CONTROLLABILITY OF BUILDINGS, A multi-input multi-output stability assessment method for buildings with fast acting heating systems

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Abstract

The paper describes a methodology to assess the controllability of a building and its servicing systems, such as heating, lighting and ventilation. The knowledge for these methods has been transferred from design processes and methods used in the design of aircraft flight control systems to establish a modelling and design process for assessing the controllability of buildings. The paper describes a holistic approach to the modelling of the nonlinear and linear dynamics of the integrated building and its systems. This model is used to analyse the controllability of the building using Nonlinear Inverse Dynamics controller design methods used in the aerospace and robotics industry. The results show that this design approach can help the architects in their decisions on which building design and services to use. Furthermore, the results demonstrate how the same method can assist the control systems designer in developing complex control systems especially for buildings designed with a Climate Adaptive Building (CAB) philosophy.

*Keywords*: Buildings, Systems, Modelling, Controllability, MIMO

1. Nomenclature

CAB = Climate Adaptive Building, SISO = single input single output

RIDE = Robust Inverse Dynamics Estimation

MIMO = multiple inputs and multiple outputs

MV = Mechanical Ventilation, NV = Natural ventilation, PSV = Passive stack ventilation

 – heat transfer rate (W)

T – temperature (K)

U – heat transfer coefficient (W/m2K)

A - area (m2)

cp – specific heat capacity (J/kgK)

V – volume (m3)

t – time (s)

d – differential operator

ke - Proportion of light power converted to heat.

P– Electrical power into lights (W)

np – number of occupancy

Gpp – heat generation rate per person (kg/s)

n – air change rate (s-1)

k - constant

I – solar radiation (W/m2)

W – humidity transfer rate (kg/s)

*v* – wind speed (m/s)

q – volume flow rate (m3/s)

Symbols:

ρ - Density of (kg/m3)

α – fraction of transmitted solar that goes into the element

σ – transmissivity

β – fraction of the plant heat transfer into the air by convection

Subscripts:

a – air

w – wall

s - solar

sa – sol-air

d – internal humidity gain

L – lights

oc – occupancy

ap – appliances

win – window

m – internal mass

nt – buoyancy

nv – wind pressure

ni – infiltration

mv – mechanical ventilation

p – plant

o – external

t – thermal

v – wind

i – infiltration, internal

pr – radiation component of the plant

pc – convection component of the plant

dr – direct

cm - comfort

2. Introduction

It is acknowledged that complex heating, ventilating and air conditioning systems in today’s Climate Adaptive Building present one of the most challenging situations to deal with from the point of view of control. In reality, building services engineers face huge problems in commissioning these complex building management systems to work reliably all year round. The actuator systems installed in a climate adaptive building will have different response characteristics and physical limits [1]. The states that are being controlled in the building will have different time constants. Depending on the requirements and application, the final decision is normally based on the required performance in terms of actuator bandwidth [2]. However all these actuation systems are highly nonlinear and the effective bandwidth of the actuation system can vary significantly with the amplitude of the input signal. The cause of this variation is normally very simple, in wet systems there is a maximum flow rate and temperature which can be achieved. Further more in D.C. motor drives (solar blinds) there is a maximum rate which is a result of having to limit the motor current to prevent damage of the motor windings. Similarly in systems such as under-floor heating the floor temperature is a restriction to prevent discomfort for the occupants walking on the floor. These complex mix of non-linearities, control algorithms, building physics, fast & slow actuator systems and disturbances (i.e. weather etc.) requires a science that can fundamentally show the factors affecting the controllability of buildings. The engineering science presented in this paper is based on ‘A Perfect Control Philosophy’ [1, 3-5]. This philosophy aims to establish for a given design, if perfect control is feasible whilst maintaining stability for the closed loop control system. Controllability of slow heating systems has been assessed previously [1] and this paper is on controllability of fast heating systems.

3. Mathematical model

The proposed 4th order model in this paper is specifically developed to test the controllability of a nonlinear multivariable system with focus on fast heating systems. The model was derived from the model originally developed by Counsell et al [1]. The dynamic model describes the energy and mass balance of air in a schematic building zone at the conceptual design stage having heating and ventilation.



Figure 1 A basic building schematic at the conceptual design stage

The reason for proposing the 4th order reduced model is because the symbolic controllability analysis of the 8th order model proposed in [1], showed that the transmission zeros corresponding to the walls are all negative and will always remain negative. Thus there is no need for analysing the individual walls for their stability and can be treated as one single external thermal mass. The CB matrix analysis also showed that the lighting has a very small coupling with the temperature control due to its zeroth order dynamics and thus can be controlled accurately with a simple Proportional + Integral controller (PI). Thus the real coupling as showed in [1] was between the heating and ventilation controls. In this paper the controllability analysis of a fast heating system for controlling temperature and mechanical ventilation of control humidity is presented. It is also shown that fast heating system with mechanical ventilation will be a better combination and thus here a fast heating system is assumed for controlling air temperature.

The 4th order model lumps the walls, floor and roof into a single external thermal mass with two resistance layers i.e. internal and external. This leads to a reduced model having four equations as follows:

1. Air temperature
2. External thermal mass: walls + floor + roof
3. Internal thermal mass: furniture + internal walls
4. Humidity level

The temperature of the zone is modelled as a single node representing an average temperature of the zone based on the work in [1]. This results in simplification stating that the air is fully stratified.

 (1)

Solar radiation: 

Lighting: 

Occupancy: 

Appliances: 

External thermal mass: 

Window: 

Internal thermal mass: 

Air change rate due to thermal forces: 

Air change rate due to external wind: 

Air tightness (infiltration): 

Mechanical ventilation: 

For fast actuation systems it was assumed that majority of the energy is directly input or taken out from the air with some energy transfer to the internal and external thermal mass. The model of a simple fast acting radiator was developed by [7]. This model was used for representing a fast acting heater and cooler. The heat transfer can be expressed as:

 (2)

For a given type of radiator or cooler, the proportion of convection and radiator can be treated as constant [7]. Therefore it is reasonable to assume:

Convection to zone air:  (3)

Radiation to the internal thermal mass:  (4)

The rate of change of wall temperature is given by:

 (5)

Where heat exchange with:

Outside: (6)

Inside: (7)

In the original model [1], an overall heat transfer coefficient was assumed. In this case-study it is also investigated the effect of the two layers of insulation, inner and outer. Thus an inner and outer U values are considered in modelling the external thermal mass.

Direct solar radiation through the window:  (8)

N.B: For heat transfer between external surroundings and the wall, the external temperature is taken to be the sol-air temperature. This allows for simpler treatment of the effect of the solar radiation and sky heat exchange with the wall.

The rate of change of thermal mass temperature is given by the following differential equation:

 (9)

Where the heat exchange with:

Air: (10)

Solar gain through the window: (11)

The direct solar gain to the walls and internal mass was excluded in [1] however for completeness it is included in this paper. The differential equation water balances on the interior volume of the building are as follows [8]:

 (12)

Where,

Wd is internal humidity gain (kg/s).

Wnt is humidity loss by thermal buoyancy:  (13)

Wnv is humidity loss by wind pressure:  (14)

Wni is humidity loss by natural air change rate:  (15)

Wmv is humidity loss by mechanical ventilation:  (16)

A feedback control system can only control (i.e. track) what it feeds back as measured system outputs. Thus, to analyse the controllability of these measurements, they must be defined and are as follows:

Measured air Temperature level is given by: Tcm = Ta

Measured Humidity level is given by: Wcm = Wa,

In order to apply the aerospace controllability science [4-5] the model needs to be represented in the state-space form [1-2]:

 (17)

 (18)

The vectors associated with these matrices are given as follows:



5. Controllability

The controllability can be assessed in terms of fundamental controller design properties, CB matrix [1], transmission zeros [2, 10] and Utrim safe criterion [3-4] (reachability).

(CB) Matrix

It was shown in [1] that with a slow acting heating system i.e. under floor heating, when air temperature Ta is controlled for achieving the required comfort temperature i.e.Tcm = Ta, the matrix (CB+sD) is not of full rank (i.e. not invertible). This indicated that perfect control of air temperature with slow heating systems is not feasible and thus extra measurements from the sensors are needed for perfect control. It was also shown that the rate of change of measured output feedback sensors i.e. feeding back rate of change of temperature resulting in a full rank CB+SD matrix which allowed for simple controls to be feasible. The findings in [1] proved that for perfect controllability of highly cross coupled heating and ventilation systems using simple controllers requires the heating system to be fast. Otherwise complex nonlinear MIMO controllers will be needed for stabilising the system and making it controllable [1]. For the system discussed in this paper the CB stability matrix is given as follows:

 (19)

The constants shown in the matrixes above are functions of building parameters and operating conditions as follows:

 (20)

The CB matrix shows that there is cross coupling between temperature and humidity control through mechanical ventilation (b12). The mechanical ventilation will affect both humidity and temperature. Whereas perimeter wall heating will only affect temperature. The reason is that heat losses due to evaporation of the water in the zone and the rate of moisture transfer from this evaporation emanate from the water contained in the contents of the zone. These factors have little influence on the thermal and moisture processes of the building indoor climate and can be considered as disturbances [8]. These factors are functions of the inside temperature and humidity. Hence direct heat input to the air node does not directly affect the water vapour content. The cross coupling of temperature is determined by the difference between operating indoor and outdoor temperatures (constant: b12). In summer this cross coupling will be smaller as temperature difference will be less. Thus in summer this will be a nearly decoupled system where each control channel can be easily controlled using simple PID controls. Where as in winter the cross coupling will be stronger as temperature difference will be larger and thus in winter the mechanical ventilation will be a significant disturbance to the heating system.

It is clear that there is cross coupling between temperature and humidity controls via the mechanical ventilation. However, as can be seen, the coupling between the temperature and CO2 controls is only one way, i.e. change in temperature control has no effect on humidity control directly. This is similar to the case study of slow systems [1]. However the difference in the case study in this paper is that due to the fast heating system the coupling is not due to rate of change of ventilation rate as was shown for slow systems [1].

The RIDE theory [3-5] states that the asymptotes for a multivariable design are given by Eigen-values:  where, g is the global gain from zero to infinity and σ is a scalar gain. Therefore the asymptotes are the solutions for s of the following determinant:

 (21)

 (22)

 (23)

Hence, the asymptotes are defined by the following building parameters:

, (24)

The asymptote for air temperature is negatively aligned and will result in stable fast transient temperature response without oscillation to reach steady state i.e. temperature control is stable and controllable.

 (25)

The asymptote for humidity is positively aligned and will result in unstable transient response increasing in the right half plane i.e. humidity control is unstable and uncontrollable. As can be seen from the terms this is caused by the negative control b42. To realign this asymptote to the negative real axis the gain σ2 will have to be made negative.



Figure 1: Asymptote directions on a root locus for a closed-loop system.

The asymptotes are a function of the operating points such as temperature difference between internal & external humidity levels. It is also interesting to note that the asymptotes are not at an angle and thus variation in the operating points will not affect the controllability as the asymptotes will always be horizontal and negatively aligned. Two fundamental conclusions can be drawn from this analysis as follows:

1. Easier to deploy high gain and high performance controls

2. Classical single input single output (SISO) controllers such as Proportional plus Integral control (PI) could be sufficient, but could be difficult to achieve fast and accurate control since asymptote are a function of operating points.

Transmission zeros

The transmission zeros determine how well the system can be stabilised when using high gain feedback control [1-3] and inverse dynamics. The linear time-invariant state-space model equations (17-18) were used for the calculation of transmission zeros and their stability analysis over the operating range. The transmission zero locations are determined by the determinant (26) [10] and can be readily solved using control system analysis software.

 (26)

When the matrices are substituted the determinant is given as follows:

 (27)

The determinant matrix can be reduced to:

 (28)

The equation is solved to find the values of *s* which are the transmission zeros. In this case study, there are two transmission zeros, which correspond the external and internal thermal mass and are given by:

 (29)

The transmission zeros are:

 (30)

 (31)

The transmission zeros are both in the left half plane and thus stable. Again they are a function of constants and thus will always remain stable. In this case they are not a function of operating conditions such as outside temperature, but are a function of thermal properties; area, mass and thermal capacitance and U values of the internal and external thermal mass.

The thermal transmittance of the walls, (the U-value) is generally required to be as low as economically possible; for purposes of energy costs saving, emissions reduction and to comply with the building regulations. If this is to be the case then the results of the transmission zero analysis (which proved that the location of the zero is a function of the heat transmitted through the fabric and the heat stored in the fabric) suggest that walls must have low thermal mass for to ensure speedy reaction to control inputs. This implies that in an Adaptive Climatic Building, reducing thermal mass is important to responsive control.

This is in contrast to the ‘passive’ approach to the design of buildings, where thermal mass is used extensively for load levelling and heat storage [11]. The results indicate that we can expect such ‘heavyweight’ buildings to be sluggish and respond slowly to control inputs. For passive buildings, this tends not to be a problem, as control of these buildings is likely to be minimal anyway. For actively controlled, adaptable buildings as are considered in the case studies, the implications of these findings are of great importance. Robust, responsive control is desired in Adaptive Climatic Buildings and it is clear that the choice of materials, and therefore the amount of thermal capacitance in these systems and the impact of this thermal mass must be given proper consideration in the design process. This analysis will allow the responsiveness of the proposed building to be quantified at an early stage and could allow for the cost effective evaluation of the impact of different material choices.

In [1], the reachability was not analysed as the model was too large for the symbolic analysis. In the light of the findings of the slow systems case study, the model was reduced and this model is just small enough for reachability analysis to be carried out symbolically.

Reachability: (Tracking limitations and Safe operation criterion)

It is shown that a good estimate of the Utrim(t) enables the control system to accurately track the reference input. For a feedback control system steady state tracking [4] of a constant reference input v (0) can be expressed as,

 (32)

If this condition is satisfied then the tracking condition will be satisfied even in the presence of a disturbance vector d (t). It is clear that the above condition depends on what is being measured? Therefore it is important that what is measured in approaches the desired output y (t) in steady state.

For a complete representation of a system given by the following state space equations:

  (33)

 (34)

The RIDE theory can be applied to derive the equivalent control Utrim(s) for the above system and is given by:

 (35)

The input Utrim(t) uses inverse dynamics to determine the actuator inputs that are required to ensure zero rate of change of the outputs. It does this by taking into account the disturbances and system dynamics that would otherwise prevent the system from operating like an ideal integrating system. This Utrim(t) term however, necessitates the measurement of all state variables and disturbances, as well as plant parameters which are typically unknown. Thus, in reality, the implementation of the equivalent control term is often ignored as it is viewed impractical and the rapid estimation of Utrim(t) has been shown to be achievable through the implementation of the RIDE controller design methodology [3-4]. However in this paper it is shown that the rapid estimated expression is used for deriving the safe criterion and the original expression can be derived symbolically for a simplified system to assess the safe operation of the control system. The safe criterion can be derived as follows:

If assumed, and this is substituted into Eq. (33-34). Then if the output Equation is differentiated (rate of change of output) and the state equation is substituted into it. Then a Laplace transform is taken to give a transfer function from input to output and from disturbance to output as follows:

 (36)

If  is substituted back into Eq. 36, and the terms u(s), y(s) are taken on one side of the equation then the other side of equation is the same Utrim(s) assumed in Eq. (35). Thus, the rapid estimated of Utrim(s) is extended for a system in equations 33-34 and is given by:

 (37)

Making sy(s) the subject gives:

 (38)

The safe operation of the control system under disturbances and actuator nonlinearities is governed by the safe criteria for Utrim [3-4]. The presence of limitations on the power of the system’s actuators results in limitations on maintaining stable tracking. In order to better understand this it is useful to derive a criterion which describes the tracking limits. When the actuator output, ua(s), has reached its upper (LU) or lower (LL) limits, the control signal to the actuator, u(s), must either remain constant or decrease in order to avoid overdriving the actuator and to maintain safe control [3-4]. This can be expressed as;

When  (39)

First consider a generic feedback control system [1] for which the control law is given by:

 (40)

It follows that if equation (40) is differentiated and equation (38) is substituted in it, then the criterion stated in eqn. (39) is as follows:

 (41)

Dividing throughout by *u*(*s*) and K(s)(CB + sD) two criteria are formed:

When  and 

 (42)

When  and 

 (43)

By combining these two expressions, a single criterion can be formed as follows:

(44)

This criterion states that providing the closed-loop response is fast i.e. K(s) is relatively large compared with rates of change of Utrim(s) and that rate of change of r(s) is made small then the limitation on tracking is that Utrim must remain between the upper and lower actuator limits.

 (45)

This criterion demonstrates that, when actuator limits are reached, safety can be ensured simply by setting the error signal to zero, providing that steady state is reachable (i.e. that the Utrim is within limits). This condition for tracking can be readily inspected using dynamic simulation. Now the original expression for Utrim can be symbolically expanded to see which properties of the building and its control system affect the value of Utrim from staying between the actuator limits. This expression for Utrim can be reduced for the system presented in this paper where D and E matrices are zero and is given by:

 (46)

The matrices can be expanded and multiplied to give:



 (47)

Simplifying further gives:

=

 (48)

The vectors can be multiplied and the two Utrim equation of temperature and humidity control are given as follows:



(49)

 (50)

Further rearranging gives:

 (51)

 (52)

As show above the Utrim of temperature control is also a function of Utrim of the humidity control. The different constants can be substituted for assessing Utrim.

 For temperature:



 (53)

This can be further reduced to:



 (54)

For temperature control the Utrim1 equation shows that the value of the Utrim depends on the difference between the sum of temperature differences and sum of casual gains. Hence in seasons where the temperature differences are small then casual gains will be the factor determining whether cooling is required or not. Thus in this case the equation would simplify to:

 (55)

Hence the important terms in this are the casual gains i.e. solar, lighting, occupancy and appliances. Any significant casual gains will make the Utrim negative. Here it is important to note that negative limit of the plant is cooling limit and positive limit is the heating limit. Thus this equation will determine for such a climate the size of the cooling plant required. If there are very high casual gains and the Utrim is more negative then the cooling limit then the system will be considered unreachable.

In cases where the temperature differences are significant such as in deep winter and in the height of summer season then the whole Utrim1 equation will have to be considered.



 (56)

Here all the factors will have to be inspected. If it is a hot summer then (Ta-To) will be a negative term. Internal temperature exchanges can be assumed small. The humidity terms in the Utrim1 equation are dependent on the internal and external humidity levels and the vapour production rate. Thus in summer it all depends on the use of the zone. If it is a high occupancy zone such as a classroom then ventilation will be ON to control humidity (i.e. (wa(s)-wo(s)) positive). This is the reason why these terms are positive in the Utrim1 equation because they will cause the temperature to drop making Utrim1 more positive due to mechanical ventilation thus reducing the chance of overheating. On the other hand if humidity control is not active i.e. ventilation is off then casual gains will have significant impact on the value of the Utrim1 in summer for temperature control. Thus depending on how large the cooling plant is will determine the reachability for temperature in summer.

In deep winter [Ta(s)-To(s)] term will be very large positive term. If at the same time the casual gains are small then the Utrim1 will be positive. If the Utrim1 value remains within the heating capacity the plant then temperature will be reachable. This will also be helped with any causal gains generated in winter. However if the plant is under sized for heating then there is a change for temperature control being unstable and set point unreachable. On the other hand in situations where the control system is required to track temperature and there is only a heating system installed and the building requires cooling, then there is also a chance of control system becoming unstable. This could happen in winter season where there is a large demand for heating in the zone and the heating is required to come on but is not managed properly causing overheating.

Another important factor is air change rates in the building. If air change rate is very high (i.e. low insulated building) and coupled with a large temperature internal external temperature difference then this will cause the Utrim1 to be more positive. In winter (Ta(s)-To(s) > 0) Utrim1 will be positive and if above the upper plant limits then will indicate a cooling requirement. In summer (Ta(s)-To(s) < 0) this will be positive and if below the lower plant limits then will indicate heating requirement.

Overall for temperature control the reachability of set point depends on the temperature difference in the building and the casual and humidity gains. The designer has to inspect these variables in Utrim1 to be able to determine the reachability of temperature control in this MIMO system.

For humidity:

 (57)

The above equation shows that the reachability of humidity depends on the humidity differences, the humidity gains, air change rates and external wind speed. If the term (wa-wo) is small and there is a high water vapour generation rate then Utrim will become positive resulting in infiltration of air from the zone. The Utrim2 will be simplified to:

 (58)

This indicates that Utrim2 will be positive meaning moisture infiltration is required by the mechanical ventilation. This is because mathematically infiltration is assumed positive and extraction as negative. Thus a higher water vapour generation will cause a greater positive value of the Utrim2resulting in more extraction of air required from the zone.

In cases where the humidity levels inside and outside are significant and cannot be ignored. Here the whole of the Utrim2will have to be considered for determining the reachability of humidity.

 (59)

However, very high generating rate, wind speeds and air change rate could cause Utrim to become negative. In winter, external humidity levels are lower than in internal and thus in this case infiltration cannot be used to increase the humidity level inside and will require extraction indicated by the negative Utrim2value. Here it is important that the plant is able to extract and infiltration depending on the situation as moisture generation is a disturbances and not a control variable.

In conclusion, it all depends on the power limits of the plant. If assuming ideally a bi-directional heating/cooling plant then negative Utrim will case the plant power to become negative causing cooling. However if there is no cooling (i.e. lower limit of the plant is zero) then the negative value will show the breach of power limits meaning that overheating is occurring and there is no way to cool thus temperature is drifting away from the set point and not reachable. The reachability of air temperature in summer depends highly on having a cooling system installed. The reachability of humidity control in depends on having a mechanical ventilation plant which can extract and infiltrate depending on the season. In summer humidity levels will be higher outside then inside and in winter higher inside than outside. Hence having a mechanical plant which can extract and infiltrate will make reachability easy. Maximum humidity generation expected in the zone can be used to size the extraction ventilation plant. Overheating of buildings and humidity discomfort can be a difficult MIMO problem in buildings and these new criteria allows the designer inspect how vulnerable the building design and its systems as a whole are to over-heating and humidity comfort.

6. Conclusions and further work

Over all in winter season the controllability of this MIMO system is a lot more complex than in the summer season. Hence in winter the temperature and humidity control are more coupled and having both heating and cooling and bi-directional mechanical ventilation will allow good control of the internal environment.

Evaluation of the matrix CB, and the relationship between the input and output has shown the extent of the cross coupling between the parameters in the system. With the analysis in this paper the matrix was proved rank full due to temperature feedback with a fast heating system. The (CB)matrix is not aligned diagonally which will make fast and accurate control very difficult with independent SISO controllers for temperature, and humidity especially in winter. This undesirable coupling changes seasonally and throughout the day as the root-locus asymptote directions are a function of the operating conditions. By assessing the state-space model (CB)matrix, the suitability of the system for high performance control has been proven to be seasonal (i.e. function of temperature difference) and not a function of the thermal mass of the construction or its U value. This analysis has shown the cross coupling which exists as a barrier to fast, non-interacting control. Any control system which is designed for this notional building must address the problem of this interaction.

Transmission zeros in the case study presented are both negative and thus stable. The transmission zero locations will allow a high performance high gain control to be successfully utilised when simultaneously controlling humidity and room temperature with a MIMO controller. As shown, the locations of the transmission zeros are a function of the building parameters.

The impact on control performance of disturbances such as changes in outside temperature and also changes in thermal characteristics such as U values can be minimised through the use of the symbolic analysis of equivalent control input (Utrim). When sizing the system actuators, such as, mechanical ventilation rate and heater power, the conditions for Utrim derived in this case study must be satisfied for safe system operation and guaranteeing reachability.

From the outcomes of this paper and the work published previously [1], primarily the model should be reduced further to include only the most fundamentals causes and effects of a building system. Although sufficient for purposes of this paper, to demonstrate how State Space modelling methods could be utilised by designers in industry, the following changes are proposed:

The different air change rates induced by, buoyancy, external wind, infiltration and mechanical ventilation can be combined into a single term for fundamentally analysing the effect of ACR on the controllability.

Humidity and CO2 control are not utilised in most buildings and normally it is just temperature control thus reducing the model for just assessing temperature control will allow for a simpler but through analysis of the controllability of systems industry.

The mechanical ventilation is normally treated as disturbance as it is not controlled but is operated with on off controls for cooling. Thus the mechanical ventilation does not have to be treated as a control element.

Thus, making the above mentioned changes in the model will make it more representative of the physical processes it seeks to model and would allow for more accurate analysis which could be used to better inform design decision making quickly.

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9. References

[1] J. Counsell, Y. Khalid, J. Brindley “CONTROLLABILITY OF BUILDINGS, A multi-input multi-output stability assessment method for buildings with slow acting heating systems” Journal of Simulation Modelling Practice and Theory, Elsevier, Volume 19, issue 4 April 2011, pages 1185-1200

[2] Franklin and Powell, Feedback Control of dynamics systems, 5th edition 2006.

[3] Counsell J.M., Optimum and safe control algorithm (OSCA) for modern missile autopilot design, PhD Thesis Dept of Engineering, Lancaster University, 1992.

[4] Bradshaw, A., Counsell, J.M., Design of autopilots for high performance missiles, IMechE J. Systems & Control 1992. pp. 75-84

[5] E Muir, A Bradshaw; Control law design for thrust vectoring fighter aircraft using robust inverse dynamics estimation (RIDE); Proc Instn. Mech. Engrs. Vol 210, 333-343, 1996.

[6] Drury B. Crawley, Jon W. Hand, Michael kummert and Brent T. Griffith, Contrasting the capabilities of building energy performance simulation programs, Building and environment volume 43, issue 4, april 2008, pages 661-673

[7] Liao, Z.; A.L.Dexter, A simplified physical model for estimating the average air temperature in multi-zone heating systems, Building and Environment, 2004, 1013-1022

[8] Daskalov, P I; Arvenitis, K G, Nonlinear adaptive temperature and humidity control in animal buildings, Biosystems Engineering, 2006, 1-24

[9] Ding, L., Bradshaw, A. and Taylor, C.J. (2006) Design of discrete-time RIDE control system, UKACC International Conference (Control-06), August, Glasgow, UK.

[10] Straete, H.J., “Physical Meaning of zeros and transmission zeros from bond graph models”, MSc Thesis 1995, Massachusetts Institute of Technology

[11] McMullan, R. Environmental Science in Building, Palgrave MacMillan, Basingstoke, 2007

**Appendix**

Temperature:









Humidity:



Substituting the nonlinear parts into the equations gives:









Rearranging the equations gives:









Identifying the coefficients:









Where the coefficients are given by:









 The state space model is given as follows:

