with the fact that the method requires no numerical optimisation means that the method can be considered to be computationally light as well as low torque, and is therefore suitable for implementation on a nano-spacecraft.

V. CONCLUSIONS

In this paper two computationally light attitude control methods for nano-spacecraft have been described. The first, based on time-delayed feedback control (TDFC), was shown to be suitable for the autonomous detumbling of a magnetically actuated spacecraft. The control law was compared to the conventionally used BDOT control and was found to be robust as noise and perturbations did not significantly affect the detumbling time of the spacecraft. However BDOT control performed poorly under these conditions. Since TDFC negates the need for a noise filter the method has low computational intensity. The second method involved repointing of the spacecraft through the planning of natural motions. This was achieved by exploiting the analytical solutions of the Euler equations and using a Lax pair integration on the Special Unitary Group $SU(2)$ in order to analytically define the corresponding quaternion kinematic representation. The resulting analytic formulas were then optimised to match prescribed boundary conditions. The method was compared via simulation to a conventional eigenaxis manoeuvre and was found to require significantly lower

torques thus reducing the computational load on the actuators. Future work will concentrate on proving the stability of the time-delayed feedback method and on the extension of the motion planning method to include constrained attitude slews e.g. repointing to a desired configuration while avoiding some specified "forbidden zones." A comparison will also be drawn with other repointing methods for axisymmetric spacecraft, including numerical optimisation using pseudospectral methods and artificial potential functions. The method will also be adapted for the more general case of an asymmetric spacecraft.

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