IAC-11-C1.4.9

NONLINEARLY STABLE EQUILIBRIA IN THE SUN-JUPITER-TROJAN-SPACECRAFT FOUR BODY PROBLEM.

Marta Ceccaroni
Advanced Concepts Space Laboratory, Glasgow, UK,
marta.ceccaroni@strath.ac.uk

James Biggs
Advanced Concepts Space Laboratory, Glasgow, UK,
james.biggs@strath.ac.uk

The Trojan asteroids have been highlighted as a main target for future discovery missions, which will enable key questions about the formation of our Solar system to be answered. Programs like the Japanese Jupiter and Trojan Asteroids Exploration Programme are already testing technology demonstrators like the IKAROS spacecraft to enable future interplanetary missions to Jupiter and the Trojans. In this paper an analytic analysis of the stability of the Low thrust Sun Jupiter Asteroid Spacecraft system, is presented, from a Hamiltonian point of view. Setting the three primaries in the stable Lagrangian equilateral triangle configuration, eight natural (i.e. with zero thrust) equilibrium points are identified, four of which are close to the asteroid, two stable and two unstable, when considering as primaries the Sun and any other two bodies of the Solar System. Artificial equilibria, which can be seen as low thrust perturbations of the natural ones, are then taken into account with the aim of identifying their linearly stable subset. The Lyapunov stability of these marginally stable points is then analysed using basic KAM (Kolmogorov Arnold Moser) theory and Arnold's stability theorem. In order to apply such theorem an iterative procedure to reduce the Hamiltonian into Birkhoff's Normal Form is applied up to fourth order, explicitly defining, at each step, the generating function of a symplectic transformation. Despite the complexity of this process, Normal Forms are a fundamental, necessary step for any application of KAM theory; such theory, transforming a non-integrable system into a sum of perturbed integrable ones, enables the computation of a high order analytical approximation of the system dynamics, plus an estimation of the discrepancy from the initial model. As an application of KAM theory, a proof of the nonlinear stability for the low thrust generated equilibrium points under non resonant conditions is found using Arnold's stability theorem. Results show that Lyapunov stability is guaranteed along the linearly stable domain with the exception of a set of points with zero measure where the conditions to apply Arnold's theorem are not satisfied.

Key words: Four Body Problems · Low-thrust Propulsion · L_4 · Nonlinear Stability · Arnold's theorem

I. INTRODUCTION

There are a number of papers in the literature that analyse the Lyapunov stability of the triangular points in the Planar Circular Restricted Three Body Problem, from nonresonant cases to the more general and complex analysis of the degenerate points.

Starting from Leontovich [1] which, in 1962, was the first to prove that the equilateral triangle position is stable for almost all admissible mass ratios, i.e. all mass ratios below Routh's Critical mass [2]. Then Deprit and Deprit-Bartholome [3] demonstrated that the set of exceptional mass ratios (for which the stability/instability condition remains unknown) is bounded between the two critical mass ratios associated with the frequency resonances $(1:2)$ and $(1:3)$ of the linearized system. Finally the full stability of the Lagrangian equilibrium for the planar degenerated cases was demonstrated by Meyer and Shmidt [4] in 1985.

From a mathematical point of view the method used in all these papers to solve the planar case is based on the use of KAM theory for the demonstration of the existence, close to the equilibrium point, of many invariant two dimensional tori. Therefore, by Arnold and Moser's theorems [5], a suitably defined neighborhood of the equilibria is proved to be invariant and therefore stable. Unfortunately the same method does not allow one to draw equivalent conclusions in the spatial case (see [6], [7] and [8]).

Among the possible methods to predict analytically the invariant tori, (or, equivalently, to construct the integrals of motion) one of the main is to calculate the normal form of the Hamiltonian. This was originally developed by Birkhoff for the non-resonant cases (no commensurability conditions between the frequencies of the harmonic term of the Hamiltonian). This method and consists of a series of canonical transformations, which are polynomial functions of position and conjugate momenta; Birkhoff's algorithm was then generalised by Gustavson to the resonant cases (see [9] and [10]). This formal procedure can be extended ad infinitum, [11] whereby one obtains a power expansion of the Hamiltonian as a function of the actions. Furthermore Siegel [12] proved that the series diverges due to the non-integrability of the Hamiltonian. However the Hamiltonian in normal form truncated to a finite degree is an integrable system, which is close to the original system therefore providing an analytical approximation of it. Finally the only attempt found to apply Arnold's theorem to the Low Thrust Planar Circular Restricted Three Body Problem was found in [13], in which the theorem is used to extend Lyapunov stability to all the nonresonant linearly stable artificial equilibria (i.e. created with the use of low thrust).

The aim of this paper is to apply the analytical tools described above to the Low Thrust stable equilateral triangle Planar Circular Four Body Problem where the spacecraft is moving in the vicinity of the Lagrangian point.
For this purpose both the natural case (i.e. with zero thrust) and the low thrust case are investigated and their linear stability studied.
Birkhoff's algorithm for normalizing Hamiltonians is thus applied, up to the fourth order, explicitly defining the generating functions of the symplectic transformations. These generating functions then enable the computation of a high order analytical approximation of the system dynamics. Finally Lyapunov (i.e. nonlinear) stability is analised by applying Arnold's stability theorem. Results show that Lyapunov stability is guaranteed along the linearly stable domain close to the lagrangian point with the exception of a set of points with zero measure (where resonances between the frequencies of the unperturbed part of the Hamiltonian occur or the so called "reversibility" condition is not satisfied).

II. THE DYNAMICAL SYSTEM

This paper deals with the autonomous Coplanar Circular Restricted Four Body Problem (CRFBP) with the massive bodies set in the stable Lagrangian equilateral triangle configuration, which is the same as in [14], although here it is treated using an energy approach, obtained by deriving and analysing the Hamiltonian of the system.
The Restricted Four Body Problem is the problem of studying the dynamics of a massless spacecraft P_S that moves under the gravitational attraction of three massive bodies P_1, P_2 P_3. In this particular case the aim is to analyse the motion of a Spacecraft which can thrust constantly in direction and magnitude. The analysis is focussed on the zone close to the smaller body, the Asteroid, which is set in the stable Lagrangian equilateral triangle configuration (i.e. in the equilibrium point L_4) with the other two massive bodies, and assumed small enough not to influence their uniformly circular motion around their barycenter.
Scaled units of measure for mass and distance are then adopted, normalized with the sum of the masses of the two main bodies and their distance respectively, while the gravitational constant $\mathcal{G}$ and the uniform rotational velocity ω of the system are set to 1. In this nondimensional units the masses of the three bodies, in increasing order, are named ϵ, μ, $1-\mu$. Moreover their respective positions, in a system of reference centered in the barycenter of the two main bodies and rotating with angular velocity $\omega = 1$, perpendicular to the plane containing the three bodies, will result as fixed, and therefore, without loss of generality, can be assumed to be $(1-\mu, 0)$, $(-\mu, 0)$ and $(Lx, Ly) = (\frac{1}{2} - \mu, \frac{\sqrt{3}}{2})$ (See Fig.1).
In this rotating system of reference this dynamical model

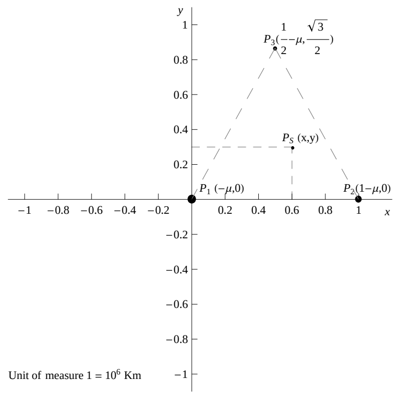

Figure 1: The non-dimensional Autonomous Coplanar CRFBP with the planets in the Lagrangian configuration.

can be described as

$$\begin{cases} \dot{\mathbf{q}} = \frac{\partial H(\mathbf{q},\mathbf{p})}{\partial \mathbf{p}} \\ \dot{\mathbf{p}} = -\frac{\partial H(\mathbf{q},\mathbf{p})}{\partial \mathbf{q}} \end{cases} \tag{1}$$

where $\mathbf{q}, \mathbf{p} \in \mathbb{R}^2$ are respectively canonical coordinates of position and conjugate momenta of the spacecraft, the dot denotes differentiation with respect to time, and where the Hamiltonian function has the form:

$$\begin{aligned} H(\mathbf{q},\mathbf{p}) = \; & \frac{p_x^2+p_y^2}{2} + (p_x q_y - p_y q_x) - \frac{(1-\mu)}{\sqrt{(q_x+\mu)^2+q_y^2}} \\ & - \frac{\mu}{\sqrt{(q_x+\mu-1)^2+q_y^2}} - \frac{\epsilon}{\sqrt{(q_x-L_x)^2+(q_y-L_y)^2}} \\ & - a_x q_x - a_y q_y \end{aligned} \tag{2}$$

where $a_x, a_y \in \mathbb{R}$ are the constant components of the acceleration due to the thrust, which can be set to zero to get back to the natural model.
To find the equilibrium points of the system we must solve (1) once $\dot{q}$ and $\ddot{q}$ have been set to be zero, namely:

$$\begin{cases} a_x = q_x - \frac{(1-\mu)(q_x+\mu)}{\sqrt{(q_x+q_{x\,e}+\mu)^2+(q_y+q_{x\,e})^2}^3} \\ \quad - \frac{\mu(q_x+\mu-1)}{\sqrt{(q_x+q_{x\,e}+\mu-1)^2+(q_y+q_{x\,e})^2}^3} \\ \quad - \frac{\epsilon(q_x-L_x)}{\sqrt{(q_x+q_{x\,e}-L_x)^2+(q_y+q_{x\,e}-L_y)^2}^3} \\ a_y = q_y - \frac{(1-\mu)q_y}{\sqrt{(q_x+q_{x\,e}+\mu)^2+(q_y+q_{x\,e})^2}^3} \\ \quad - \frac{\mu q_y}{\sqrt{(q_x+q_{x\,e}+\mu-1)^2+(q_y+q_{x\,e})^2}^3} \\ \quad - \frac{\epsilon(q_y-L_y)}{\sqrt{(q_x+q_{x\,e}-L_x)^2+(q_y+q_{x\,e}-L_y)^2}^3} \end{cases} \tag{3}$$

To find the natural equilibria (i.e. with zero thrust), it is sufficient to set the accelerations due to the thrust to be zero in (3) and solve the system.
By the stability result for unequal masses of [15], if we set our bodies to be the Sun and any other two objects of the Solar System, this zero thrust case admits eight solutions, equilibrium points of the dynamical model, four of which are close to the Asteroid (see Fig. 3). Moreover it can be proved that there exist a lower limit for the mass ratio $\frac{m_1}{m_1+m_2} = 1-\mu$ such as, for all the values bigger than that, the points M_6 and

M_7 are always stable [15], which is satisfied, for example by the Sun Jupiter Asteroid Spacecraft system.

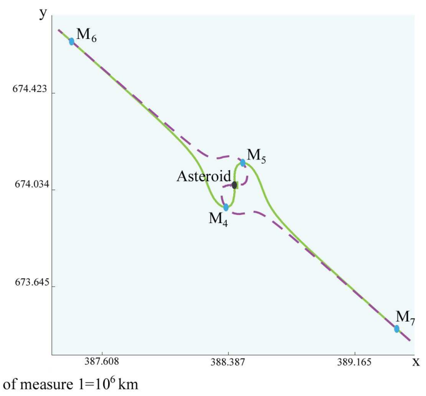

Figure 2: The four natural equilibria of the system close to the asteroid, M_4 and M_5 are unstable, M_6 and M_7 are linearly stable.

All the other solutions, found for each $(a_x, a_y) \neq 0$ are artificial equilibrium points, displaced from the natural ones, created incorporating in the model a low-thrust propulsion as shown in [14].
The linear stability of these artificial equilibrium points is then briefly illustrated here as it is fundamental for the stability analysis.
Hereafter, for simplicity of notation, $(q_{x_e}, q_{y_e}, p_{x_e}, p_{y_e}) = (q_{x_e}, q_{y_e}, 0, 0)$ will indicate any generic equilibria, solution of the system (3). Moreover, for convenience, the system of reference is translated to a generic equilibria by the transformation:

$$\begin{cases} q_x{}' = q_x - q_{x_e} \\ q_y{}' = q_y - q_{y_e} \end{cases} \tag{4}$$

but, to simplify notation, the indices above $q_x{}'$ and $q_y{}'$ will be ignored.
After this translation the two degrees of freedom Hamiltonian describing the dynamical system is:

$$\begin{aligned} H(\mathbf{q},\mathbf{p}) = {} & \tfrac{p_x^2+p_y^2}{2} + (p_x q_y - p_y q_x) \\ & - \tfrac{(1-\mu)}{\sqrt{(q_x+q_{x_e}+\mu)^2+(q_y+q_{x_e})^2}} \\ & - \tfrac{\mu}{\sqrt{(q_x+q_{x_e}+\mu-1)^2+(q_y+q_{x_e})^2}} \\ & - \tfrac{\epsilon}{\sqrt{(q_x+q_{x_e}-L_x)^2+(q_y+q_{x_e}-L_y)^2}} \end{aligned} \tag{5}$$

Which can therefore be expanded as a power series of q and p in a neighborhood of the origin phase space (i.e. the equilibrium point)

$$H(\mathbf{q},\mathbf{p}) = \sum_{j\geq 2} H^{(j)}(\mathbf{q},\mathbf{p}) \tag{6}$$

where the term $H^{(j)}$ is the homogenous components of the series with degree j:

$$H^{(j)}(\mathbf{q},\mathbf{p}) = \sum_{k+l+m+n=j} h_{k,l,m,n}\, q_x^k q_y^l p_x^m p_y^n \tag{7}$$

$h_{k,l,m,n} \in \mathbb{R}$, constants.
The linearized Hamiltonian of the system, i.e. the collection of all the second order terms, will therefore be:

$$H^{(2)}(\mathbf{q},\mathbf{p}) = \frac{(p_x^2+p_y^2)}{2} + p_x q_y - p_y q_x + \frac{(\alpha q_x^2+\beta q_y^2)}{2} + \chi q_x q_y \tag{8}$$

with the three coefficients $\alpha, \beta, \chi \in \mathbb{R}$:

$$\begin{aligned} \alpha = {} & -\frac{\mu(-q_{y_e}^2+2(-1+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^5} - \frac{(1-\mu)(-q_{y_e}^2+2(+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^5} \\ & - \frac{\epsilon(-(q_{y_e}-Ly)^2+2(q_{x_e}-Lx)^2)}{\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^5}; \\ \beta = {} & -\frac{\mu(2q_{y_e}^2-(-1+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^5} - \frac{(1-\mu)(2q_{y_e}^2-(+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^5} \\ & - \frac{\epsilon(2(q_{y_e}-Ly)^2-(q_{x_e}-Lx)^2)}{\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^5}; \\ \chi = {} & -\frac{3\mu(q_{y_e}^2(-1+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^5} - \frac{3(1-\mu)(q_{y_e}^2(+q_{x_e}+\mu)^2)}{\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^5} \\ & - \frac{3\epsilon((q_{y_e}-Ly)^2(q_{x_e}-Lx)^2)}{\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^5}; \end{aligned} \tag{9}$$

The coefficients $h^{(3)}_{k,l,m,n}$ and $h^{(4)}_{k,l,m,n}$ of the developed third and fourth orders are listed in the Appendix A and B respectively.

III. LINEAR STABILITY

In order to study the linear stability of the system, the matrix Σ the associated with the linearized Hamiltonian (8) is considered, which is the matrix such that:

$$H^{(2)} = \frac{1}{2} < (q_x, q_y, p_x, p_y), \Sigma \begin{pmatrix} q_x \\ q_y \\ p_x \\ p_y \end{pmatrix} > \tag{10}$$

where $< \cdot, \cdot >$ stands for the scalar product.
Therefore:

$$\Sigma = \begin{pmatrix} \alpha & \chi & 0 & -1 \\ \chi & \beta & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

Thus, calling J the skew symmetric matrix

$$J = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{12}$$

the matrix $\mathcal{A}$ associated to the system, i.e. the matrix such that

$$\begin{pmatrix} \dot{q}_x \\ \dot{q}_y \\ \dot{p}_x \\ \dot{p}_y \end{pmatrix} = \mathcal{A} \begin{pmatrix} q_x \\ q_y \\ p_x \\ p_y \end{pmatrix} \tag{13}$$

will be:

$$\mathcal{A} = J^{-1}\Sigma = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -\alpha & -\chi & 0 & 1 \\ -\chi & -\beta & -1 & 0 \end{pmatrix} \tag{14}$$

whose characteristic polynomial is :

$$\Gamma^4 + (2 + a + b)\Gamma^2 + 1 - a - b + ab - c^2 \tag{15}$$

and therefore the eigenvalues of the linear system (characteristic frequencies of the unperturbed harmonic part) are:

$$\begin{aligned} \Gamma_{1,3} &= \pm\sqrt{\tfrac{(2+\alpha+\beta+\sqrt{(2+\alpha+\beta)^2-4(1-\alpha)(1-\beta)+4\chi^2})}{2}}; \\ \Gamma_{2,4} &= \pm\sqrt{\tfrac{(2+\alpha+\beta-\sqrt{(2+\alpha+\beta)^2-4(1-\alpha)(1-\beta)+4\chi^2})}{2}}; \end{aligned} \tag{16}$$

As, assuming the two conditions:

$$\begin{cases} (2+\alpha+\beta)^2 - 4(1-\alpha)(1-\beta) + 4\chi^2 > 0 \\ 2+\alpha+\beta+\sqrt{(2+\alpha+\beta)^2 - 4(1-\alpha)(1-\beta) + 4\chi^2} < 0 \end{cases} \tag{17}$$

yields four purely imaginary eigenvalues, all the points satisfying (17), are linearly stable, and therefore, under this conditions, the four eigenvalues can be rearranged as:

$$\begin{aligned} \Gamma_{1,3} &= \pm i\lambda; \\ \Gamma_{2,4} &= \pm i\nu; \end{aligned} \tag{18}$$

with

$$\begin{aligned} \lambda &= \sqrt{\tfrac{-(2+\alpha+\beta+\sqrt{(2+\alpha+\beta)^2-4(1-\alpha)(1-\beta)+4\chi^2})}{2}}; \\ \nu &= \sqrt{\tfrac{-(2+\alpha+\beta-\sqrt{(2+\alpha+\beta)^2-4(1-\alpha)(1-\beta)+4\chi^2})}{2}}; \end{aligned} \tag{19}$$

The domain of the linearly stable point close to the Asteroid is shown by the points outside the "four leaf clover" (as coined in [14]) in Figure 3.
In the Figure, the zone that satisfies the conditions in (17), the linearly stable zone, is the intersection of the zone outside the dashed, dark green line and that outside the continuous, light green line, which respectively represents the solution of the first and the second equations of system (17)

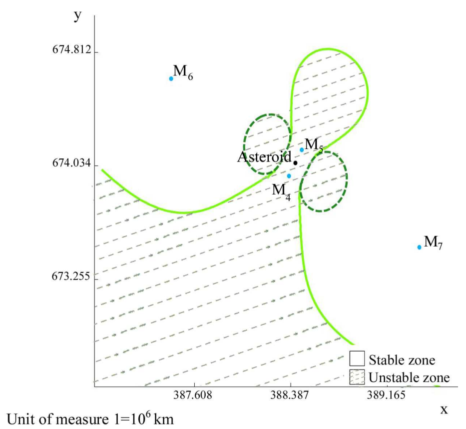

Figure 3: The linearly stable subset of the artificial equilibrium points.

Finally we want to construct a symplectic transformation of coordinates $(\mathbf{q}, \mathbf{p}) \rightarrow (\bar{\mathbf{q}}, \bar{\mathbf{p}})$ to rearrange the linearized Hamiltonian into the standard form

$$\bar{H}^{(2)}(\bar{\mathbf{q}}, \bar{\mathbf{p}}) = \frac{\lambda}{2}(\bar{q}_x^2 + \bar{p}_x^2) + \frac{\mu}{2}(\bar{q}_y^2 + \bar{p}_y^2) \tag{20}$$

which will be exploited in the next Section.
Calling $\mathbf{w}_1, \mathbf{w}_2 \in \mathbb{R}^4$ the eigenvalues corresponding to the eigenvectors Γ_1 and Γ_2, solution of the system:

$$\mathcal{A}\mathbf{w}_i = \Gamma_i \mathbf{w}_i, \;\; i = 1, 2 \tag{21}$$

and $\mathbf{u}_i = Re(\mathbf{w}_i)$, $\mathbf{v}_i = Im(\mathbf{w}_i)$,$i = 1, 2$.
The matrix $\mathcal{M}$ of the required transformation is therefore constructed as:

$$\mathcal{M} = \begin{pmatrix} m_1\mathbf{u}_1 & m_2\mathbf{u}_2 & m_1\mathbf{v}_1 & m_2\mathbf{v}_2 \\ \vdots \;\;, & \vdots \;\;, & \vdots \;\;, & \vdots \;\;, \end{pmatrix} \tag{22}$$

with

$$\frac{1}{m_i^2} = \mathbf{u}_i^T J \mathbf{v}_i, \;\; i = 1, 2 \tag{23}$$

Which is symplectic as, by direct calculation, it is easy to show that $M^T J M^{-1} = J$ and which brings the linearized Hamiltonian into the desired form (25) as

$$\Sigma' = \mathcal{M}^{-1}\Sigma\mathcal{M} = \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \nu & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \nu \end{pmatrix} \tag{24}$$

This transformation brings the Hamiltonian in the form:

$$\bar{H}(\bar{\mathbf{q}}, \bar{\mathbf{p}}) = \frac{\lambda}{2}(\bar{q}_x^2 + \bar{p}_x^2) + \frac{\mu}{2}(\bar{q}_y^2 + \bar{p}_y^2) + \sum_{j \geq 3} \bar{H}^{(j)}(\bar{\mathbf{q}}, \bar{\mathbf{p}}) \tag{25}$$

where the new coefficients $\bar{h}^{(3)}_{k,l,m,n}$ and $\bar{h}^{(4)}_{k,l,m,n}$ of the third and fourth order polynomials $\bar{H}^{(3)}$ and $\bar{H}^{(4)}$ are in Appendices C and D

V. BIRKHOFF'S NORMAL FORM

Normal Forms are a fundamental, necessary step for any application of KAM (Kolmogorov Arnold Moser) theory, which, transforming a non-integrable system into the sum integrable ones, plus an estimable perturbative part, enable a deep knowledge of the dynamics of the system itself.
In a more mathematical way:

Definition 1. *A two degrees of freedom Hamiltonian in the form (6) is said to be in normal form to degree s if:*

$$D_{(\boldsymbol{q},\boldsymbol{p})} H^{(j)}(\boldsymbol{q}, \boldsymbol{p}) = 0, \;\; \forall 2 \leq j \leq s \tag{26}$$

Where the partial differential operator $D_{(\boldsymbol{q},\boldsymbol{p})}\cdot$ is equivalent to the Poisson Brackets $\left\{\bar{H}^{(2)}, \cdot\right\}$, i.e.:

$$\begin{aligned} D_{(\boldsymbol{q},\boldsymbol{p})}\cdot &:= \left\{\bar{H}^{(2)}, \cdot\right\} \\ &= \lambda\left(q_x \frac{\partial \cdot}{\partial p_x} - p_x \frac{\partial \cdot}{\partial q_x}\right) + \mu\left(q_y \frac{\partial \cdot}{\partial p_y} - p_y \frac{\partial \cdot}{\partial q_y}\right) \end{aligned} \tag{27}$$

The possibility of rearranging a Hamiltonian into a Normal Form is insured by the main result of the work by Birkhoff and Gustavson [9], which can be summarized into the following iterative theorem (See also [10] and [3]), and will therefore reported without the proof:

Theorem 1. *Let a two degrees of freedom Hamiltonian (6) with the harmonic part in the standard form (25), be in normal form to degree $s-1$, $s \geq 3$, and let*

- (A_s) *(restricted condition of irrationality)*$\forall s \leq 4$ *be the condition such that:*

$$k_1\lambda + k_2\nu \neq 0,\ \ \forall k_1, k_2 \in \mathbb{Z},\ |k_1| + |k_2| \leq n \quad (28)$$

Then, if (A_s) is satisfied, there exist at least one canonical generating function $G^{(s)}(\bar{\boldsymbol{q}}, \hat{\boldsymbol{p}}) = \bar{\boldsymbol{q}}\hat{\boldsymbol{p}} + W^{(s)}(\bar{\boldsymbol{q}}, \hat{\boldsymbol{p}})$ (with $W^{(s)}(\bar{\boldsymbol{q}}, \hat{\boldsymbol{p}})$ homogeneous polynomial of degree s), such that, after the canonical transformation from $(\bar{\boldsymbol{q}}, \bar{\boldsymbol{p}})$ to $(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}})$ with

$$\begin{aligned} \hat{\boldsymbol{q}} &= \bar{\boldsymbol{q}} + \tfrac{\partial W^{(s)}(\bar{\boldsymbol{q}}, \hat{\boldsymbol{p}})}{\partial \hat{\boldsymbol{p}}} \\ \bar{\boldsymbol{p}} &= \hat{\boldsymbol{p}} + \tfrac{\partial W^{(s)}(\bar{\boldsymbol{q}}, \hat{\boldsymbol{p}})}{\partial \bar{\boldsymbol{q}}} \end{aligned} \quad (29)$$

the hamiltonian $\hat{H}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}}) := \bar{H}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}})$ is in normal form up to the degree s. Furthermore such change of coordinates decomposes the Hamiltonian into the sum

$$\hat{H}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}}) = \hat{H}_0 + R^{(s+1)} \quad (30)$$

where $\hat{H}_0$ is a sum, up to the degree s, of polynomials in the variables $I_x = \frac{\hat{q}_x^2 + \hat{p}_x^2}{2}$ and $I_y = \frac{\hat{q}_y^2 + \hat{p}_y^2}{2}$ of the form:

$$\hat{H}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}}) = \lambda I_x + \nu I_y + \tfrac{1}{2}(AI_x^2 + 2BI_xI_y + CI_y^2) + ... \quad (31)$$

and $R^{(s+1)}$ is a power of series of $\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}}$ starting from the degree $s+1$.

A Hamiltonian of the form (31) is said to be in the Birkhoff-Gustavson Normal Form and the iterative algorithm to explicitly construct the canonical transformations leading to it (see [11] and [3]) will now be applied up to the fourth order for our Hamiltonian (5).
Such process will allow the derivation of some properties of the normal form and pave the way to apply Arnold's stability theorem in the next Sections.
Notice that, the Birkhoff-Gustavson Normal Form is Normal up to degree 4, as:

$$\begin{aligned}
&D_{(\hat{\mathbf{q}}, \hat{\mathbf{p}})}\hat{H}^{(2)}(\hat{\mathbf{q}}, \hat{\mathbf{p}}) = \lambda(\hat{q}_x\lambda\hat{p}_x - \hat{p}_x\lambda\hat{q}_x) + \nu(\hat{q}_y\nu\hat{p}_y - \hat{p}_y\nu\hat{q}_y) \\
&\quad = 0; \\
&D_{(\hat{\mathbf{q}}, \hat{\mathbf{p}})}\hat{H}^{(3)}(\hat{\mathbf{q}}, \hat{\mathbf{p}}) = 0; \\
&D_{(\hat{\mathbf{q}}, \hat{\mathbf{p}})}\hat{H}^{(4)}(\hat{\mathbf{q}}, \hat{\mathbf{p}}) = \lambda\left[\hat{q}_xA\left(\tfrac{\hat{q}_x^2+\hat{p}_x^2}{2}\right)\hat{p}_x + \hat{q}_xB(\hat{q}_y^2 + \hat{p}_y^2)\hat{p}_x\right. \\
&\quad \left. -\hat{p}_xA\left(\tfrac{\hat{q}_x^2+\hat{p}_x^2}{2}\right)\hat{q}_x - \hat{p}_xB(\hat{q}_y^2 + \hat{p}_y^2)\hat{q}_x\right] \\
&\quad +\nu\left[\hat{q}_yC\left(\tfrac{\hat{q}_y^2+\hat{p}_y^2}{2}\right)\hat{p}_y + \hat{q}_yB(\hat{q}_x^2 + \hat{p}_x^2)\hat{p}_y\right. \\
&\quad \left. -\hat{p}_yC\left(\tfrac{\hat{q}_y^2+\hat{p}_y^2}{2}\right)\hat{q}_y - \hat{p}_yB(\hat{q}_x^2 + \hat{p}_x^2)\hat{q}_y\right] \\
&\quad = 0;
\end{aligned} \quad (32)$$

which is a fourth degree Normal Form by ***Definition 1***.
As the second order of the Hamiltonian is already in the desired Normal Form by the symplectic transformation found in Section II, the first step is to determine the generating function $G^{(3)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}}) = \bar{\mathbf{q}}\tilde{\mathbf{p}} + W^{(3)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}})$ from the coordinates $(\bar{\mathbf{q}}, \bar{\mathbf{p}})$ to $(\tilde{\mathbf{q}}, \tilde{\mathbf{p}})$ (with $W^{(3)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}}) = \sum_{k+l+m+n=3} w^{(3)}_{k,l,m,n}\bar{\mathbf{q}}_x^k\bar{\mathbf{q}}_y^l\tilde{\mathbf{p}}_x^m\tilde{\mathbf{p}}_x^n$), such that the Hamiltonian in the new coordinates $\tilde{H}^{(3)}(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}) \equiv 0$.
This, in other words, means that we must determine explicitly the coefficients $w^{(3)}_{k,l,m,n},\ k+l+m+n = 3$ of the transformation. As, by definition:

$$H(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \frac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) = \tilde{H}(\bar{\mathbf{q}} + \frac{\partial W^{(3)}}{\partial \tilde{\mathbf{p}}}, \tilde{\mathbf{p}}) \quad (33)$$

then, by developing the left and right terms of the equation:

$$\begin{aligned}
&\bar{H}^{(2)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \tfrac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) + \bar{H}^{(3)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \tfrac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) \\
&+\bar{H}^{(4)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \tfrac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) + \sum_{j\geq 5}\bar{H}^{(j)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \frac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) \\
&= \tilde{H}^{(2)}(\bar{\mathbf{q}} + \tfrac{\partial W^{(3)}}{\partial \tilde{\mathbf{p}}}, \tilde{\mathbf{p}}) + \tilde{H}^{(4)}(\bar{\mathbf{q}} + \tfrac{\partial W^{(3)}}{\partial \tilde{\mathbf{p}}}, \tilde{\mathbf{p}}) \\
&+ \sum_{j\geq 5}\tilde{H}^{(j)}(\bar{\mathbf{q}} + \frac{\partial W^{(3)}}{\partial \tilde{\mathbf{p}}}, \tilde{\mathbf{p}})
\end{aligned} \quad (34)$$

which is:

$$\begin{aligned}
&\tfrac{\lambda}{2}\left[\bar{q}_x^2 + \left(\tilde{p}_x + \tfrac{\partial W^{(3)}}{\partial \bar{q}_x}\right)^2\right] + \tfrac{\nu}{2}\left[\bar{q}_y^2 + \left(\tilde{p}_y + \tfrac{\partial W^{(3)}}{\partial \bar{q}_y}\right)^2\right] \\
&+ \sum_{k+l+m+n=3}\bar{h}^{(3)}_{k,l,m,n}\bar{q}_x^k\bar{q}_y^l(\tilde{p}_x + \frac{\partial W^{(3)}}{\partial \bar{q}_x})^m(\tilde{p}_y + \frac{\partial W^{(3)}}{\partial \bar{q}_y})^n \\
&+ \sum_{k+l+m+n=4}\bar{h}^{(4)}_{k,l,m,n}\bar{q}_x^k\bar{q}_y^l(\tilde{p}_x + \frac{\partial W^{(3)}}{\partial \bar{q}_x})^m(\tilde{p}_y + \frac{\partial W^{(3)}}{\partial \bar{q}_y})^n \\
&+ \sum_{j\geq 5}\bar{H}^{(j)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}} + \frac{\partial W^{(3)}}{\partial \bar{\mathbf{q}}}) \\
&= \sum_{k+l+m+n=2}\tilde{h}^{(2)}_{k,l,m,n}\left(\bar{q}_x + \frac{\partial W^{(3)}}{\partial \tilde{p}_x}\right)^k\left(\bar{q}_y + \frac{\partial W^{(3)}}{\partial \tilde{p}_y}\right)^l\tilde{p}_x^m\tilde{p}_y^n \\
&+ \sum_{k+l+m+n=4}\tilde{h}^{(4)}_{k,l,m,n}\left(\bar{q}_x + \frac{\partial W^{(3)}}{\partial \tilde{p}_x}\right)^k\left(\bar{q}_y + \frac{\partial W^{(3)}}{\partial \tilde{p}_y}\right)^l\tilde{p}_x^m\tilde{p}_y^n \\
&+ \sum_{j\geq 5}\tilde{H}^{(j)}(\bar{\mathbf{q}} + \frac{\partial W^{(3)}}{\partial \tilde{\mathbf{p}}}, \tilde{\mathbf{p}})
\end{aligned} \quad (35)$$

Equating term by term up to fourth order, recalling that $W^{(3)}$ is a third degree polynomial, yields the three equations:

$$\begin{aligned}
&\bullet\ \tfrac{\lambda}{2}(\bar{q}_x^2 + \tilde{p}_x^2) + \tfrac{\nu}{2}(\bar{q}_y^2 + \tilde{p}_y^2) = \tilde{H}^{(2)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}}); \\
&\bullet\ \lambda(\tilde{p}_x\tfrac{\partial W^{(3)}}{\partial \bar{q}_x}) + \nu(\tilde{p}_y\tfrac{\partial W^{(3)}}{\partial \bar{q}_y}) \\
&\quad + \sum_{k+l+m+n=3}\bar{h}^{(3)}_{k,l,m,n}\bar{q}_x^k\bar{q}_y^l\tilde{p}_x^m\tilde{p}_y^n \\
&\quad = \mathcal{H}_2; \\
&\bullet\ \tfrac{\lambda}{2}(\tfrac{\partial W^{(3)}}{\partial \bar{q}_x})^2 + \tfrac{\nu}{2}(\tfrac{\partial W^{(3)}}{\partial \bar{q}_y})^2 + \mathcal{H}_3 \\
&\quad + \sum_{k+l+m+n=4}\bar{h}^{(4)}_{k,l,m,n}\bar{q}_x^k\bar{q}_y^l\tilde{p}_x^m\tilde{p}_y^n \\
&\quad = \mathcal{H}_2^* + \sum_{k+l+m+n=4}\tilde{h}^{(4)}_{k,l,m,n}\bar{q}_x^k\bar{q}_y^l\tilde{p}_x^m\tilde{p}_y^n;
\end{aligned} \quad (36)$$

Where $\mathcal{H}_2$ is the collection of the third order terms arising from:

$$\sum_{k+l+m+n=2}\tilde{h}^{(2)}_{k,l,m,n}\left(\bar{q}_x + \frac{\partial W^{(3)}}{\partial \tilde{p}_x}\right)^k\left(\bar{q}_y + \frac{\partial W^{(3)}}{\partial \tilde{p}_y}\right)^l\tilde{p}_x^m\tilde{p}_y^n$$

Where $\mathcal{H}_2^*$ is the collection of the fouth order terms arising from:

$$\sum_{k+l+m+n=2}\tilde{h}^{(2)}_{k,l,m,n}\left(\bar{q}_x + \frac{\partial W^{(3)}}{\partial \tilde{p}_x}\right)^k\left(\bar{q}_y + \frac{\partial W^{(3)}}{\partial \tilde{p}_y}\right)^l\tilde{p}_x^m\tilde{p}_y^n$$

And analogously $\mathcal{H}_3$ is the collection of the fourth order terms arising from:

$$\sum_{k+l+m+n=3} \bar{h}^{(3)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l (\tilde{p}_x + \frac{\partial W^{(3)}}{\partial \tilde{q}_x})^m (\tilde{p}_y + \frac{\partial W^{(3)}}{\partial \tilde{q}_y})^n$$

The first equation states that the symplectic transformation generated by the third degree polynomial $W^{(3)}$ does not effect the second order of the Hamiltonian therefore conserving its Normal Form. Furthermore it states that :

$$\begin{array}{l} \tilde{h}^{(2)}_{2,0,0,0} = \frac{\lambda}{2}; \\ \tilde{h}^{(2)}_{0,2,0,0} = \frac{\nu}{2}; \\ \tilde{h}^{(2)}_{0,0,2,0} = \frac{\lambda}{2}; \\ \tilde{h}^{(2)}_{0,0,0,2} = \frac{\nu}{2}; \end{array} \tag{37}$$

and all the other are zero.

Therefore the second equation allows to determine explicitly the coefficients $w^{(3)}_{k,l,m,n}$ of the transformation as:

$$\lambda(\tilde{p}_x \tfrac{\partial W^{(3)}}{\partial \bar{q}_x} - \tilde{p}_y \tfrac{\partial W^{(3)}}{\partial \bar{q}_y}) + \nu(\tilde{p}_y \tfrac{\partial W^{(3)}}{\partial \bar{q}_y} - \tilde{p}_x \tfrac{\partial W^{(3)}}{\partial \bar{q}_x}) + \sum_{k+l+m+n=3} \bar{h}^{(3)}_{k,l,m,n} \bar{q}_x^k \bar{q}_y^l \tilde{p}_x^m \tilde{p}_y^n = 0 \tag{38}$$

The coefficients $\bar{h}^{(3)}_{k,l,m,n},\ \ k+l+m+n=3$ have already been determined in Section III and listed in Appendix C, thus, with a few algebraic manipulations, it is possible to explicit the $w^{(3)}_{k,l,m,n},\ k+l+m+n=3$ therefore determining the transformation $G^{(3)}(\bar{\mathbf{q}}, \tilde{\mathbf{p}})$. Such coefficients are listed in Appendix F.

Finally the third equation determines the transformed coefficients of the fourth order $\tilde{h}^{(4)}_{k,l,m,n},\ \ k+l+m+n=4$ by solving:

$$\begin{aligned} &\tfrac{\lambda}{2}((\tfrac{\partial W^{(3)}}{\partial \bar{q}_x})^2 - (\tfrac{\partial W^{(3)}}{\partial \bar{q}_y})^2) + \tfrac{\nu}{2}((\tfrac{\partial W^{(3)}}{\partial \bar{q}_y})^2 - (\tfrac{\partial W^{(3)}}{\partial \bar{q}_x})^2) \\ &+ \sum_{k+l+m+n=4} \bar{h}^{(4)}_{k,l,m,n} \bar{q}_x^k \bar{q}_y^l \tilde{p}_x^m \tilde{p}_y^n + \mathcal{H}_3 \\ &= \sum_{k+l+m+n=4} \tilde{h}^{(4)}_{k,l,m,n} \bar{q}_x^k \bar{q}_y^l \tilde{p}_x^m \tilde{p}_y^n \end{aligned} \tag{39}$$

The coefficients $\tilde{h}^{(4)}_{k,l,m,n},\ k+l+m+n=4$ are listed in Appendix E.

Now the Hamiltonian of the system is in Normal Form up to the third order, namely it is in the form:

$$\begin{aligned} \tilde{H}(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}) &= \tfrac{\lambda}{2}\left(\tfrac{\tilde{q}_x^2 + \tilde{p}_x^2}{2}\right) + \tfrac{\nu}{2}\left(\tfrac{\tilde{q}_y^2 + \tilde{p}_y^2}{2}\right) \\ &+ \sum_{k+l+m+n=4} \tilde{h}^{(4)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l \tilde{p}_x^m \tilde{p}_y^n + \sum_{j \geq 5} \tilde{H}^{(j)}(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}) \end{aligned} \tag{40}$$

Iterating an analogous procedure for the fourth order it is then possible to find the coefficients $w^{(4)}_{k,l,m,n}$ of the transformation $G(\tilde{\mathbf{q}}, \hat{\mathbf{p}}) = \tilde{\mathbf{q}}\hat{\mathbf{p}} + W^{(4)}(\tilde{\mathbf{q}}, \hat{\mathbf{p}})$ (with $W^{(4)}(\tilde{\mathbf{q}}, \hat{\mathbf{p}}) = \sum_{k+l+m+n=4} w^{(4)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l \hat{p}_x^m \hat{p}_y^n$) which uniquely identify the symplectic change of variables $(\tilde{\mathbf{q}}, \tilde{\mathbf{p}}) \to (\hat{\mathbf{q}}, \hat{\mathbf{p}})$ that reduces the Hamiltonian to the Birkhoff's normal form up to degree four (31).

Once again, by definition, we set:

$$\tilde{H}(\tilde{\mathbf{q}}, \hat{\mathbf{p}} + \frac{\partial W^{(4)}}{\partial \tilde{\mathbf{q}}}) = \hat{H}(\tilde{\mathbf{q}} + \frac{\partial W^{(4)}}{\partial \hat{\mathbf{p}}}, \hat{\mathbf{p}}) \tag{41}$$

which developed is:

$$\begin{aligned} &\tfrac{\lambda}{2}\left[\tilde{q}_x^2 + \left(\hat{p}_x + \tfrac{\partial W^{(4)}}{\partial \tilde{q}_x}\right)^2\right] + \tfrac{\nu}{2}\left[\tilde{q}_y^2 + \left(\hat{p}_y + \tfrac{\partial W^{(4)}}{\partial \tilde{q}_y}\right)^2\right] \\ &+ \sum_{k+l+m+n=4} \tilde{h}^{(4)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l (\hat{p}_x + \frac{\partial W^{(4)}}{\partial \tilde{q}_x})^m (\hat{p}_y + \frac{\partial W^{(4)}}{\partial \tilde{q}_y})^n \\ &+ \sum_{j \geq 5} \tilde{H}^{(j)}(\tilde{\mathbf{q}}, \hat{\mathbf{p}} + \frac{\partial W^{(4)}}{\partial \tilde{\mathbf{q}}}) \\ &= \sum_{k+l+m+n=2} \hat{h}^{(2)}_{k,l,m,n} \left(\tilde{q}_x + \frac{\partial W^{(4)}}{\partial \hat{p}_x}\right)^k \left(\tilde{q}_y + \frac{\partial W^{(4)}}{\partial \hat{p}_y}\right)^l \hat{p}_x^m \hat{p}_y^n \\ &+ \sum_{k+l+m+n=3} \hat{h}^{(3)}_{k,l,m,n} \left(\tilde{q}_x + \frac{\partial W^{(4)}}{\partial \hat{p}_x}\right)^k \left(\tilde{q}_y + \frac{\partial W^{(4)}}{\partial \hat{p}_y}\right)^l \hat{p}_x^m \hat{p}_y^n \\ &+ \tfrac{A}{8}\left[\left(\tilde{q}_x + \tfrac{\partial W^{(4)}}{\partial \hat{p}_x}\right)^2 + \hat{p}_x^2\right]^2 - \tfrac{B}{4}\left[\left(\tilde{q}_x + \tfrac{\partial W^{(4)}}{\partial \hat{p}_x}\right)^2 + \hat{p}_x^2\right] \\ &\left[\left(\tilde{q}_y + \tfrac{\partial W^{(4)}}{\partial \hat{p}_y}\right)^2 + \hat{p}_y^2\right] + \tfrac{C}{8}\left[\left(\tilde{q}_y + \tfrac{\partial W^{(4)}}{\partial \hat{p}_y}\right)^2 + \hat{p}_y^2\right]^2 \\ &+ \sum_{j \geq 5} \hat{H}^{(j)}(\tilde{\mathbf{q}} + \frac{\partial W^{(4)}}{\partial \hat{\mathbf{p}}}, \hat{\mathbf{p}}) \end{aligned} \tag{42}$$

therefore, equalling term by term (this time $W^{(4)}$ is a third degree polynomial):

$$\begin{aligned} \bullet\ &\tfrac{\lambda}{2}\left(\tilde{q}_x^2 + \hat{p}_x^2\right) + \tfrac{\nu}{2}\left(\tilde{q}_y^2 + \hat{p}_y^2\right) \\ &= \sum_{k+l+m+n=2} \hat{h}^{(2)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l \hat{p}_x^m \hat{p}_y^n; \\ \bullet\ &0 = \sum_{k+l+m+n=3} \hat{h}^{(3)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l \hat{p}_x^m \hat{p}_y^n; \\ \bullet\ &\lambda\left(\hat{p}_x \tfrac{\partial W^{(4)}}{\partial \tilde{q}_x}\right) + \nu\left(\hat{p}_y \tfrac{\partial W^{(4)}}{\partial \tilde{q}_y}\right) \\ &+ \sum_{k+l+m+n=4} \tilde{h}^{(4)}_{k,l,m,n} \tilde{q}_x^k \tilde{q}_y^l \hat{p}_x^m \hat{p}_y^n \\ &= \hat{\mathcal{H}}_2 + \tfrac{A}{2} I_x^2 + B I_x I_y + \tfrac{C}{2} I_y^2; \end{aligned} \tag{43}$$

Where $\hat{\mathcal{H}}_2$ is the collection of the fourth order term arising from:

$$\sum_{k+l+m+n=2} \hat{h}^{(2)}_{k,l,m,n} \left(\tilde{q}_x + \frac{\partial W^{(4)}}{\partial \hat{p}_x}\right)^k \left(\tilde{q}_y + \frac{\partial W^{(4)}}{\partial \hat{p}_y}\right)^l \hat{p}_x^m \hat{p}_y^n$$

The first two equations of (43) suggest that the symplectic change of coordinates does not affect the two orders that are already in Normal Form and that among all the coefficients $\hat{h}^{(2)}_{k,l,m,n},\ \ k+l+m+n=2$ and $\hat{h}^{(3)}_{k,l,m,n},\ \ k+l+m+n=3$ the only non zero are:

$$\begin{array}{l} \hat{h}^{(2)}_{2,0,0,0} = \frac{\lambda}{2}; \\ \hat{h}^{(2)}_{0,2,0,0} = \frac{\nu}{2}; \\ \hat{h}^{(2)}_{0,0,2,0} = \frac{\lambda}{2}; \\ \hat{h}^{(2)}_{0,0,0,2} = \frac{\nu}{2}; \end{array} \tag{44}$$

The solving coefficients of the third equation of (43), instead, allows to find explicitly transformation $G^{(4)}(\tilde{\mathbf{q}}, \hat{\mathbf{p}})$ and moreover to identify the coefficients A, B, C which will be used in the next Section for the application of Arnold's stability theorem.

Such coefficients $w^{(4)}_{k,l,m,n},\ k+l+m+n=4$ are listed in Appendix G, while the three coefficients A, B, C, instead result to be:

$$\begin{aligned} A &= 3\tilde{h}^{(4)}_{0,0,4,0} + \tilde{h}^{(4)}_{2,0,2,0} + 3\tilde{h}^{(4)}_{4,0,0,0} \\ B &= \tilde{h}^{(4)}_{0,0,2,2} + \tilde{h}^{(4)}_{0,2,2,0} + \tilde{h}^{(4)}_{2,0,0,2} + \tilde{h}^{(4)}_{2,2,0,0} \\ C &= 3\tilde{h}^{(4)}_{0,0,0,4} + \tilde{h}^{(4)}_{0,2,0,2} + 3\tilde{h}^{(4)}_{0,4,0,0} \end{aligned} \tag{45}$$

IV. APPLICATION OF ARNOLD'S THEOREM AND CONCLUSIONS

The computation of the eigenvalues of the linearized system determines the linear stability of the equilibrium points of the system. But, while the linear instability is sufficient to imply the general instability, a linearly stable point can be either nonlinear stable or unstable e.g. in case of a *center* with $\lambda = \pm ai,\ \ a \in \mathbb{R}$ (where nonlinear analysis is required to determine nonlinear stability) . Arnold's stability theorem can guarantee, under certain nonresonance and reversibility conditions, the Lyapunov stability for a subset of the linearly stable points. However, the points where resonance or non reversibility occurs, see for example in [4], is beyond the scope of this paper. We now apply Arnold's stability theorem in the nondegenerate case, far from the resonances.

Theorem 2. *(Arnold) Considering a Hamiltonian in Birkhoff's normal form up to degree n, real analytic in a neighborhood of a marginally stable equilibrium set at the origin of the phase space.*
If the two conditions

- (A_∞) *(general nonresonance condition):*

$$k_1\lambda + k_2\nu \neq 0,\quad \forall k_1, k_2 \in \mathbb{N}, \tag{46}$$

- (B) *(reversibility condition):*
  $A\lambda^2 + 2B\lambda\nu + C\nu^2 \neq 0$

are verified, then, on each energy manifold $H = h$, in the neighborhood of the equilibrium, the linearly stable equilibria is Lyapunov stable.

However, as Moser showed, this theorem is still valid under the weaker conditions (A_4) and (B); the fourth order Normal Form Hamiltonian (31) found in the previous Sections, with A, B, C as in (45), is therefore enough to apply this simplified stability theorem.
The result is that Arnold's stability theorem guarantees the Lyapunov stability along the linearly stable domain close to the Asteroid with the exception of a set of points with zero measure where either condition (A_s) or (B) are not satisfied and the theorem cannot deduce stability.

The resulting curves, which must be excluded from the domain of the linearly stable equilibria in order to satisfy the required condition (A_4) and (B), are plotted in Fig 4 and Fig 5, respectively the points where resonances between the frequencies or non reversibility occur.
In Fig 4 the resonance arcs of points which do not satisfy the first condition, going from the more internal to the external ones represent the $(1:1)$ resonance (in black), the $(2:1)$ and $(1:2)$ resonances (in purple), the $(3:1)$ and $(1:3)$ resonances (in cyan) and the $(4:1)$ and $(1:4)$ resonances (in blue).
On the other hand the curve resulting from the linearly stable points excluded to satisfy the second condition is plotted in Fig. 5.

Therefore, considering both the conditions necessary to apply Arnold's theorem (i.e. considering both the graphs) we can conclude that the set of the Lyapunov stable equilibrium points and of the linearly stable ones, only differ for a set with zero measure (the resonant arcs in Figure 4 and the non reversible curve in Fig. 5). The stability over these curves, instead, cannot be verified using the Arnold's theorem and a higher order Normalized Hamiltonian must be take into account.

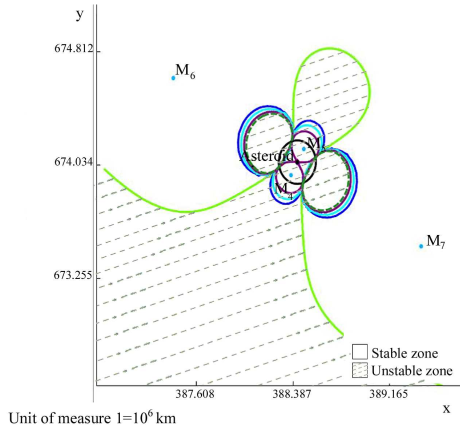

Figure 4: The points where frequency resonances occur where Arnold's theorem cannot guarantee the Lyapunov stability.

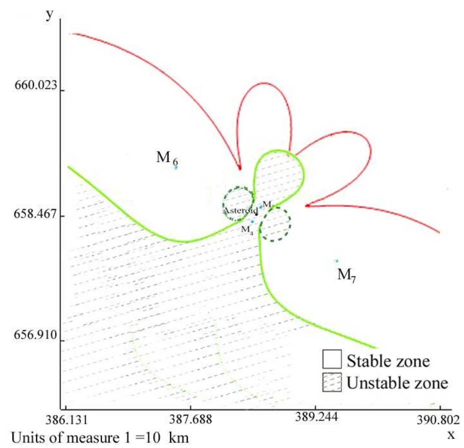

Figure 5: Condition (B) is satisfied by all the linearly stable points except by those on the red line.

References

[1] Leontovich, A.: 'On the stability of the lagrange periodic solutions of the restricted three-body problem'. Dokl. Math. (in Russian) 3, 425-428 (1962)

[2] Routh, E.J.: 'On Laplace's three particles, with a supplement on the stability of steady motion', Proc. Lond. Math. Soc., 6, pp. 86-97 (1875)

[3] Deprit, A., Deprit-Bartholome, A.: 'Stability of the triangular Lagrangian points', Astr. J., Vol. 72, p. 173 (1967)

[4] Meyer, K.R., Schmidt D.S.: 'The Stability of the Lagrange Triangular Point and a Theorem of Arnold', Journal of differential equations 62, 222-236 (1986)

[5] Rink, B., Tuwankotta, T.: 'Stability in Hamiltonian Systems: Applications to the restricted three-body problem' London Mathematical Society Lecture Note Series (2005)

[6] Giorgilli, A., Delshams, A., Fontich, E., Galgani, L., Simo, C.: 'Effective stability for a Hamiltonian system near an elliptic equilibrium point, with an application to the restricted three body problem', Jour. of Diff. Eq. Volume 77, Issue 1, January 1989, pp. 167-198

[7] Markeev, A.: 'Stability of the triangular lagrangian solutions of the restricted three-body problem in the threedimensional circular case'. Sov. Astron. 15(4), pp. 682-686 (1972)

[8] Celletti, A.,Giorgilli, A.: 'On the stability of the lagrangian points in the spatial restricted problem of threebody', Celest. Mech. Dyn. Astron. 50, 31-58 (1991)

[9] Birkhoff,G.D.: 'Dynamical Systems', Vol. IX, American Mathematical Society Colloquium Publications, New York, (1927).

[10] Arnold, V., I.: 'The stability of the equilibrium position of a Hamiltonian system of ordinary differential equations in the general elliptic case', Soviet Math. Dokl. 2, pp.247-249 (1961)

[11] Robnik ,M.: 'The algebraic quantisation of the Birkhoff-Gustavson normal form', J. Phys. A: Math. Gen. 17 pp.109-130 (1984)

[12] Siegel C.L.: 'On integrals of canonical systems', Ann. Math. 42, pp. 806-822 (1941).

[13] Bombardelli C., Pelaez, J.: 'On the stability of artificial equilibrium points in the circular restricted three-body problem', Cele. Mech. and Dyn. Astr, Online First

[14] Ceccaroni, M., Biggs, J.: 'Extension of low-thrust propulsion to the Autonomous Coplanar Circular Restricted Four Body Problem with application to future Trojan Asteroid missions', in 61^{st} International Astronautical Congress, IAC-10-1.1.3, Prague (2010)

[15] Baltagiannis, A.N., Papadakis K.E.: 'Equilibrium Points and Their Stability In The Restricted Four Body Problem', International Journal of Bifurcation and Chaos, World Scientific Publishing Company, accepted (2011)

APPENDIX A

$$h^{(3)}_{3,0,0,0} = -\frac{\mu(-1+q_{x\,e}+\mu)[-2(-1+q_{x\,e}+\mu)^2+3q_{y\,e}^2]}{2\sqrt{(q_{x\,e}+\mu-1)^2+q_{y\,e}^2}^7} - \frac{(1-\mu)(q_{x\,e}+\mu)[-2(q_{x\,e}+\mu)^2+3q_{y\,e}^2]}{2\sqrt{(q_{x\,e}+\mu)^2+q_{y\,e}^2}^7} - \frac{\epsilon(q_{x\,e}-Lx)[-2(q_{x\,e}-Lx)^2+3(q_{y\,e}-Ly)^2]}{2\sqrt{(q_{x\,e}-Lx)^2+(q_{y\,e}-Ly)^2}^7};$$

$$h^{(3)}_{0,3,0,0} = -\frac{\mu q_{y\,e}[3(-1+q_{x\,e}+\mu)^2-2q_{y\,e}^2]}{2\sqrt{(q_{x\,e}+\mu-1)^2+q_{y\,e}^2}^7} - \frac{(1-\mu)q_{y\,e}[3(q_{x\,e}+\mu)^2-2q_{y\,e}^2]}{2\sqrt{(q_{x\,e}+\mu)^2+q_{y\,e}^2}^7} - \frac{\epsilon(q_{y\,e}-Ly)[3(q_{x\,e}-Lx)^2-2(q_{y\,e}-Ly)^2]}{2\sqrt{(q_{x\,e}-Lx)^2+(q_{y\,e}-Ly)^2}^7};$$

$$h^{(3)}_{2,1,0,0} = -\frac{3\mu q_{y\,e}[q_{y\,e}^2-4(-1+q_{x\,e}+\mu)^2]}{2\sqrt{(q_{x\,e}+\mu-1)^2+q_{y\,e}^2}^7} - \frac{3(1-\mu)q_{y\,e}[q_{y\,e}^2-4(q_{x\,e}+\mu)^2]}{2\sqrt{(q_{x\,e}+\mu)^2+q_{y\,e}^2}^7} - \frac{3\epsilon(q_{y\,e}-Ly)[(q_{y\,e}-Ly)^2-4(q_{x\,e}-Lx)^2]}{2\sqrt{(q_{x\,e}-Lx)^2+(q_{y\,e}-Ly)^2}^7};$$

$$h^{(3)}_{1,2,0,0} = -\frac{3\mu(-1+q_{x\,e}+\mu)[-4q_{y\,e}^2+(-1+q_{x\,e}+\mu)^2]}{2\sqrt{(q_{x\,e}+\mu-1)^2+q_{y\,e}^2}^7} - \frac{3(1-\mu)(q_{x\,e}+\mu)[-4q_{y\,e}^2+(q_{x\,e}+\mu)^2]}{2\sqrt{(q_{x\,e}+\mu)^2+q_{y\,e}^2}^7} - \frac{3\epsilon(q_{x\,e}-Lx)[-4(q_{y\,e}-Ly)^2+(q_{x\,e}-Lx)^2]}{2\sqrt{(q_{x\,e}-Lx)^2+(q_{y\,e}-Ly)^2}^7};$$

and all the other set to zero

APPENDIX B

$$h^{(4)}_{4,0,0,0} = -\frac{\mu\{35(-1+q_{x_e}+\mu)^4-30(-1+q_{x_e}+\mu)^2[q_{y_e}^2+(-1+q_{x_e}+\mu)^2]+3[q_{y_e}^2+(-1+q_{x_e}+\mu)^2]^2\}}{8\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^9} - \frac{(1-\mu)\{35(q_{x_e}+\mu)^4-30(q_{x_e}+\mu)^2[q_{y_e}^2+(q_{x_e}+\mu)^2]+3[q_{y_e}^2+(q_{x_e}+\mu)^2]^2\}}{8\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^9}$$
$$-\frac{\epsilon\{35(q_{x_e}-Lx)^4-30(q_{x_e}-Lx)^2[(q_{y_e}-Ly)^2+(q_{x_e}-Lx)^2]+3[(q_{y_e}-Ly)^2+(q_{x_e}-Lx)^2]^2\}}{8\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^9};$$

$$h^{(4)}_{0,4,0,0} = -\frac{\mu\{35q_{y_e}^4-30q_{y_e}^2[q_{y_e}^2+(-1+q_{x_e}+\mu)^2]+3[q_{y_e}^2+(-1+q_{x_e}+\mu)^2]^2\}}{8\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^9} - \frac{(1-\mu)\{35q_{y_e}^4-30q_{y_e}^2[q_{y_e}^2+(q_{x_e}+\mu)^2]+3[q_{y_e}^2+(q_{x_e}+\mu)^2]^2\}}{8\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^9}$$
$$-\frac{\epsilon\{35(q_{y_e}-Ly)^4-30(q_{y_e}-Ly)^2[(q_{y_e}-Ly)^2+(q_{x_e}-Lx)^2]+3[(q_{y_e}-Ly)^2+(q_{x_e}-Lx)^2]^2\}}{8\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)}^9};$$

$$h^{(4)}_{3,1,0,0} = -\frac{\mu\{5q_{y_e}[-1+q_{x_e}+\mu][4(-1+q_{x_e}+\mu)^2-3q_{y_e}^2]\}}{2\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^9} - \frac{(1-\mu)\{5q_{y_e}(q_{x_e}+\mu)[4(q_{x_e}+\mu)^2-3q_{y_e}^2]\}}{2\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^9} - \frac{\epsilon\{5(q_{y_e}-Ly)(q_{x_e}-Lx)[7(q_{x_e}-Lx)^2-3(q_{y_e}-Ly)^2]\}}{2\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^9};$$

$$h^{(4)}_{1,3,0,0} = -\frac{\mu\{5q_{y_e}(-1+q_{x_e}+\mu)[4q_{y_e}^2-3(-1+q_{x_e}+\mu)^2]\}}{2\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^9} - \frac{(1-\mu)\{5q_{y_e}(q_{x_e}+\mu)[4q_{y_e}^2-3(q_{x_e}+\mu)^2]\}}{2\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^9} - \frac{\epsilon\{5(q_{y_e}-Ly)(q_{x_e}-Lx)[7(q_{y_e}-Ly)^2-3(q_{x_e}-Lx)^2]\}}{2\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^9};$$

$$h^{(4)}_{2,2,0,0} = -\frac{\mu\{-12[(q_{x_e}+\mu-1)^2+q_{y_e}^2]^2+105q_{y_e}^2(q_{x_e}+\mu-1)^2\}}{4\sqrt{(q_{x_e}+\mu-1)^2+q_{y_e}^2}^9} - \frac{(1-\mu)\{-12[(q_{x_e}+\mu)^2+q_{y_e}^2]^2+105q_{y_e}^2(q_{x_e}+\mu)^2\}}{4\sqrt{(q_{x_e}+\mu)^2+q_{y_e}^2}^9} - \frac{\epsilon\{-12[(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2]^2+105(q_{y_e}-Ly)^2(q_{x_e}-Lx)^2\}}{4\sqrt{(q_{x_e}-Lx)^2+(q_{y_e}-Ly)^2}^9};$$

and all the other set to zero

APPENDIX C

$$\begin{aligned}
\bar{h}^{(3)}_{3,0,0,0} = {} & \frac{h^{(3)}{}_{3,0,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\lambda^{(3/2)}(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}} + \frac{{h^{(2)}}^{3}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}} \\
& + \frac{{h^{(2)}}^{2}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}} \\
& + \frac{h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{2,1,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}};
\end{aligned}$$

$$\begin{aligned}
\bar{h}^{(3)}_{0,3,0,0} = {} & \frac{{h^{(2)}}^{3}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^3}{\nu^{(3/2)}({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}} + \frac{h^{(3)}{}_{3,0,0,0}({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)^{(3/2)}}{\nu^{(3/2)}(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}} \\
& + \frac{{h^{(2)}}^{2}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^2}{\nu^{(3/2)}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}} + \frac{h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{2,1,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}}{\nu^{(3/2)}(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}};
\end{aligned}$$

$$\bar{h}^{(3)}_{0,0,3,0} = \frac{8h^{(3)}{}_{0,3,0,0}\lambda^(3/2)(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)(3/2)};$$

$$\bar{h}^{(3)}_{0,0,0,3} = \frac{8h^{(3)}{}_{0,3,0,0}\nu^{(3/2)}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^3}{({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)(3/2)(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}};$$

$$\begin{aligned}
\bar{h}^{(3)}_{2,1,0,0} = {} & \frac{(h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{2,1,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})} \\
& + \frac{(3{h^{(2)}}^{3}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{-3+h^{(2^2)}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})} \\
& + \frac{(2{h^{(2)}}^{2}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})} \\
& + \frac{(3h^{(3)}{}_{3,0,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2})}{(\lambda(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})} \\
& + \frac{({h^{(2)}}^{2}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2})}{(\lambda(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})} \\
& + \frac{(2h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{2,1,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2})}{(\lambda(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})};
\end{aligned}$$

$$
\begin{aligned}
\bar{h}^{(3)}_{2,0,1,0} = {} & \frac{6{h^{(2)}}^2_{1,1,0,0}h^{(3)}_{0,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}} \\
& +\frac{4h^{(2)}_{1,1,0,0}h^{(3)}_{1,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}} \\
& +\frac{2h^{(3)}_{2,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{2,0,0,1} = {} & \frac{(2h^{(3)}_{2,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4})} \\
& +\frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(3)}_{0,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4})} \\
& +\frac{(4h^{(2)}_{1,1,0,0}h^{(3)}_{1,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{1,2,0,0} = {} & \frac{(2h^{(2)}_{1,1,0,0}h^{(3)}_{2,1,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(2{h^{(2)}}^2_{1,1,0,0}h^{(3)}_{1,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+h^{(2^2)}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{({h^{(2)}}^2_{1,1,0,0}h^{(3)}_{1,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(3{h^{(2)}}^3_{1,1,0,0}h^{(3)}_{0,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(3h^{(3)}_{3,0,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2))}{(\sqrt{\lambda}\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(h^{(2)}_{1,1,0,0}h^{(3)}_{2,1,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{0,2,1,0} = {} & \frac{(4h^{(2)}_{1,1,0,0}h^{(3)}_{1,2,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(3)}_{0,3,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(2h^{(3)}_{2,1,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{0,2,0,1} = {} & \frac{6{h^{(2)}}^2_{1,1,0,0}h^{(3)}_{0,3,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3}{\sqrt{\nu}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}} \\
& +\frac{4h^{(2)}_{1,1,0,0}h^{(3)}_{1,2,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2}{\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}} \\
& +\frac{2h^{(3)}_{2,1,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}}{\sqrt{\nu}(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}};
\end{aligned}
$$

$$\bar{h}^{(3)}_{1,0,2,0} = \frac{12{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{0,3,0,0}\sqrt{\lambda}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{(1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^3(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}$$

$$+\frac{4{h^{(3)}}_{1,2,0,0}\sqrt{\lambda}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}}{(1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}};$$

$$\bar{h}^{(3)}_{0,1,2,0} = \frac{(12{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{0,3,0,0}\lambda(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)(1-2{h^{(2)}}_{2,0,0,0}+\nu^2))}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})}$$
$$+\frac{(4{h^{(3)}}_{1,2,0,0}\lambda(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})};$$

$$\bar{h}^{(3)}_{0,0,2,1} = \frac{(24{h^{(3)}}_{0,3,0,0}\lambda(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2{h^{(2)}}_{2,0,0,0}+\nu^2))}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})};$$

$$\bar{h}^{(3)}_{1,0,0,2} = \frac{(4{h^{(3)}}_{1,2,0,0}\sqrt{1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}\nu(1-2{h^{(2)}}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(12{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{0,3,0,0}\sqrt{1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}\nu(1-2{h^{(2)}}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)\sqrt{-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4))};$$

$$\bar{h}^{(3)}_{0,1,0,2} = \frac{12{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{0,3,0,0}\sqrt{\nu}(1-2{h^{(2)}}_{2,0,0,0}+\nu^2)^3}{({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}}$$
$$+\frac{4{h^{(3)}}_{1,2,0,0}\sqrt{\nu}(1-2{h^{(2)}}_{2,0,0,0}+\nu^2)^2}{\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}};$$

$$\bar{h}^{(3)}_{0,0,1,2} = \frac{24{h^{(3)}}_{0,3,0,0}\sqrt{\lambda}\sqrt{1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}\nu(1-2{h^{(2)}}_{2,0,0,0}+\nu^2)^2)}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)\sqrt{-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4))};$$

$$\bar{h}^{(3)}_{1,1,1,0} = \frac{(12{h^{(2)}}^2_{1,1,0,0}{h^{(3)}}_{0,3,0,0}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)(1-2{h^{(2)}}_{2,0,0,0}+\nu^2))}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})}$$
$$+\frac{(4{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{1,2,0,0}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)(1-2{h^{(2)}}_{2,0,0,0}+\nu^2))}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})}$$
$$+\frac{(4{h^{(2)}}_{1,1,0,0}{h^{(3)}}_{1,2,0,0}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)^2(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})}$$
$$+\frac{(4{h^{(3)}}_{2,1,0,0}(1-4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{((1-2{h^{(2)}}_{0,2,0,0}+\lambda^2)(-3+4{h^{(2)}}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4{h^{(2)}}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}\sqrt{-3+{h^{(2)}}^2_{1,1,0,0}+4{h^{(2)}}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4{h^{(2)}}_{2,0,0,0}\nu^2+\nu^4})};$$

$$
\begin{aligned}
\bar{h}^{(3)}_{1,1,0,1} = {} & \frac{(4h^{(3)}{}_{2,1,0,0}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))};\\
& +\frac{(4h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))}\\
& +\frac{(4h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{1,2,0,0}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(h^{(2)2}{}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))}\\
& +\frac{(12h^{(2)2}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(h^{(2)2}{}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{1,0,1,1} = {} & \frac{(24h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{h^{(2)2}{}_{1,1,0,0}+4\nu^2}\sqrt{-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})}\\
& +\frac{(8h^{(3)}{}_{1,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{\nu}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)\sqrt{h^{(2)2}{}_{1,1,0,0}+4\nu^2}\sqrt{-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(3)}_{0,1,1,1} = {} & \frac{(8h^{(3)}{}_{1,2,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))}\\
& +\frac{(24h^{(2)}{}_{1,1,0,0}h^{(3)}{}_{0,3,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)\sqrt{-3+4h^{(2)}{}_{0,2,0,0}+4h^{(2)2}{}_{0,2,0,0}+h^{(2)2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4}(h^{(2)2}{}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4h^{(2)2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}
$$

APPENDIX D

$$
\begin{aligned}
\bar{h}^{(4)}_{4,0,0,0} = {} & \frac{(h^{(4)}{}_{4,0,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)} \\
& + \frac{({h^{(2)}}^{4}{}_{1,1,0,0}h^{(4)}{}_{0,4,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda^2(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^4(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)} \\
& + \frac{({h^{(2)}}^{3}{}_{1,1,0,0}h^{(4)}{}_{1,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda^2(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)} \\
& + \frac{({h^{(2)}}^{2}{}_{1,1,0,0}h^{(4)}{}_{2,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda^2(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)} \\
& + \frac{(h^{(2)}{}_{1,1,0,0}h^{(4)}{}_{3,1,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda^2(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{0,4,0,0} = {} & \frac{{h^{(2)}}^{2}{}_{1,1,0,0}h^{(4)}{}_{2,2,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^2}{\nu^2(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2} \\
& + \frac{({h^{(2)}}^{4}{}_{1,1,0,0}h^{(4)}{}_{0,4,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^4)}{(\nu^2({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)^2(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2)} \\
& + \frac{({h^{(2)}}^{3}{}_{1,1,0,0}h^{(4)}{}_{1,3,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^3)}{(\nu^2({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2)} \\
& + \frac{(h^{(2)}{}_{1,1,0,0}h^{(4)}{}_{3,1,0,0}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2))}{(\nu^2(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2)} \\
& + \frac{(h^{(4)}{}_{4,0,0,0}({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)^2)}{(\nu^2(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2)};
\end{aligned}
$$

$$
\bar{h}^{(4)}_{0,0,4,0} = \frac{(16h^{(4)}{}_{0,4,0,0}\lambda^2(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^4(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^2)};
$$

$$
\bar{h}^{(4)}_{0,0,0,4} = \frac{(16h^{(4)}{}_{0,4,0,0}\nu^2(1-2h^{(2)}{}_{2,0,0,0}+\nu^2)^4)}{(({h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2)^2(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)^2)};
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{3,1,0,0} = {} & \frac{(h^{(2)}{}_{1,1,0,0}h^{(4)}{}_{3,1,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda^{(3/2)}(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})} \\
& + \frac{(4{h^{(2)}}^{4}{}_{1,1,0,0}h^{(4)}{}_{0,4,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})} \\
& + \frac{(3{h^{(2)}}^{3}{}_{1,1,0,0}h^{(4)}{}_{1,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})} \\
& + \frac{(2{h^{(2)}}^{2}{}_{1,1,0,0}h^{(4)}{}_{2,2,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}{}_{2,0,0,0}+\nu^2))}{(\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})} \\
& + \frac{(4h^{(4)}{}_{4,0,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2})}{(\lambda^{(3/2)}(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})} \\
& + \frac{({h^{(2)}}^{3}{}_{1,1,0,0}h^{(4)}{}_{1,3,0,0}(1-4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}{}_{1,1,0,0}+4\nu^2})}{(\lambda^{(3/2)}(1-2h^{(2)}{}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}{}_{0,2,0,0}+4{h^{(2)}}^{2}{}_{0,2,0,0}+{h^{(2)}}^{2}{}_{1,1,0,0}+2\lambda^2-4h^{(2)}{}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^{2}{}_{1,1,0,0}+4h^{(2)}{}_{2,0,0,0}+4{h^{(2)}}^{2}{}_{2,0,0,0}+2\nu^2-4h^{(2)}{}_{2,0,0,0}\nu^2+\nu^4)})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{3,0,1,0} = &+\frac{(8{h^{(2)}}^3_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^4(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}\\
&+\frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}\\
&+\frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}\\
&+\frac{(2h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{3,0,0,1} = &+\frac{(2h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda^{(}3/2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}\\
&+\frac{(8{h^{(2)}}^3_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda^{(}3/2)(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}\\
&+\frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda^{(}3/2)(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}\\
&+\frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda^{(}3/2)(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(3/2)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{1,3,0,0} = &\frac{({h^{(2)}}^3_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(4{h^{(2)}}^4_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(2{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{2,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(3{h^{(2)}}^3_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(3h^{(2)}_{1,1,0,0}h^{(4)}_{3,1,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(2{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{2,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(4h^{(4)}_{4,0,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^{(3/2)})}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}\\
&+\frac{(h^{(2)}_{1,1,0,0}h^{(4)}_{3,1,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^{(3/2)})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(}3/2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})};
\end{aligned}
$$

$$\bar{h}^{(4)}_{0,3,1,0} = \frac{(8{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\nu^{(3/2)}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(6{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu^{(3/2)}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})}$$
$$+\frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\nu^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(2h^{(4)}_{3,1,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\nu^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};$$

$$\bar{h}^{(4)}_{0,3,0,1} = \frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)}\frac{(8{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^4)}{(\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^2(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)}$$
$$+\frac{(6{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)}+\frac{(2h^{(4)}_{3,1,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{(\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)};$$

$$\bar{h}^{(4)}_{1,0,3,0} = \frac{(32h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^4(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}+\frac{(8h^{(4)}_{1,3,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)};$$

$$\bar{h}^{(4)}_{0,1,3,0} = \frac{(32h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\lambda^{(3/2)}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+\frac{(8h^{(4)}_{1,3,0,0}\lambda^{(3/2)}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};$$

$$\bar{h}^{(4)}_{0,0,3,1} = \frac{(64h^{(4)}_{0,4,0,0}\lambda^{(3/2)}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};$$

$$\bar{h}^{(4)}_{1,0,0,3} = \frac{(8h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(32h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\nu^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};$$

$$\bar{h}^{(4)}_{0,1,0,3} = \frac{(32h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^4)}{(({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^2(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)}+\frac{(8h^{(4)}_{1,3,0,0}\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)};$$

$$\bar{h}^{(4)}_{0,0,1,3} = \frac{(64h^{(4)}_{0,4,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\,\nu^{(}3/2)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+h^{(2^2)}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)}})};$$

$$\begin{aligned}
\bar{h}^{(4)}_{2,2,0,0} = {} & \frac{(3h^{(2)}_{1,1,0,0}h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(3{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(4{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{({h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(6{h^{(2)}}^{4}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(3{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(6h^{(4)}_{4,0,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{({h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(3h^{(2)}_{1,1,0,0}h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}$$

$$\begin{aligned}
\bar{h}^{(4)}_{2,0,2,0} = {} & \frac{(24{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^4(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}\\
& + \frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}\\
& + \frac{(4h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^2)};
\end{aligned}$$

$$\begin{aligned}
\bar{h}^{(4)}_{2,0,0,2} = {} & \frac{(4h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(24{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}$$

$$\begin{aligned}
\bar{h}^{(4)}_{0,2,2,0} = {} & \frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(24{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{0,4,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}\\
& + \frac{(4h^{(4)}_{2,2,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}$$

$$\bar{h}^{(4)}_{0,2,0,2} = \frac{4h^{(4)}_{2,2,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2}{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2} + \frac{(24{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^4)}{(({h^{(2)}}^2_{1,1,0,0}+4\nu^2)^2(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)} + \frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^2)};$$

$$\bar{h}^{(4)}_{0,0,2,2} = \frac{96h^{(4)}_{0,4,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)};$$

$$\bar{h}^{(4)}_{2,1,1,0} = \frac{(24{h^{(2)}}^3_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(12{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(8h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(6h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};$$

$$\bar{h}^{(4)}_{2,1,0,1} = \frac{(6h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+ \frac{(6{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+ \frac{(8h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+ \frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+ \frac{(24{h^{(2)}}^3_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+ \frac{(12{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\lambda(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^2_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};$$

$$\bar{h}^{(4)}_{2,0,1,1} = \frac{(48{h^{(2)}}^2_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(24h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+ \frac{(8h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^2_{0,2,0,0}+{h^{(2)}}^2_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^2_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^2_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^2_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};$$

$$
\begin{aligned}
\bar{h}^{(4)}_{1,2,1,0} =\ & \frac{(12{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(8h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(24{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(6{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))} \\
& +\frac{(6h^{(4)}_{3,1,0,0}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{1,2,0,1} =\ & \frac{(6{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(24{h^{(2)}}^{3}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(8h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(12{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(6h^{(4)}_{3,1,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(4h^{(2)}_{1,1,0,0}h^{(4)}_{2,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{0,2,1,1} =\ & \frac{(48{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}({h^{(2)}}^{2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(24h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})} \\
& +\frac{(8h^{(4)}_{2,2,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}(1-2h^{(2)}_{2,0,0,0}+\nu^2)\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{\nu}(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};
\end{aligned}
$$

$$
\begin{aligned}
\bar{h}^{(4)}_{1,1,2,0} =\ & \frac{(48{h^{(2)}}^{2}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})} \\
& +\frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})} \\
& +\frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})} \\
& +\frac{(8h^{(4)}_{2,2,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{{h^{(2)}}^{2}_{1,1,0,0}+4\nu^2})}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4{h^{(2)}}^{2}_{0,2,0,0}+{h^{(2)}}^{2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}\sqrt{(-3+{h^{(2)}}^{2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4{h^{(2)}}^{2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};
\end{aligned}
$$

$$\bar{h}^{(4)}_{1,0,2,1} = \frac{(96h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^3(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(3/2)\sqrt{h^{(2)2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})}$$
$$+\frac{(24h^{(4)}_{1,3,0,0}\sqrt{\lambda}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)^{(3/2)}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(3/2)\sqrt{h^{(2)2}_{1,1,0,0}+4\nu^2}\sqrt{(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)})};$$

$$\bar{h}^{(4)}_{0,1,2,1} = \frac{(24h^{(4)}_{1,3,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(96h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\lambda(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(h^{(2)2}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};$$

$$\bar{h}^{(4)}_{1,1,0,2} = \frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}(h^{(2)2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(48h^{(2)2}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}(h^{(2)2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(8h^{(4)}_{2,2,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{h^{(2)2}_{1,1,0,0}+4\nu^2}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(12h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{(\sqrt{\lambda}(1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{h^{(2)2}_{1,1,0,0}+4\nu^2}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};$$

$$\bar{h}^{(4)}_{1,0,1,2} = \frac{(96h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(h^{(2)2}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(24h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)\nu(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(h^{(2)2}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};$$

$$\bar{h}^{(4)}_{0,1,1,2} = \frac{(96h^{(2)}_{1,1,0,0}h^{(4)}_{0,4,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^3)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}(h^{(2)2}_{1,1,0,0}+4\nu^2)^{(3/2)}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})}$$
$$+\frac{(24h^{(4)}_{1,3,0,0}\sqrt{\lambda}\sqrt{1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4}\sqrt{\nu}(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)\sqrt{(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)}\sqrt{h^{(2)2}_{1,1,0,0}+4\nu^2}(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4)^{(3/2)})};$$

$$\bar{h}^{(4)}_{1,1,1,1} = \frac{(24h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(16h^{(4)}_{2,2,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2))}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(96h^{(2)2}_{1,1,0,0}h^{(4)}_{0,4,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)^2(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(h^{(2)2}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))}$$
$$+\frac{(24h^{(2)}_{1,1,0,0}h^{(4)}_{1,3,0,0}(1-4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(1-2h^{(2)}_{2,0,0,0}+\nu^2)^2)}{((1-2h^{(2)}_{0,2,0,0}+\lambda^2)(-3+4h^{(2)}_{0,2,0,0}+4h^{(2)2}_{0,2,0,0}+h^{(2)2}_{1,1,0,0}+2\lambda^2-4h^{(2)}_{0,2,0,0}\lambda^2+\lambda^4)(h^{(2)2}_{1,1,0,0}+4\nu^2)(-3+h^{(2)2}_{1,1,0,0}+4h^{(2)}_{2,0,0,0}+4h^{(2)2}_{2,0,0,0}+2\nu^2-4h^{(2)}_{2,0,0,0}\nu^2+\nu^4))};$$

APPENDIX E

$$\tilde{h}^{(4)}_{4,0,0,0} = \bar{h}^{(4)}_{4,0,0,0} + \frac{(-\bar{h}^{(3)^2}_{3,0,0,0} + (2\bar{h}^{(3)}_{0,0,3,0} + \bar{h}^{(3)}_{2,0,1,0})^2)}{2\lambda} + \frac{(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)^2}{2\nu(4\lambda^2-\nu^2)^2} - \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 - \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{2,1,0,0}\nu^2)^2}{2\nu(-4\lambda^2+\nu^2)^2};$$

$$\tilde{h}^{(4)}_{0,4,0,0} = \bar{h}^{(4)}_{0,4,0,0} + \frac{(-\bar{h}^{(3)^2}_{0,3,0,0} + (-2\bar{h}^{(3)}_{0,0,0,3} - \bar{h}^{(3)}_{0,2,0,1})^2)}{2\nu} + \frac{(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)^2}{2\lambda(\lambda^2-4\nu^2)^2} - \frac{(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2 - \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)^2}{2\lambda(\lambda^2-4\nu^2)^2};$$

$$\tilde{h}^{(4)}_{0,0,4,0} = \bar{h}^{(4)}_{0,0,4,0} - \frac{(-\bar{h}^{(3)^2}_{0,0,3,0} + (\bar{h}^{(3)}_{1,0,2,0} + 2\bar{h}^{(3)}_{3,0,0,0})^2)}{2\lambda} - \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 + \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{0,1,2,0}\nu^2)^2}{2\nu(-4\lambda^2+\nu^2)^2} + \frac{(-(\bar{h}^{(3)}_{0,0,2,1} + \bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2) + 2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)^2}{2\nu(4\lambda^2-\nu^2)^2};$$

$$\tilde{h}^{(4)}_{0,0,0,4} = \bar{h}^{(4)}_{0,0,0,4} - \frac{(-\bar{h}^{(3)^2}_{0,0,0,3} + (\bar{h}^{(3)}_{0,1,0,2} + 2\bar{h}^{(3)}_{0,3,0,0})^2)}{2\nu} - \frac{(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2 + \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)^2}{2\lambda(\lambda^2-4\nu^2)^2} + \frac{(-(\bar{h}^{(3)}_{0,0,1,2} + \bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2) + \bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)^2}{2\lambda(\lambda^2-4\nu^2)^2};$$

$$\begin{aligned}\tilde{h}^{(4)}_{3,1,0,0} = \bar{h}^{(4)}_{3,1,0,0} &+ \frac{\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda - 2\bar{h}^{(3)}_{2,1,0,0}\lambda - \bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} - \frac{2(-2\bar{h}^{(3)}_{0,0,3,0} - \bar{h}^{(3)}_{2,0,1,0})(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)}\\ &+ \frac{2(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} + \frac{(-\bar{h}^{(3)}_{0,1,1,1}\lambda + 2\bar{h}^{(3)}_{1,0,0,2}\nu - 2\bar{h}^{(3)}_{1,2,0,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 - \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{2,1,0,0}\nu^2)}{(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{3,0,1,0} = \bar{h}^{(4)}_{3,0,1,0} &+ \frac{(2\bar{h}^{(3)}_{3,0,0,0}(-2\bar{h}^{(3)}_{0,0,3,0} - \bar{h}^{(3)}_{2,0,1,0}) + 2\bar{h}^{(3)}_{0,0,3,0}\bar{h}^{(3)}_{3,0,0,0})}{\lambda} + \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda - 2\bar{h}^{(3)}_{2,1,0,0}\lambda - \bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(4\lambda^2-\nu^2)^2}\\ &+ \frac{(-2\bar{h}^{(3)}_{0,0,2,1}\lambda + 2\bar{h}^{(3)}_{2,0,0,1}\lambda - \bar{h}^{(3)}_{1,1,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 - \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{2,1,0,0}\nu^2)}{(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{3,0,0,1} = \bar{h}^{(4)}_{3,0,0,1} &+ \frac{\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda - 2\bar{h}^{(3)}_{2,0,0,1}\lambda + \bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{(\bar{h}^{(3)}_{0,1,1,1}\lambda - 2\bar{h}^{(3)}_{1,0,0,2}\nu + 2\bar{h}^{(3)}_{1,2,0,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)}\\ &- \frac{2(-2\bar{h}^{(3)}_{0,0,3,0} - \bar{h}^{(3)}_{2,0,1,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 - \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)}\\ &+ \frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 - \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{2,1,0,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2} + \bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2) + \bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(-4\lambda^2+\nu^2)(\lambda^2-4\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{1,3,0,0} = \bar{h}^{(4)}_{1,3,0,0} &+ \frac{\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{0,1,1,1}\lambda - 2\bar{h}^{(3)}_{1,0,0,2}\nu + 2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} - \frac{2(-2\bar{h}^{(3)}_{0,0,0,3} - \bar{h}^{(3)}_{0,2,0,1})(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\ &+ \frac{(-2\bar{h}^{(3)}_{0,1,2,0}\lambda + 2\bar{h}^{(3)}_{2,1,0,0}\lambda + \bar{h}^{(3)}_{1,0,1,1}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2 - \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} + \frac{2(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,3,1,0} = \bar{h}^{(4)}_{0,3,1,0} &+ \frac{\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{1,1,0,1}\lambda + 2\bar{h}^{(3)}_{0,0,1,2}\nu - 2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} + \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda - 2\bar{h}^{(3)}_{2,1,0,0}\lambda - \bar{h}^{(3)}_{1,0,1,1}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)}\\ &- \frac{2(-2\bar{h}^{(3)}_{0,0,0,3} - \bar{h}^{(3)}_{0,2,0,1})(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2 - \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\ &+ \frac{2(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2 - \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(-(4\lambda^2-\nu^2)(\bar{h}^{(3)}_{0,0,2,1} + \bar{h}^{(3)}_{2,0,0,1}) + 2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)(\lambda^2-4\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,3,0,1} = \bar{h}^{(4)}_{0,3,0,1} &+ \frac{(2\bar{h}^{(3)}_{0,3,0,0}(-2\bar{h}^{(3)}_{0,0,0,3} - \bar{h}^{(3)}_{0,2,0,1}) + 2\bar{h}^{(3)}_{0,0,0,3}\bar{h}^{(3)}_{0,3,0,0})}{\nu} + \frac{(\bar{h}^{(3)}_{0,1,1,1}\lambda - 2\bar{h}^{(3)}_{1,0,0,2}\nu + 2\bar{h}^{(3)}_{1,2,0,0}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2 - \bar{h}^{(3)}_{1,1,0,1}\lambda\nu - 2\bar{h}^{(3)}_{0,0,1,2}\nu^2 - 2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(\lambda^2-4\nu^2)^2}\\ &+ \frac{(-\bar{h}^{(3)}_{1,1,0,1}\lambda - 2\bar{h}^{(3)}_{0,0,1,2}\nu + 2\bar{h}^{(3)}_{0,2,1,0}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2 - \bar{h}^{(3)}_{0,1,1,1}\lambda\nu + 2\bar{h}^{(3)}_{1,0,0,2}\nu^2 + 2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{1,0,3,0} = \bar{h}^{(4)}_{1,0,3,0} &+ \frac{(2\bar{h}^{(3)}_{0,0,3,0}(\bar{h}^{(3)}_{1,0,2,0} + 2\bar{h}^{(3)}_{3,0,0,0}) - 2\bar{h}^{(3)}_{0,0,3,0}\bar{h}^{(3)}_{3,0,0,0})}{\nu} + \frac{(-2\bar{h}^{(3)}_{0,0,2,1}\lambda + 2\bar{h}^{(3)}_{2,0,0,1}\lambda - \bar{h}^{(3)}_{1,1,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2 + 2\bar{h}^{(3)}_{2,1,0,0}\lambda^2 + \bar{h}^{(3)}_{1,0,1,1}\lambda\nu - \bar{h}^{(3)}_{0,1,2,0}\nu^2)}{(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)}\\ &- \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda - 2\bar{h}^{(3)}_{2,1,0,0}\lambda - \bar{h}^{(3)}_{1,0,1,1}\nu)(-(\bar{h}^{(3)}_{0,0,2,1} + \bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2) + 2\bar{h}^{(3)}_{0,0,2,1}\lambda^2 + 2\bar{h}^{(3)}_{2,0,0,1}\lambda^2 + \bar{h}^{(3)}_{1,1,1,0}\lambda\nu - \bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(4\lambda^2-\nu^2)^2};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,1,3,0} = \bar{h}^{(4)}_{0,1,3,0} &+ \frac{\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} + \frac{(-\bar{h}^{(3)}_{1,1,0,1}\lambda-2\bar{h}^{(3)}_{0,0,1,2}\nu+2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{(-4\lambda^2+\nu^2)(-\lambda^2+4\nu^2)}\\
&-\frac{2(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(-(4\lambda^2-\nu^2)(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)}\\
&-\frac{2(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,0,3,1} = \bar{h}^{(4)}_{0,0,3,1} &+ \frac{\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{2(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)}\\
&-\frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)}\\
&-\frac{(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(-\lambda^2+4\nu^2)(4\lambda^2-\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{1,0,0,3} = \bar{h}^{(4)}_{1,0,0,3} &+ \frac{\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} + \frac{(-2\bar{h}^{(3)}_{0,0,2,1}\lambda+2\bar{h}^{(3)}_{2,0,0,1}\lambda-\bar{h}^{(3)}_{1,1,1,0}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)}\\
&-\frac{2(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)(\lambda^2-4\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,1,0,3} = \bar{h}^{(4)}_{0,1,0,3} &+ \frac{(2\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})-2\bar{h}^{(3)}_{0,0,0,3}\bar{h}^{(3)}_{0,3,0,0})}{\nu} + \frac{(-\bar{h}^{(3)}_{1,1,0,1}\lambda-2\bar{h}^{(3)}_{0,0,1,2}\nu+2\bar{h}^{(3)}_{0,2,1,0}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)}\\
&-\frac{(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(\lambda^2-4\nu^2)^2};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{0,0,1,3} = \bar{h}^{(4)}_{0,0,1,3} &+ \frac{\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} + \frac{2(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\nu(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)}\\
&-\frac{(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(4\lambda^2-\nu^2)(\lambda^2-4\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{2,2,0,0} = \bar{h}^{(4)}_{2,2,0,0} &- \frac{\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)^2}{2(\lambda^2-4\nu^2)^2} - \frac{\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)^2}{2(4\lambda^2-\nu^2)^2} - \frac{(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&+\frac{2\nu(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)^2}{\lambda^2(\lambda^2-4\nu^2)^2} + \frac{\bar{h}^{(3)}_{3,0,0,0}(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{(-2\bar{h}^{(3)}_{0,0,0,3}-\bar{h}^{(3)}_{0,2,0,1})(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} + \frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)^2}{\nu^2(4\lambda^2-\nu^2)^2} + \frac{\bar{h}^{(3)}_{0,3,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{2,0,2,0} = \bar{h}^{(4)}_{2,0,2,0} &- \frac{\bar{h}^{(3)}_{0,0,3,0}(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})+\bar{h}^{(3)}_{3,0,0,0}(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})-(-2{\bar{h}^{(3)}_{0,0,3,0}}^2+2{\bar{h}^{(3)}_{3,0,0,0}}^2)}{\lambda} + \frac{\nu(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)^2-\nu(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)^2}{2(4\lambda^2-\nu^2)^2}\\
&-\frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)^2}\\
&-\frac{(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)^2};\end{aligned}$$

$$\begin{aligned}\tilde{h}^{(4)}_{2,0,0,2} = \bar{h}^{(4)}_{2,0,0,2} &+ \frac{\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)^2}{2(\lambda^2-4\nu^2)^2} - \frac{\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)^2}{2(4\lambda^2-\nu^2)^2} + \frac{\bar{h}^{(3)}_{3,0,0,0}(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&+\frac{\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} + \frac{(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)}\\
&+\frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)^2}{\nu^2(-4\lambda^2+\nu^2)^2} + \frac{(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})(-(\lambda^2-4\nu^2)(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{2\nu(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)^2}{\lambda^2(\lambda^2-4\nu^2)^2};\end{aligned}$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{0,2,2,0} = \bar{h}^{(4)}_{0,2,2,0} &+ \frac{\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)^2}{2(4\lambda^2-\nu^2)^2} - \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)^2}{2(-\lambda^2+4\nu^2)^2} + \frac{\bar{h}^{(3)}_{0,3,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} \\
&+ \frac{\bar{h}^{(3)}_{0,0,3,0}(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} + \frac{(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} + \frac{2\nu(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)^2}{\lambda^2(\lambda^2-4\nu^2)^2} \\
&+ \frac{(-2\bar{h}^{(3)}_{0,0,0,3}-\bar{h}^{(3)}_{0,2,0,1})(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} - \frac{2\lambda(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)^2}{\nu^2(4\lambda^2-\nu^2)^2};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{0,2,0,2} = \bar{h}^{(4)}_{0,2,0,2} &+ \frac{(\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,0,0,3}+\bar{h}^{(3)}_{0,2,0,1})-\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})-2\bar{h}^{(3)\,2}_{0,0,0,3}+2\bar{h}^{(3)\,2}_{0,3,0,0})}{\nu} + \frac{\lambda(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)^2}{2(\lambda^2-4\nu^2)^2} - \frac{\lambda(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)^2}{2(-\lambda^2+4\nu^2)^2} \\
&- \frac{(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2} \\
&- \frac{(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{0,0,2,2} = \bar{h}^{(4)}_{0,0,2,2} &+ \frac{\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)^2}{2(4\lambda^2-\nu^2)^2} + \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)^2}{2(-\lambda^2+4\nu^2)^2} + \frac{(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} \\
&- \frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)^2}{\nu^2(-4\lambda^2+\nu^2)^2} + \frac{(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} \\
&- \frac{2\nu(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)^2}{\lambda^2(\lambda^2-4\nu^2)^2} - \frac{\bar{h}^{(3)}_{0,0,3,0}(-(\lambda^2-4\nu^2)(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} \\
&- \frac{\bar{h}^{(3)}_{0,0,0,3}(-(4\lambda^2-\nu^2)(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{2,1,1,0} = \bar{h}^{(4)}_{2,1,0,1} &- \frac{2\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)+(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} - \frac{\nu(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} \\
&+ \frac{2\nu(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} - \frac{4\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} \\
&+ \frac{2(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} + \frac{(-\bar{h}^{(3)}_{1,1,0,1}\lambda-2\bar{h}^{(3)}_{0,0,1,2}\nu+2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{(-4\lambda^2+\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{2\bar{h}^{(3)}_{3,0,0,0}(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{2,1,0,1} = \bar{h}^{(4)}_{2,1,1,0} &- \frac{(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} - \frac{\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{(4\lambda^2-\nu^2)^2} + \frac{\bar{h}^{(3)}_{3,0,0,0}(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} \\
&+ \frac{2\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2} - \frac{2\bar{h}^{(3)}_{0,3,0,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} \\
&- \frac{2\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} + \frac{4\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu^2(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)} \\
&+ \frac{2\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{2,0,1,1} = \bar{h}^{(4)}_{2,0,1,1} &- \frac{2\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} - \frac{(-2\bar{h}^{(3)}_{0,0,3,0}-\bar{h}^{(3)}_{2,0,1,0})(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{\nu(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} \\
&+ \frac{2\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} + \frac{(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{4\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} - \frac{2(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} \\
&+ \frac{2\nu(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{1,2,1,0} = \bar{h}^{(4)}_{1,2,1,0} &- \frac{(-2\bar{h}^{(3)}_{0,0,0,3}-\bar{h}^{(3)}_{0,2,0,1})(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} + \frac{\bar{h}^{(3)}_{0,3,0,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} - \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{2\bar{h}^{(3)}_{3,0,0,0}(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} - \frac{2\bar{h}^{(3)}_{0,0,3,0}(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} \\
&+ \frac{4\nu(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda^2(\lambda^2-4\nu^2)^2} + \frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)^2} \\
&+ \frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(-(4\lambda^2-\nu^2)(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)^2};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{1,2,0,1} = \bar{h}^{(4)}_{1,2,0,1} &- \frac{2\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)+(-2\bar{h}^{(3)}_{0,0,0,3}-\bar{h}^{(3)}_{0,2,0,1})(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} - \frac{\lambda(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{4\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} + \frac{(-2\bar{h}^{(3)}_{0,0,2,1}\lambda+2\bar{h}^{(3)}_{2,0,0,1}\lambda-\bar{h}^{(3)}_{1,1,1,0}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} \\
&+ \frac{2\lambda(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} + \frac{2(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} \\
&- \frac{2\bar{h}^{(3)}_{0,3,0,0}(-(\lambda^2-4\nu^2)(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{0,2,1,1} = \bar{h}^{(4)}_{0,2,1,1} &+ \frac{\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} - \frac{2\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} - \frac{(-2\bar{h}^{(3)}_{0,0,0,3}-\bar{h}^{(3)}_{0,2,0,1})(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} \\
&+ \frac{(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} - \frac{4\bar{h}^{(3)}_{0,3,0,0}(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} \\
&- \frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\nu(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} + \frac{2\bar{h}^{(3)}_{0,3,0,0}(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} \\
&+ \frac{2\lambda(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{1,1,2,0} = \bar{h}^{(4)}_{1,1,2,0} &- \frac{2\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} + \frac{(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} - \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)} \\
&+ \frac{(-\bar{h}^{(3)}_{0,1,1,1}\lambda+2\bar{h}^{(3)}_{1,0,0,2}\nu-2\bar{h}^{(3)}_{1,2,0,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} + \frac{2\nu(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} \\
&+ \frac{2\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} + \frac{4\bar{h}^{(3)}_{0,0,3,0}(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} \\
&- \frac{2(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{1,0,2,1} = \bar{h}^{(4)}_{1,0,2,1} &- \frac{2\bar{h}^{(3)}_{3,0,0,0}(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{4\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} - \frac{2\nu(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} \\
&+ \frac{2\bar{h}^{(3)}_{0,0,3,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} + \frac{2(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} \\
&- \frac{(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}^{(4)}_{0,1,2,1} = \bar{h}^{(4)}_{0,1,2,1} &+ \frac{\bar{h}^{(3)}_{0,0,3,0}(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} + \frac{\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{(4\lambda^2-\nu^2)^2} + \frac{(\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0})(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} \\
&- \frac{2\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)} + \frac{2\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)} \\
&- \frac{2\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)} + \frac{2\bar{h}^{(3)}_{0,3,0,0}(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)} \\
&+ \frac{4\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu^2(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)};
\end{aligned}
$$

$$\begin{aligned}
\tilde{h}^{(4)}_{1,1,0,2} = \bar{h}^{(4)}_{1,1,0,2} &- \frac{2\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} + \frac{(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} - \frac{\lambda(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{(4\lambda^2-\nu^2)(-\lambda^2+4\nu^2)}\\
&+\frac{2\bar{h}^{(3)}_{0,0,0,3}(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} + \frac{(-2\bar{h}^{(3)}_{0,1,2,0}\lambda+2\bar{h}^{(3)}_{2,1,0,0}\lambda+\bar{h}^{(3)}_{1,0,1,1}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)}\\
&+\frac{2\lambda(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(\lambda^2-4\nu^2)(-4\lambda^2+\nu^2)} + \frac{4\bar{h}^{(3)}_{0,0,0,3}(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\nu\lambda(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)};
\end{aligned}$$

$$\begin{aligned}
\tilde{h}^{(4)}_{1,0,1,2} = \bar{h}^{(4)}_{1,0,1,2} &+ \frac{\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} + \frac{(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} + \frac{\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)}\\
&-\frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)} - \frac{2\bar{h}^{(3)}_{0,0,3,0}(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&+\frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)} + \frac{2\bar{h}^{(3)}_{3,0,0,0}(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&+\frac{4\nu(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda^2(\lambda^2-4\nu^2)^2};
\end{aligned}$$

$$\begin{aligned}
\tilde{h}^{(4)}_{0,1,1,2} = \bar{h}^{(4)}_{0,1,1,2} &+ \frac{\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)} - \frac{2\bar{h}^{(3)}_{0,3,0,0}(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2} + \frac{(\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0})(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2}\\
&-\frac{2\lambda(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(-4\lambda^2+\nu^2)(-\lambda^2+4\nu^2)} + \frac{2\bar{h}^{(3)}_{0,0,0,3}(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)}\\
&-\frac{4\bar{h}^{(3)}_{0,0,0,3}(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)} - \frac{(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{(\lambda^2-4\nu^2)(4\lambda^2-\nu^2)}\\
&+\frac{2(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)(\lambda^2-4\nu^2)};
\end{aligned}$$

$$\begin{aligned}
\tilde{h}^{(4)}_{1,1,1,1} = \bar{h}^{(4)}_{1,1,1,1} &- \frac{2\bar{h}^{(3)}_{3,0,0,0}(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)}{\lambda^2-4\nu^2} - \frac{2\bar{h}^{(3)}_{0,3,0,0}(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)}{4\lambda^2-\nu^2} - \frac{2\bar{h}^{(3)}_{0,0,0,3}(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)}{4\lambda^2-\nu^2} - \frac{2\bar{h}^{(3)}_{0,0,3,0}(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)}{-\lambda^2+4\nu^2}\\
&-\frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)}{\nu(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)} + \frac{2\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)}\\
&+\frac{2\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2} - \frac{2\nu(\bar{h}^{(3)}_{0,1,1,1}\lambda-2\bar{h}^{(3)}_{1,0,0,2}\nu+2\bar{h}^{(3)}_{1,2,0,0}\nu)(-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)^2}\\
&+\frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)^2} + \frac{2\lambda(2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu)(2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2-\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{2,1,0,0}\nu^2)}{\nu(4\lambda^2-\nu^2)(-4\lambda^2+\nu^2)}\\
&+\frac{2\nu(\bar{h}^{(3)}_{1,1,0,1}\lambda+2\bar{h}^{(3)}_{0,0,1,2}\nu-2\bar{h}^{(3)}_{0,2,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0})(\lambda^2-4\nu^2)+\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2)}{\lambda(\lambda^2-4\nu^2)(-\lambda^2+4\nu^2)}\\
&+\frac{2\lambda(2\bar{h}^{(3)}_{0,0,2,1}\lambda-2\bar{h}^{(3)}_{2,0,0,1}\lambda+\bar{h}^{(3)}_{1,1,1,0}\nu)(-(\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1})(4\lambda^2-\nu^2)+2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2)}{\nu(4\lambda^2-\nu^2)(4\lambda^2-\nu^2)};
\end{aligned}$$

APPENDIX F

$$w^{(3)}_{3,0,0,0} = -\frac{2\bar{h}^{(3)}_{0,0,3,0}+\bar{h}^{(3)}_{2,0,1,0}}{3\lambda};$$

$$w^{(3)}_{0,3,0,0} = -\frac{2\bar{h}^{(3)}_{0,0,0,3}+\bar{h}^{(3)}_{0,2,0,1}}{3\nu};$$

$$w^{(3)}_{0,0,3,0} = \frac{\bar{h}^{(3)}_{1,0,2,0}+2\bar{h}^{(3)}_{3,0,0,0}}{3\lambda};$$

$$w^{(3)}_{0,0,0,3} = \frac{\bar{h}^{(3)}_{0,1,0,2}+2\bar{h}^{(3)}_{0,3,0,0}}{3\nu};$$

$$w^{(3)}_{2,1,0,0} = -\frac{2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2}{\nu(4\lambda^2-\nu^2)};$$

$$w^{(3)}_{2,0,1,0} = \frac{\bar{h}^{(3)}_{3,0,0,0}}{\lambda};$$

$$w^{(3)}_{2,0,0,1} = -\frac{2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2}{\nu(4\lambda^2-\nu^2)};$$

$$w^{(3)}_{1,2,0,0} = -\frac{\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2}{\lambda(\lambda^2-4\nu^2)};$$

$$w^{(3)}_{0,2,1,0} = -\frac{-\bar{h}^{(3)}_{1,2,0,0}\lambda^2-\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2}{\lambda(\lambda^2-4\nu^2)};$$

$$w^{(3)}_{0,2,0,1} = \frac{\bar{h}^{(3)}_{0,3,0,0}}{\nu};$$

$$w^{(3)}_{1,0,2,0} = -\frac{\bar{h}^{(3)}_{0,0,3,0}}{\lambda};$$

$$w^{(3)}_{0,1,2,0} = -\frac{\bar{h}^{(3)}_{0,0,2,1}+\bar{h}^{(3)}_{2,0,0,1}}{\nu} + \frac{2\bar{h}^{(3)}_{0,0,2,1}\lambda^2+2\bar{h}^{(3)}_{2,0,0,1}\lambda^2+\bar{h}^{(3)}_{1,1,1,0}\lambda\nu-\bar{h}^{(3)}_{2,0,0,1}\nu^2}{\nu(4\lambda^2-\nu^2)};$$

$$w^{(3)}_{0,0,2,1} = -\frac{2\bar{h}^{(3)}_{0,1,2,0}\lambda^2+2\bar{h}^{(3)}_{2,1,0,0}\lambda^2+\bar{h}^{(3)}_{1,0,1,1}\lambda\nu-\bar{h}^{(3)}_{0,1,2,0}\nu^2)/(\nu(-4\lambda^2+\nu^2)}{;}$$

$$w^{(3)}_{1,0,0,2} = -\frac{\bar{h}^{(3)}_{0,0,1,2}+\bar{h}^{(3)}_{0,2,1,0}}{\lambda} + \frac{\bar{h}^{(3)}_{0,2,1,0}\lambda^2-\bar{h}^{(3)}_{1,1,0,1}\lambda\nu-2\bar{h}^{(3)}_{0,0,1,2}\nu^2-2\bar{h}^{(3)}_{0,2,1,0}\nu^2}{\lambda(\lambda^2-4\nu^2)};$$

$$w^{(3)}_{0,1,0,2} = -\frac{\bar{h}^{(3)}_{0,0,0,3}}{\nu};$$

$$w^{(3)}_{0,0,1,2} = -\frac{-\bar{h}^{(3)}_{1,0,0,2}\lambda^2+\bar{h}^{(3)}_{0,1,1,1}\lambda\nu+2\bar{h}^{(3)}_{1,0,0,2}\nu^2+2\bar{h}^{(3)}_{1,2,0,0}\nu^2}{\lambda(\lambda^2-4\nu^2)};$$

$$w^{(3)}_{1,1,1,0} = -\frac{2\bar{h}^{(3)}_{0,1,2,0}\lambda-2\bar{h}^{(3)}_{2,1,0,0}\lambda-\bar{h}^{(3)}_{1,0,1,1}\nu}{4\lambda^2-\nu^2};$$

$$w^{(3)}_{1,1,0,1} = -\frac{\bar{h}^{(3)}_{0,1,1,1}\lambda - 2\bar{h}^{(3)}_{1,0,0,2}\nu + 2\bar{h}^{(3)}_{1,2,0,0}\nu}{\lambda^2 - 4\nu^2};$$

$$w^{(3)}_{1,0,1,1} = -\frac{2\bar{h}^{(3)}_{0,0,2,1}\lambda - 2\bar{h}^{(3)}_{2,0,0,1}\lambda + \bar{h}^{(3)}_{1,1,1,0}\nu}{4\lambda^2 - \nu^2};$$

$$w^{(3)}_{0,1,1,1} = -\frac{\bar{h}^{(3)}_{1,1,0,1}\lambda + 2\bar{h}^{(3)}_{0,0,1,2}\nu - 2\bar{h}^{(3)}_{0,2,1,0}\nu}{-\lambda^2 + 4\nu^2};$$

APPENDIX G

$$w^{(4)}_{4,0,0,0} = \frac{w^{(4)}_{2,0,2,0}}{2} - \frac{\tilde{h}^{(4)}_{3,0,1,0}}{4\lambda};$$

$$w^{(4)}_{0,4,0,0} = \frac{w^{(4)}_{0,2,0,2}}{2} - \frac{\tilde{h}^{(4)}_{0,3,0,1}}{4\nu};$$

$$w^{(4)}_{0,0,4,0} = \frac{w^{(4)}_{2,0,2,0}}{2} + \frac{\tilde{h}^{(4)}_{1,0,3,0}}{4\lambda};$$

$$w^{(4)}_{0,0,0,4} = \frac{w^{(4)}_{0,2,0,2}}{2} + \frac{\tilde{h}^{(4)}_{0,1,0,3}}{4\nu};$$

$$w^{(4)}_{2,0,0,2} = w^{(4)}_{2,2,0,0} - \frac{(\tilde{h}^{(4)}_{0,1,2,1}\lambda^2+\tilde{h}^{(4)}_{2,1,0,1}\lambda^2-\tilde{h}^{(4)}_{1,0,1,2}\lambda\nu+\tilde{h}^{(4)}_{1,2,1,0}\lambda\nu-2\tilde{h}^{(4)}_{2,1,0,1}\nu^2)}{4\nu(-\lambda^2+\nu^2)};$$

$$w^{(4)}_{0,2,2,0} = w^{(4)}_{2,2,0,0} - \frac{(-2\tilde{h}^{(4)}_{1,2,1,0}\lambda^2-\tilde{h}^{(4)}_{0,1,2,1}\lambda\nu+\tilde{h}^{(4)}_{2,1,0,1}\lambda\nu+\tilde{h}^{(4)}_{1,0,1,2}\nu^2+\tilde{h}^{(4)}_{1,2,1,0}\nu^2)}{4\lambda(\lambda^2-\nu^2)};$$

$$w^{(4)}_{0,0,2,2} = w^{(4)}_{2,2,0,0} - \frac{-\tilde{h}^{(4)}_{0,1,2,1}\lambda-\tilde{h}^{(4)}_{2,1,0,1}\lambda-\tilde{h}^{(4)}_{1,0,1,2}\nu-\tilde{h}^{(4)}_{1,2,1,0}\nu}{4\lambda\nu};$$

$$w^{(4)}_{3,1,0,0} = \frac{(6\tilde{h}^{(4)}_{0,1,3,0}\lambda^3+3\tilde{h}^{(4)}_{2,1,1,0}\lambda^3-2\tilde{h}^{(4)}_{1,0,2,1}\lambda^2\nu-7\tilde{h}^{(4)}_{3,0,0,1}\lambda^2\nu-\tilde{h}^{(4)}_{2,1,1,0}\lambda\nu^2+\tilde{h}^{(4)}_{3,0,0,1}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{3,0,1,0} = -\frac{3\tilde{h}^{(4)}_{0,0,4,0}+\tilde{h}^{(4)}_{2,0,2,0}-5\tilde{h}^{(4)}_{4,0,0,0}}{8\lambda};$$

$$w^{(4)}_{3,0,0,1} = -\frac{(6\tilde{h}^{(4)}_{0,0,3,1}\lambda^3+3\tilde{h}^{(4)}_{2,0,1,1}\lambda^3+2\tilde{h}^{(4)}_{1,1,2,0}\lambda^2\nu+7\tilde{h}^{(4)}_{3,1,0,0}\lambda^2\nu-\tilde{h}^{(4)}_{2,0,1,1}\lambda\nu^2-\tilde{h}^{(4)}_{3,1,0,0}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{1,3,0,0} = -\frac{(\tilde{h}^{(4)}_{0,3,1,0}\lambda^3-\tilde{h}^{(4)}_{1,2,0,1}\lambda^2\nu-2\tilde{h}^{(4)}_{0,1,1,2}\lambda\nu^2-7\tilde{h}^{(4)}_{0,3,1,0}\lambda\nu^2+6\tilde{h}^{(4)}_{1,0,0,3}\nu^3+3\tilde{h}^{(4)}_{1,2,0,1}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{0,3,1,0} = -\frac{(-\tilde{h}^{(4)}_{1,3,0,0}\lambda^3-\tilde{h}^{(4)}_{0,2,1,1}\lambda^2\nu+2\tilde{h}^{(4)}_{1,1,0,2}\lambda\nu^2+7\tilde{h}^{(4)}_{1,3,0,0}\lambda\nu^2+6\tilde{h}^{(4)}_{0,0,1,3}\nu^3+3\tilde{h}^{(4)}_{0,2,1,1}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{0,3,0,1} = -\frac{3\tilde{h}^{(4)}_{0,0,0,4}+\tilde{h}^{(4)}_{0,2,0,2}-5\tilde{h}^{(4)}_{0,4,0,0}}{8\nu};$$

$$w^{(4)}_{1,0,3,0} = \frac{-5\tilde{h}^{(4)}_{0,0,4,0}+\tilde{h}^{(4)}_{2,0,2,0}+3\tilde{h}^{(4)}_{4,0,0,0}}{8\lambda};$$

$$w^{(4)}_{0,1,3,0} = -\frac{(-3\tilde{h}^{(4)}_{1,1,2,0}\lambda^3-6\tilde{h}^{(4)}_{3,1,0,0}\lambda^3-7\tilde{h}^{(4)}_{0,0,3,1}\lambda^2\nu-2\tilde{h}^{(4)}_{2,0,1,1}\lambda^2\nu+\tilde{h}^{(4)}_{1,1,2,0}\lambda\nu^2+\tilde{h}^{(4)}_{0,0,3,1}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{0,0,3,1} = -\frac{(-3\tilde{h}^{(4)}_{1,0,2,1}\lambda^3-6\tilde{h}^{(4)}_{3,0,0,1}\lambda^3+7\tilde{h}^{(4)}_{0,1,3,0}\lambda^2\nu+2\tilde{h}^{(4)}_{2,1,1,0}\lambda^2\nu+\tilde{h}^{(4)}_{1,0,2,1}\lambda\nu^2-\tilde{h}^{(4)}_{0,1,3,0}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{1,0,0,3} = -\frac{(\tilde{h}^{(4)}_{0,0,1,3}\lambda^3+\tilde{h}^{(4)}_{1,1,0,2}\lambda^2\nu-7\tilde{h}^{(4)}_{0,0,1,3}\lambda\nu^2-2\tilde{h}^{(4)}_{0,2,1,1}\lambda\nu^2-3\tilde{h}^{(4)}_{1,1,0,2}\nu^3-6\tilde{h}^{(4)}_{1,3,0,0}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{0,1,0,3} = \frac{-5\tilde{h}^{(4)}_{0,0,0,4}+\tilde{h}^{(4)}_{0,2,0,2}+3\tilde{h}^{(4)}_{0,4,0,0}}{8\nu};$$

$$w^{(4)}_{0,0,1,3} = -\frac{(-\tilde{h}^{(4)}_{1,0,0,3}\lambda^3+\tilde{h}^{(4)}_{0,1,1,2}\lambda^2\nu+7\tilde{h}^{(4)}_{1,0,0,3}\lambda\nu^2+2\tilde{h}^{(4)}_{1,2,0,1}\lambda\nu^2-3\tilde{h}^{(4)}_{0,1,1,2}\nu^3-6\tilde{h}^{(4)}_{0,3,1,0}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{2,1,1,0} = -\frac{(-9\tilde{h}^{(4)}_{3,1,0,0}\lambda^3-6\tilde{h}^{(4)}_{0,0,3,1}\lambda^2\nu-3\tilde{h}^{(4)}_{2,0,1,1}\lambda^2\nu-2\tilde{h}^{(4)}_{1,1,2,0}\lambda\nu^2+3\tilde{h}^{(4)}_{3,1,0,0}\lambda\nu^2+\tilde{h}^{(4)}_{2,0,1,1}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{2,1,0,1} = \frac{(\tilde{h}^{(4)}_{0,0,2,2}\lambda^2-\tilde{h}^{(4)}_{0,2,2,0}\lambda^2+\tilde{h}^{(4)}_{2,0,0,2}\lambda^2-\tilde{h}^{(4)}_{2,2,0,0}\lambda^2+\tilde{h}^{(4)}_{1,1,1,1}\lambda\nu-2\tilde{h}^{(4)}_{2,0,0,2}\nu^2+2\tilde{h}^{(4)}_{2,2,0,0}\nu^2)}{4\nu(-\lambda^2+\nu^2)};$$

$$w^{(4)}_{2,0,1,1} = -\frac{(-2\lambda\nu(3\tilde{h}^{(4)}_{0,1,3,0}\lambda-\tilde{h}^{(4)}_{1,0,2,1}\nu)-(-3\tilde{h}^{(4)}_{3,0,0,1}\lambda+\tilde{h}^{(4)}_{2,1,1,0}\nu)(3\lambda^2-\nu^2))}{(-9\lambda^4+10\lambda^2\nu^2-\nu^4)};$$

$$w^{(4)}_{1,2,1,0} = -\frac{(2\tilde{h}^{(4)}_{0,2,2,0}\lambda^2-2\tilde{h}^{(4)}_{2,2,0,0}\lambda^2-\tilde{h}^{(4)}_{1,1,1,1}\lambda\nu-\tilde{h}^{(4)}_{0,0,2,2}\nu^2-\tilde{h}^{(4)}_{0,2,2,0}\nu^2+\tilde{h}^{(4)}_{2,0,0,2}\nu^2+\tilde{h}^{(4)}_{2,2,0,0}\nu^2)}{4\lambda(\lambda^2-\nu^2)};$$

$$w^{(4)}_{1,2,0,1} = -\frac{(\tilde{h}^{(4)}_{0,2,1,1}\lambda^3-2\tilde{h}^{(4)}_{1,1,0,2}\lambda^2\nu+3\tilde{h}^{(4)}_{1,3,0,0}\lambda^2\nu-6\tilde{h}^{(4)}_{0,0,1,3}\lambda\nu^2-3\tilde{h}^{(4)}_{0,2,1,1}\lambda\nu^2-9\tilde{h}^{(4)}_{1,3,0,0}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{0,2,1,1} = -\frac{(-2\lambda\nu(-\tilde{h}^{(4)}_{0,1,1,2}\lambda+3\tilde{h}^{(4)}_{1,0,0,3}\nu)-(\tilde{h}^{(4)}_{1,2,0,1}\lambda-3\tilde{h}^{(4)}_{0,3,1,0}\nu)(-\lambda^2+3\nu^2))}{(-\lambda^4+10\lambda^2\nu^2-9\nu^4)};$$

$$w^{(4)}_{1,1,2,0} = -\frac{(9\tilde{h}^{(4)}_{0,1,3,0}\lambda^3-3\tilde{h}^{(4)}_{1,0,2,1}\lambda^2\nu-6\tilde{h}^{(4)}_{3,0,0,1}\lambda^2\nu-3\tilde{h}^{(4)}_{0,1,3,0}\lambda\nu^2+2\tilde{h}^{(4)}_{2,1,1,0}\lambda\nu^2+\tilde{h}^{(4)}_{1,0,2,1}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{1,0,2,1} = -\frac{(9\tilde{h}^{(4)}_{0,0,3,1}\lambda^3+3\tilde{h}^{(4)}_{1,1,2,0}\lambda^2\nu+6\tilde{h}^{(4)}_{3,1,0,0}\lambda^2\nu-3\tilde{h}^{(4)}_{0,0,3,1}\lambda\nu^2+2\tilde{h}^{(4)}_{2,0,1,1}\lambda\nu^2-\tilde{h}^{(4)}_{1,1,2,0}\nu^3)}{9\lambda^4-10\lambda^2\nu^2+\nu^4};$$

$$w^{(4)}_{0,1,2,1} = \frac{(\tilde{h}^{(4)}_{0,0,2,2}\lambda^2-\tilde{h}^{(4)}_{0,2,2,0}\lambda^2+\tilde{h}^{(4)}_{2,0,0,2}\lambda^2-\tilde{h}^{(4)}_{2,2,0,0}\lambda^2-\tilde{h}^{(4)}_{1,1,1,1}\lambda\nu-2\tilde{h}^{(4)}_{0,0,2,2}\nu^2+2\tilde{h}^{(4)}_{0,2,2,0}\nu^2)}{4\nu(-\lambda^2+\nu^2)};$$

$$w^{(4)}_{1,1,0,2} = -\frac{(\tilde{h}^{(4)}_{0,1,1,2}\lambda^3-3\tilde{h}^{(4)}_{1,0,0,3}\lambda^2\nu+2\tilde{h}^{(4)}_{1,2,0,1}\lambda^2\nu-3\tilde{h}^{(4)}_{0,1,1,2}\lambda\nu^2-6\tilde{h}^{(4)}_{0,3,1,0}\lambda\nu^2+9\tilde{h}^{(4)}_{1,0,0,3}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{1,0,1,2} = -\frac{(2\tilde{h}^{(4)}_{0,0,2,2}\lambda^2-2\tilde{h}^{(4)}_{2,0,0,2}\lambda^2+\tilde{h}^{(4)}_{1,1,1,1}\lambda\nu-\tilde{h}^{(4)}_{0,0,2,2}\nu^2-\tilde{h}^{(4)}_{0,2,2,0}\nu^2+\tilde{h}^{(4)}_{2,0,0,2}\nu^2+\tilde{h}^{(4)}_{2,2,0,0}\nu^2)}{4\lambda(\lambda^2-\nu^2)};$$

$$w^{(4)}_{0,1,1,2} = -\frac{(-\tilde{h}^{(4)}_{1,1,0,2}\lambda^3-3\tilde{h}^{(4)}_{0,0,1,3}\lambda^2\nu+2\tilde{h}^{(4)}_{0,2,1,1}\lambda^2\nu+3\tilde{h}^{(4)}_{1,1,0,2}\lambda\nu^2+6\tilde{h}^{(4)}_{1,3,0,0}\lambda\nu^2+9\tilde{h}^{(4)}_{0,0,1,3}\nu^3)}{\lambda^4-10\lambda^2\nu^2+9\nu^4};$$

$$w^{(4)}_{1,1,1,1} = -\frac{\tilde{h}^{(4)}_{0,1,2,1}\lambda-\tilde{h}^{(4)}_{2,1,0,1}\lambda-\tilde{h}^{(4)}_{1,0,1,2}\nu+\tilde{h}^{(4)}_{1,2,1,0}\nu}{2(\lambda^2-\nu^2)};$$

and the three remainig coefficients $w^{(4)}_{2,0,2,0} = 0$, $w^{(4)}_{2,2,0,0} = 0$ and $w^{(4)}_{0,2,0,2} = 0$ arbitrarily set to zero.