Efficient Implementations of Complex and Real Valued Filter Banks for Comparative Subband Processing with an Application to Adaptive Filtering

Stephan Weiß, Lutz Lampe[1]*, and Robert W. Stewart*

Signal Processing Division, University of Strathclyde, Glasgow G1 1XW, Scotland
e-mail: {weiss,lutz,bob}@spd.eee.strath.ac.uk.

Abstract. In this paper, we discuss efficient methods to implement correlation type algorithms in subbands. Based on a polyphase representation, a modulated GDFT filter bank with arbitrary integer decimation ratio can be performed at low cost, yielding complex subband signals. Real valued subbands can be achieved by appropriate postprocessing of the complex filter bank. Together with consideration of the computational order of the algorithm applied for subband processing, complex or real implementation may be better suited, for which we present criteria. For the example of subband adaptive filtering as applied in acoustic echo cancellation, we compare a number of algorithms in terms of the most efficient implementation.

1. Introduction

Often the processing of signals is performed in frequency bands. This signal decomposition allows a lowering the sampling rate in the subbands, thus reducing the computational complexity of implemented systems, and leads to a convenient parallelization. For comparing the subbands of different signals as e.g. performed in adaptive filtering, critical decimation by a factor N (where $N = K$ the number of uniform subbands), requires either cross-terms between overlapping frequency bands [4], or the use of gap filter banks [11]. The cross-terms are necessary to compensate for information leaked into adjacent bands in the decimation stage, whereas in the case of gap filters there is a loss of spectral information which may not be acceptable. Oversampled filter banks can resolve this problem by introducing spectral redundancy. While complex subband signals can be decimated at any integer rate $N < K$, thus allowing a choice of a high decimation rate close to K while suppressing aliasing in the subband signals, for real valued subband signals oversampling is problematic and requires either non-uniform filter banks [5] or single side band (SSB) modulation into the baseband prior to decimation [2, 1, 7].

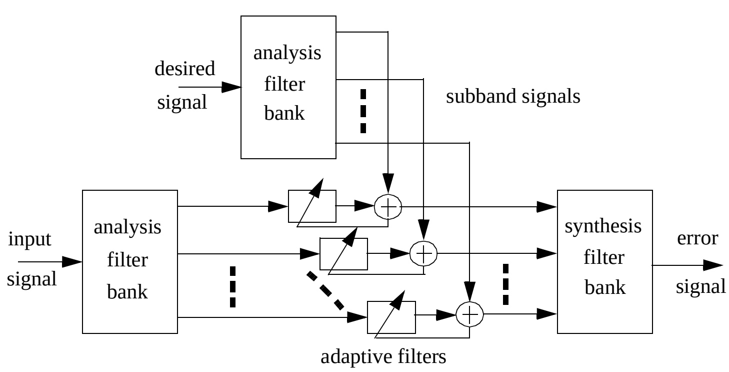

Fig. 1: Subband adaptive filter setup, where the adaptive filters try to produce output signals such, that when subtracted from the desired signals, the error signal is minimized (e.g. in a least mean square sense); for the comparison of filter outputs and desired signals, the subband signals should be alias free.

In this paper we endeavour to present an efficient polyphase implementation of a complex valued oversampled generalized DFT (GDFT) filter bank [2], extending it to an arbitrary integer decimation $N < K$. We use a polyphase factorization based on [3], which allow a filtering of the input signal by a polyphase network followed by a GDFT transformation, which can be efficiently implemented using an FFT [9]. This approach can be extended by a complex modulation and real part operation on the subband signals to yield a highly efficient SSB modulated, real valued filter bank. Although complex processing can be shown to be computationally more efficient in most cases, there exist adaptive filtering algorithms where non-complex arithmetic offers computational savings, like the affine projection algorithm [7] widely used in acoustic echo cancellation.

[1]Visiting from Telecommunications Institute, Universität Erlangen-Nürnberg, Germany

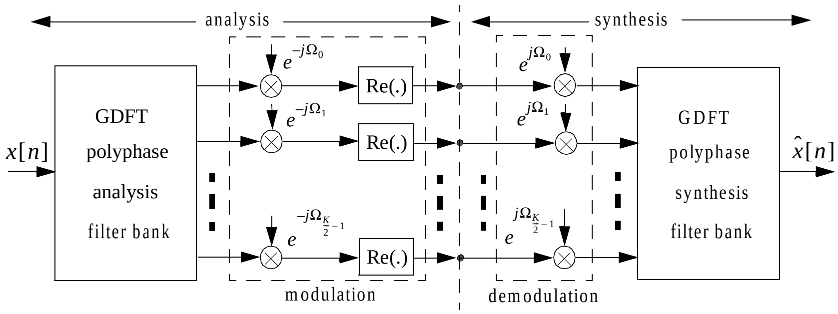

Fig. 2: GDFT analysis and synthesis filter bank with modulation (in dashed boxes) to achieve real valued subband signals.

2. Filter Bank Implementation

2.1 Complex Valued GDFT Filter Banks

Generalized DFT (GDFT, [2]) filter banks are characterized by a DFT-like modulation of a real valued prototype lowpass filter, $p[n]$,

$$h_k[n] = e^{j\frac{2\pi}{K}(k+1/2)(n-(L_p-1)/2)} \cdot p[n], \qquad k = 0(1)K, \;\; n = 0(1)L_p - 1, \tag{1}$$

where K denotes the number of complex subbands and L_p the length of $p[n]$. The offset values $k_0 = \frac{1}{2}$ and $n_0 = -(L_p - 1)/2$ are chosen such as to obtain linear phase filters $h_k[n]$ and to achieve a uniform coveraged of the normalized angular frequency interval $\Omega = [0; \pi]$ by $K/2$ analytic bandpass filters. For real input, the remaining subbands will be complex conjugate and their processing therefore redundant.
The synthesis filters $g_k[n]$ can be obtained by time reversion of the analysis filter, i.e. $g_k[n] = \tilde{h}_k[n] = h_k^*[L_p - n + 1]$. Thus, all filters can be derived from one single prototype $p[n]$, which has to be designed appropriately.

Polyphase Representation. To describe the analysis bank decimated by a factor N, a matrix $\mathbf{H}(z)$ can be created,

$$\mathbf{H}(z) = \begin{bmatrix} H_{0|0}(z) & H_{0|1}(z) & \cdots & H_{0|N-1}(z) \\ H_{1|0}(z) & H_{1|1}(z) & & H_{1|N-1}(z) \\ \vdots & & \ddots & \vdots \\ H_{K-1|0}(z) & H_{K-1|1}(z) & \cdots & H_{K-1|N-1}(z) \end{bmatrix} \quad \text{where} \quad H_k(z) = \sum_{j=0}^{N-1} z^{-j} H_{k|j}(z^N), \tag{2}$$

holding the $H_{k|j}(z)$, $j = 0(1)N - 1$ polyphase components of the kth analysis filter. With a similar polyphase decomposition of the input signal $x[n]$, $X(z) = \sum_{j=0}^{N-1} z^{-j} X_j(z^N)$

$$\underline{X}(z) = [X_0(z), X_1(z), \ldots X_{N-1}(z)]^T \tag{3}$$

the analysis bank operation is denoted as $\underline{Y}(z) = \mathbf{H}(z) \cdot \underline{X}(z)$, where $\underline{Y}(z)$ contains the K subband signals. With careful filter design, the polyphase matrix $\mathbf{H}(z)$ is paraunitary, i.e. the input signal $x[n]$ can be perfectly reconstructed in the synthesis bank with the above choice of synthesis filter $g_k[n]$ [3].

Polyphase Factorization. Let M be the least common multiple (lcm) of the periodicity of the GDFT transform in (1), $2K$, and the decimation ratio N, $M = \text{lcm}(2K, N)$, with $M = J\cdot 2K = L\cdot N, \quad J, L \in \mathbb{Z}$. To exploit common calculations between filters $H_k(z)$, the polyphase matrix $\mathbf{H}(z)$ can be written in terms of the M polyphase components of the prototype filter $P(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$,

$$H_{k|n}(z) = \sum_{l=0}^{L-1} z^{-l} \cdot t_{k,lN+n} \cdot P_{lN+n}(z^L). \tag{4}$$

If the periodicity $2K$ of the transform coefficients $t_{k,n}$ is considered, it is possible to formulate a dense matrix notation

$$\mathbf{H}(z) = \mathbf{T}_{\text{GDFT}} \cdot \mathbf{P}(z) \tag{5}$$

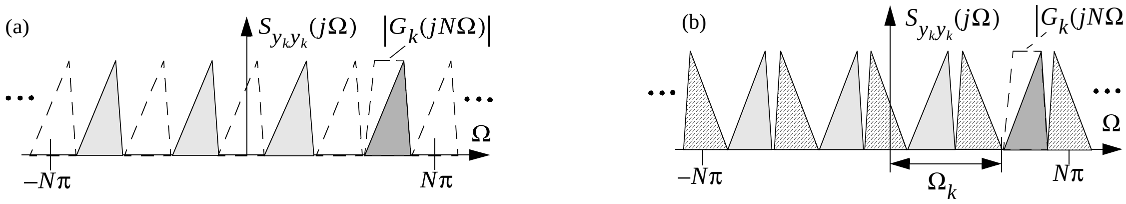

Fig. 3: (a) PSD of kth subband signal of GDFT filter bank after decimation by $N = 8$; the filled image spectra only mark the case $N = 4$, leaving gaps; (b) postprocessing of case $N = 4$ with a modulation as indicated in Fig. 2: gaps have been filled with reverse images due to the real operation; note that for both methods, the correct spectrum can be extracted after upsampling by N in the synthesis using $G_k(j\Omega)$.

similar to [3], with a GDFT transform matrix

$$\mathbf{P}(z) = [\mathbf{I}_{2K} \ldots \mathbf{I}_{2K}] \cdot \text{diag}\left\{P_0(z^L), P_1(z^L), \ldots P_{M-1}(z^L)\right\} \cdot \left[\mathbf{I}_N, z^{-1}\mathbf{I}_N, \ldots z^{-L+1}\mathbf{I}_N\right]^T. \quad (6)$$

The GDFT matrix $\mathbf{T}_{\text{GDFT},r}$ in (5) can be further factorized to mainly reduce to a DFT matrix, which can be efficiently implemented using standard FFT algorithms.

Computational Complexity. The appeal lies in the fact, that all filtering is performed using with only real valued quantities, followed by a complex modulation (and vice versa in the synthesis). For real inputs, only half of $\mathbf{H}(z)$ needs to be evaluated, the overall computational complexity $C_{\text{bank}}^{\text{cmplx}}$ can be further reduced, to yield

$$C_{\text{bank}}^{\text{cmplx}} = \frac{1}{N}\left(4K\log_2 K + 6K + L_p\right) \quad (7)$$

real multiplications per fullband sample.

2.2 Single Sideband Modulation by GDFT Modification

Based on GDFT filter banks, SSB modulation can be performed by decimating the analytic subbands signals by only $N/2$. This leaves gaps as depicted in Fig. 3(a). After complex modulation with $e^{-j\Omega_k}$ to dock the passband signal at 0Hz, the symmetrization caused by the real operation as seen in Fig. 2 fills the gaps with reverse image spectra. Note from Fig. 3(b) that after demodulation with $e^{j\Omega_k}$ as shown in Fig. 2, the appropriate spectral interval gets selected for reconstruction. Compared to complex GDFT subbands, the SSB requires an extra K multiplies for de-/modulation, and therefore due to the reduced factor N approximately twice the number of multiplications over (7), $C_{\text{bank}}^{\text{real}} \approx 2 \cdot C_{\text{bank}}^{\text{cmplx}}$. Furthermore, the alternative Weaver method [1] would still be more costly since polyphase implementation is not an option.

3. Subband Processing

Assume some processing task is performed on L_f samples in a real valued subband implementation by a filter of length L_f. If the computational complexity of performing the operations associated with this processing is of order $\mathcal{O}(L_f^I)$, the number of arithmetic operations can be written in the form of a polynomial in L_f:

$$C_{\text{proc}}^{\text{real}}(L_f) = \sum_{i=0}^{I} c_i^{\text{real}} \cdot L_f^i = \sum_{i=0}^{I} C_{\text{proc},i}^{\text{real}} \quad (8)$$

For complex valued subband processing with identical filter bank quality and performance measures [10], the doubling of the decimation factor results in shorter filters of length $L_f/2$ and half the processing speed, although one complex multiplication requires 4 real multiplications. Therefore the ratio of computational complexity between processing with real and complex valued subband signals for the ith term in sum (8), $C_{\text{proc},i}^{\text{real}}$, can be derived as

$$C_{\text{proc},i}^{\text{real}} \propto L_f^i \;, \quad C_{\text{proc},i}^{\text{cmplx}} \propto 4 \cdot \left(\frac{1}{2} \cdot \left(\frac{L_f}{2}\right)^i\right) \;, \quad \longrightarrow \frac{C_{\text{proc},i}^{\text{cmplx}}}{C_{\text{proc},i}^{\text{real}}} = \frac{1}{2^{i-1}}. \quad (9)$$

Algorithm	Comp. Load
Normalized LMS	$3 + 2L_f$
Fast APA, order p	$20p + 2L_f$
Recurs. Least Squares	$4L_f + 3L_f^2$

Tab. 1: Number of multiplications of different algorithms in dependence of the filter length L_f for real valued implementation.

Thus, the computational complexity of algorithms with complex arithmetic compares by

$$C_{\text{proc}}^{\text{cmplx}}(L_f/2) = \sum_{i=0}^{I} c_i^{\text{cmplx}} \cdot \frac{L_f^i}{2} = \sum_{i=0}^{I} 2^{1-i} \cdot c_i^{\text{real}} \cdot L_f^i \tag{10}$$

Generally, where an algorithm or application exhibits high computational complexity ($I > 1$), a complex valued implementation will be prefered. However, for algorithms of $\mathcal{O}(L_f)$ with a high number of overhead calculations c_0^r, a real valued approach can be more efficient if the savings out-weigh the additional cost in the filter bank calculation described in Sec. 2.

Subband Adaptive Filtering. Tab. 1 lists the complexities of a number of popular DSP algorithms for real valued processing. The complex implementations of NLMS, RLS, and affine projection algorithm (APA) exist and simply require a 4 times higher load in terms of computations [6, 8]. For RLS implementations, complex processing can roughly half the processing load over real valued calculations, while for LMS-type algorithms, processing is approximately equal in both real or complex subbands. However, for the latter, the lower processing gain for the filter bank calculation would favour an implementation in complex subbands. For the APA, the load independ of the filter length can be so high that a real valued implementation is prefered.

5. Conclusion

We have presented a highly efficient polyphase implementation for complex valued GDFT filter banks. A modification can be applied to produce SSB modulation-like real valued subband signals, bypassing the Weaver method. By also taking into account the computational load of subband processing which differs for real and complex implementation due to a higher possible decimation for complex signals, an overall optimum implementation can be found using. Examples have been given for a number of adaptive algorithms.

References:

[1] CHU, P.L.: Weaver SSB subband acoustic echo canceller, Proc. IEEE Workshop Applications of Signal Processing to Audio and Acoustics, 1993.

[2] CROCHIERE, R.E., and RABINER, L.R.: Multirate Digital Signal Processing (Prentice Hall, 1983).

[3] CVETKOVIĆ, Z., and VETTERLI, M.: Oversampled filter banks, to appear in IEEE Trans SP.

[4] GILLOIRE, A. and VETTERLI, M.: Adaptive filtering in subbands with critical sampling: analysis, experiments and applications to acoustic echo cancelation, IEEE Trans SP, 1992, 40(8):1862–1875.

[5] HARTENECK, M. and STEWART, R.W.: Filterbank design for oversampled filter banks without aliasing in the subbands, IEE Elec. Lett., 1997, 38(18), pp.1538–1539.

[6] HAYKIN, S.: Adaptive filter theory (Prentice Hall, 1991).

[7] MAKINO, S., NOEBAUER, J., HANEDA, Y., and NAKAGAWA, A.: SSB subband echo canceller using low-order projection algorithm, Proc. ICASSP, 1996, Atlanta, GA, II:945–948.

[8] NAKAGAWA, A., HANEDA, Y., and MAKINO, S.: Subband acoustic echo canceller using two different analysis filters and 8th order projection algorithm, Proc. IWAENC, 1997, London, pp:140–143.

[9] WEISS, S., LAMPE, L., and STEWART, R.W.: Efficient subband adaptive filtering with oversampled GDFT filter banks, Proc. IEE Colloq. Adapt. DSP Mobile Comms Sys, 1997, London, pp.4.1–4.9.

[10] WEISS, S., STEWART, R.W., STENGER, A., and RABENSTEIN, R.: Performance limitations of subband adaptive filters, Submitted to EUSIPCO, 1998, Rhodos, Greece.

[11] YAMADA, Y., OCHI, H., and KIYA, H.: A subband adaptive filter allowing maximally decimation, IEEE J Selected Areas Comms, 1994, 12(9), pp.1548–1552.