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Design of Near Perfect Reconstruction Oversampled Filter Banks for Subband Adaptive Filters

Moritz Harteneck, Stephan Weiss, and Robert W. Stewart

Abstract—in this brief, a design algorithm for real-valued and complex-valued oversampled filter banks which yield a low level of inband alias and enable simple subband adaptive structures is presented. The filter banks are either based on complex modulation of an inband alias, or on the direct or modulated setups of real-valued filter banks. If real-valued filter banks are required, then the different channels will have different subsampling ratios so that the bandpass sampling theorem is not violated. This brief also presents design examples of real-valued and complex-valued filter banks.

Index Terms—Adaptive filtering, filter bank design, oversampled near-perfect reconstruction filter banks.

I. INTRODUCTION

Over the last two decades, adaptive filtering has been of considerable interest for many applications, such as acoustic echo cancellation [1], noise reduction, equalization, and beamforming [2]. Often, the adaptive system has to “model” a long-duration impulse response, as in the case of, e.g., the identification of room acoustics [3], [4]. Hence, with the high number of adaptive filter weights required, popular adaptive algorithms based on least-mean-squares and least squares techniques [5] become very computationally complex and exhibit a slow convergence.

One possibility to combat these problems is the use of adaptive algorithms, together with multirate techniques, to split the full-band problem into smaller subband problems. If the well-researched critically sampled perfect reconstruction filter banks are used to decompose the input signals, the subband signals are contaminated by aliasing, which requires adaptive filters in between adjacent bands to compensate for this distortion [6]. Thus, multichannel adaptive algorithms are being used in the subbands which exhibit a higher computational complexity and a slower convergence for correlated input signals. If the perfect reconstruction condition is dropped, critically sampled structures are obtainable, which do not need cross-adaptive filters, as spectral gaps ensure that aliasing in the subbands does not occur [7], [8]. This spectral loss however prohibits exact system identification and can significantly distort signals passing through the filter bank. Applications in data communications or high-quality audio can be very sensitive to such errors.

Another possibility to achieve the decomposition of the input signals is to use oversampled filter banks (OSFB’s), i.e., filter banks which generate redundancy in the subbands. One class of OSFB’s is obtained via complex modulation from a real-valued low-pass prototype filter. If the prototype filter possesses a high stopband attenuation, then aliasing in the subbands is sufficiently suppressed. The subband signals in this class of OSFB’s are all subsampled by the same subsampling ratio S. This filter bank can be modified to perform a single-sideband (SSB) modulation-like filter bank, yielding real-valued subband signals [9]–[11]. The trick is to modulate the bandpass signals into the baseband prior to decimation.

Fig. 1 shows the concatenation of an analysis and a synthesis bank. On the analysis side, the input signal x(k) is split into L frequency bands by analysis filters h_l(k) and decimated by a factor of S_l in the lth branch, giving the subband signal v_l(k). On the synthesis side, each corresponding branch, consisting of an upsampler and a synthesis filter f_l(k), restores the original fullband sampling rate and interpolates the subband signal. Summing over all branches then forms the output signal ̂x(k). Such a filter bank possesses perfect or near-perfect reconstruction property, if the output signal ̂x(k) is essentially a copy of the input signal x(k) delayed by Δ samples, i.e., ̂x(k) ≈ x(k − Δ). Writing the transfer function of the filter bank shown in Fig. 1 in the frequency domain gives

\[
\hat{X}(z) = \sum_{l=0}^{L-1} \sum_{m=0}^{M_l-1} \frac{1}{S_l} F_l(z) \hat{H}_l \left( zW_{S_l}^m \right) X \left( zW_{S_l}^m \right)
\]

(1)

where ̂X(z), \hat{H}_l(z), F_l(z), and X(z) are the z-transforms of ̂x(k), h_l(k), f_l(k), and x(k), respectively, and W_{S_l} is a modulation factor e^{−j2πz/S_l}. The transfer functions T_{l,m}(z) describe the transfer of the different aliasing terms m of each channel l. For the adaptive filters to work in the subbands without the need for information from adjacent subbands [6], the aliasing terms for m > 0 have to be approximately zero, i.e.,

\[
T_{l,m}(z) \approx 0, \quad \forall l = 0, \ldots, L-1; \quad m = 1, \ldots, S_l.
\]

Thus avoiding problems with bandpass sampling [12]. Since the prototype design is identical to the design for complex modulated filter banks, it is not discussed further in this paper. Another OSFB class consists of real-valued nonuniform filter banks [13] which have different subsampling ratios S_l in each channel to satisfy the bandpass sampling theorem and avoid inband aliasing. In this case, the analysis filters possess different bandwidths which might be advantageous in, e.g., psychoacoustic coding [14].

In this paper, we present a fast-converging and efficient design algorithm for OSFB’s with near-perfect reconstruction property which suppresses aliasing in the subbands, and is thus well suited for subband adaptive filtering. Therefore, Section II, will introduce two classes of real- and complex-valued OSFB’s. Section III discusses an iterative least-squares design algorithm, for which examples will be shown in Section IV. Our notation uses a normalized sampling frequency f_s = 2. Lower-case italic letters denote scalar values, lower- and upper-case boldface letters represent vector and matrix quantities, respectively, and k is the discrete time index.

II. TYPES OF FILTER BANKS

The authors are with the Signal Processing Division, Department of Electrical and Electronic Engineering, University of Strathclyde, Glasgow G1 1XW, Scotland, U.K.

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Note that if (2) is satisfied, (1) simplifies to
\[ \hat{X}(z) = \sum_{l=0}^{L-1} \frac{1}{S_i} F_l(z) H_l(z) X(z) \] (3)
i.e., perfect reconstruction and power complementarity become equivalent claims. To minimize the alias level \( T_{i,m}(z) \) in the subband signals, complex- and real-valued setups are possible. These setups are outlined in Section II-A and II-B, respectively.

A. Complex-Valued Filter Banks

One way to implement an OSFB where inband aliasing is reduced to the level of the stopband attenuation of the analysis filters is to use a complex-valued set of analysis filters, where the filters have the prototype from zero to one, be covered by the analysis bank, since the remaining subbands are complex conjugate copies and therefore redundant. The oversampling ratio (OSR) for this filter bank equates to \((2L)/S\), where \(S\) is the common subsampling ratio for all channels. Note that to avoid aliasing in the subbands, the prototype filter \( p(k) \) has to be a real-valued low-pass filter strictly limited to the frequency range \([-1/S; 1/S]\). This type of OSFB approximates an implementation of Weil–Heisenberg frames which are studied in detail in [15].

Collecting the coefficients of the prototype \( p(k) \) and the analysis and synthesis filters \( h_i(k) \) and \( f_i(k) \) in vectors \( p \), \( h \), and \( f \):
\[
\begin{align*}
p & = [p(0) \quad p(1) \quad \cdots \quad p(L_p - 1)]^T \quad (6a) \\
h & = [h_0(0) \quad h_1(0) \quad \cdots \quad h_0(L_p - 1)]^T \quad (6b)
\end{align*}
\]
where superscript \( T \) denotes transpose, and the modulation in (4) can be expressed by a convenient matrix-vector notation
\[ h_i = f_i = M \cdot p. \] (7)
The matrices \( M_i \) are diagonal matrices with modulation factors (c.f. (4)) on their main diagonal. For further details on this type of filter bank and efficient polyphase implementations, refer to [10], [16], [11].

B. Real-Valued Filter Banks

Another way to obtain a filter bank satisfying (2) is to use a real-valued nonuniform OSFB and choose a setup where the bandpass sampling theorem is not violated [12], [13]. For a real-valued signal in the \( i \)th channel to be undistorted due to decimation by a factor \( S_i \), an interpretation of the bandpass sampling theorem requires the undecimated subband signal’s band to lie entirely between any two consecutive frequency points among \( f = i/S_i, i = 1, 2, \ldots, S_i - 1 \).

Extending this to OSFB’s means that every analysis filter \( h_i(k) \) has to be placed in the frequency domain, such that the resulting signal does not violate the bandpass sampling theorem when subsampled by a factor \( S_i \) and that the analysis filters have to be placed such that all frequencies are covered by at least one filter in order to allow reconstruction.

Fig. 3(a) shows the simplest setup of analysis filters of a nonuniform real-valued OSFB. In this filter bank, the low-pass and high-pass channels, produced by \( H_0(z) \) and \( H_2(z) \), are subsequently decimated by two and therefore are alias-free. The bandpass channel \( H_1(z) \) is subsampled by a factor of three and also remains undistorted. Hence, the OSR for this filter bank equates to \( \sum_{i=0}^{S-1} (1/S_i) = 133\% \).

Another possibility to obtain real-valued OSFB’s is to start from a cosine-modulated filter bank with \( L_0 \) channels, where the bandwidth of the analysis filters is reduced such that no aliasing occurs during decimation, and to fill the resulting spectral gaps by additional analysis filters which are derived from a second prototype to allow reconstruction. Filling the gaps with additional channels leads to a filter bank with a total of \( L = 2L_0 + 1 \) channels. All filters are derived from the two prototypes \( p_0(k) \) and \( p_1(k) \), as shown in Table I.

Fig. 3(b) shows, as an example, a nine-channel filter bank which is modulated according to Table I. Channels 1, 3, 5, and 7 are derived from the prototype \( p_0(k) \) and can be subsampled by 4. To avoid aliasing, the prototype filter \( p_0(k) \) has to have a bandwidth of less than \( 1/8 \), i.e., be limited to the frequency range \( f \in [-1/8; 1/8] \). Channels 0, 2, 4, 6, and 8, however, are derived from the prototype \( p_1(k) \) and are subsampled by 8, 6, 5, 6, and 8, respectively. These decimation ratios are chosen such that the allowed bandwidth of the prototype \( p_1(k) \) is as large as possible and results in this example in the passband of the prototype to be restricted to the region \([-0.0833:0.0833]\). The OSR of this filter bank is 178%.

Analogous to Section II-A, the modulation can be expressed in matrix notation. More information on this type of filter bank can be found in [13], [17]–[19].
TABLE I

<table>
<thead>
<tr>
<th>band l</th>
<th>analysis filter $h_l(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\sqrt{S_0} \cdot p_1(k)$</td>
</tr>
<tr>
<td>1, 3, ..., $2L_0 - 1$</td>
<td>$2\sqrt{S_0} \cdot p_0(k) \cdot \cos \left( k \cdot \frac{\pi}{2} \right)$</td>
</tr>
<tr>
<td>2, 4, ..., $2L_0 - 2$</td>
<td>$2\sqrt{S_0} \cdot p_1(k) \cdot \cos \left( k \cdot \frac{\pi}{2} \right)$</td>
</tr>
<tr>
<td>$2L_0$</td>
<td>$\sqrt{S_2 \cdot L_0} \cdot p_1(k) \cdot \cos (k \cdot \pi)$</td>
</tr>
</tbody>
</table>

If a filter bank with a higher number of channels is required, the presented cosine modulation approach can be used, starting with a higher $L_0$, or different setups can be found which satisfy the bandpass sampling theorem. Alternatively, a previously designed filter bank can be iterated which, however, leads in general to suboptimal solutions.

III. DESIGN ALGORITHM

A common choice in the design of filter banks is to choose the synthesis filters to be time-reversed and complex-conjugate versions of the analysis filters. This yields a reduction in the free design parameters. According to the theory of frames [20], this also leads to very close approximation of signal decompositions with a fixed energy translation between time and subband domain, regardless of the input signal.

To design OSFB’s, as presented in Section II, an iterative least-squares algorithm [22], [23] is used. The performance criterion to be minimized by the algorithm is a combination of the filter bank reconstruction error $\xi_1$ and the stopband energy $\xi_2$ of the analysis filters $h_l(k)$

$$\xi = \xi_1 + \gamma \cdot \xi_2$$

where $\gamma$ is a positive weighting factor which trades off between the importance of the reconstruction error and the stopband attenuation. In the following, we discuss separately formulations for both $\xi_1$ and $\xi_2$, and finally present the algorithm in Section III-C.

A. Reconstruction Error

If inband aliasing is sufficiently suppressed, the impulse response $t(k)$ of the overall filter bank, as given in Fig. 1, is the time-domain formulation of (3), and can be written as a convolution of the analysis filters and the synthesis filters

$$t(k) = \sum_{l=0}^{L-1} t_l(k) * f_l(k)$$

where “*” denotes convolution and the factor $1/S_l$ takes account of the power loss in the channels due to decimation. Using matrix notation, the convolution in the lth analysis filter can be calculated by [24]

$$t_1 = \begin{bmatrix}
    t_0(0) & t_1(1) & \cdots & t_0(2) \\
    t_1(0) & t_0(1) & \cdots & t_1(1) \\
    \vdots & \vdots & \ddots & \vdots \\
    t_{L_0}(0) & 0 & \cdots & 0 \\
    0 & t_{L_0}(1) & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & t_{L_0}(L_0 - 1)
\end{bmatrix} \begin{bmatrix}
    f_0(0) \\
    f_1(0) \\
    \vdots \\
    f_{L_0}(0) \\
    f_0(1) \\
    \vdots \\
    f_{L_0}(L_0 - 1)
\end{bmatrix} = H \cdot f_1.$$
TABLE II

<table>
<thead>
<tr>
<th>Step</th>
<th>Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:</td>
<td>initialize filters h(i), set i = 1</td>
</tr>
<tr>
<td>2:</td>
<td>build H(i - 1) from h(i - 1)</td>
</tr>
<tr>
<td>3:</td>
<td>minimize with respect to h(i)</td>
</tr>
<tr>
<td></td>
<td>( \xi = \left| \begin{bmatrix} H(i - 1) \ \gamma P \end{bmatrix} h(i) - \begin{bmatrix} v \ 0 \end{bmatrix} \right|_2^2 )</td>
</tr>
<tr>
<td>4:</td>
<td>apply relaxation</td>
</tr>
<tr>
<td></td>
<td>h(i) = ( \tau h(i) + (1 - \tau) h(i - 1) )</td>
</tr>
<tr>
<td>5:</td>
<td>check stop condition</td>
</tr>
<tr>
<td></td>
<td>if ( | h(i) - h(i - 1) | &lt; \epsilon ) then stop</td>
</tr>
<tr>
<td>6:</td>
<td>i = i + 1, goto 2</td>
</tr>
</tbody>
</table>

The design algorithm can be conveniently performed by a QR decomposition [27].

![Fig. 4. Four-channel complex-valued OSFB.](image)

![Fig. 5. Real-valued OSFB's using (a) three channels and (b) nine channels.](image)


text

For the nine channel real-valued OSFB as presented in Section II-B, which is derived from two prototypes \( p_0 \) and \( p_1 \), the performance criterion simplifies to

\[
\xi = \left\| \begin{bmatrix} \sum_{\ell=0}^{L} H_M \ell \sum_{\ell=0}^{1} H_M \ell \gamma P_L \\ 0 \end{bmatrix} - \begin{bmatrix} v \\ 0 \end{bmatrix} \right\|_2^2
\]

where the matrices \( P_0 \) and \( P_1 \) describe the frequency requirements on the prototypes \( p_0 \) and \( p_1 \), respectively, and the set \( S_1 \) denotes the set of filters derived from the prototype \( p_1 \).

The performance criterion for the complex-valued OSFB presented in Section II-A, which is derived from only one real-valued prototype \( p \), further simplifies to

\[
\xi = \left\| \begin{bmatrix} \frac{1}{S} \sum_{\ell=0}^{L-1} H_M \ell \gamma P_L \\ 0 \end{bmatrix} - \begin{bmatrix} v \\ 0 \end{bmatrix} \right\|_2^2
\]

where \( S \) is the common subsampling ratio and \( P \) describes the frequency requirements on the prototype.

To perform the minimization of (14), an iterative least-squares technique [22], [26], [23] is used where the matrix \( H \), which is being built from the analysis filters, is substituted by the matrix being built from an old set of analysis filters. This linearization gives a quadratic approximation of the performance surface. To ensure convergence, a relaxation step is added to the algorithm. The design algorithm can now be formulated iteratively as shown in Table II, where \( \tau \) is the relaxation parameter, which has to be chosen between zero and one and \( \epsilon \) quantifies the stop condition. The minimization of Step 2 can be conveniently performed by a QR decomposition [27].

IV. DESIGN EXAMPLES

In this section, design examples of OSFB’s targeted for subband adaptive filtering are shown. The filter banks have been designed to yield a reconstruction error and an inband aliasing of about −80 dB, giving a resolution of approximately 13 bits. These specifications are targeted at audio applications, but higher quality designs are easily achievable with the presented algorithms [13]. The parameters settings in the design algorithm are \( \tau = 0.5 \) and \( \gamma = 10 \).

Fig. 4 shows the transfer functions of the analysis and synthesis filters of a four-channel OSFB. The filter bank is designed for alias-free decimation by \( S = 6 \) [11], giving an OSR of 133%. To achieve the specifications, the design algorithm needs 14 iterations to produce the prototype filter with a required number of \( L_p = 192 \) taps.
the specifications, the analysis filters required 172 taps. The design algorithm needs 11 iterations to achieve the minimization.

V. CONCLUSION

In this paper, a design algorithm for OSFB’s has been presented, which is designed for the implementation of subband adaptive filters. Therefore, two classes of filter banks, a real-valued and a complex-valued class, have been introduced, which satisfy the requirement of a low inband aliasing level, such that single-channel adaptive filters operating in each subband are possible. An algorithm for the design of these filter banks has been presented which is based on an iterative least squares technique to minimize the performance criterion. Finally, design examples for the real-valued case and the complex-valued case have been given.

Using these filter banks, together with adaptive filters, significant computational savings are achievable. For example, using least-squares adaptive filters, the subband approach with real-valued filter banks reduces the computational requirements to 28.5% and 8.5%, compared with a fullband algorithm for the presented three channel and the nine channel OSFB’s, respectively. Using the presented complex-valued filter bank, the structure needs about 7.4% of the computational complexity of a fullband implementation. These savings can be even more significant if the number of channels and the associated decimation factors are increased. Note that the given percentages refer to the cost of adaptive filtering only. However, the cost of filter bank computations can be kept low by judicious implementation using polyphase factorizations [15], [28].

Simulations using the filter banks, together with adaptive filters, which are not presented in this paper, have shown that the performance in terms of minimum mean-squared error is limited by the amount of inband aliasing and reconstruction error, which come from the used filter bank [29]. For colored-noise input signals, however, in general these subband structures have shown an improvement in terms of convergence speed.

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