

A hybrid ant algorithm for scheduling independent jobs in heterogeneous computing environments

Graham Ritchie and John Levine

Centre for Intelligent Systems and their Applications
School of Informatics, University of Edinburgh
Appleton Tower, Crichton Street, Edinburgh, EH8 9LE
Graham.Ritchie@ed.ac.uk

Department of Computer and Information Sciences
University of Strathclyde
Livingstone Tower, 26 Richmond Street, Glasgow, G1 1XH
John.Levine@cis.strath.ac.uk

Abstract

The efficient scheduling of independent computational jobs in a heterogeneous computing (HC) environment is an important problem in domains such as grid computing. Finding optimal schedules for such an environment is (in general) an NP-hard problem, and so heuristic approaches must be used. In this paper we describe an ant colony optimisation (ACO) algorithm that, when combined with local and tabu search, can find shorter schedules on benchmark problems than other techniques found in the literature.

Introduction & Motivation

The efficient scheduling of independent computational jobs in a heterogeneous computing (HC) environment such as a computational grid is clearly important if good use is to be made of such a valuable resource. However, finding optimal schedules in such a system has been shown, in general, to be NP-hard (it is a generalised reformulation of SS8 from (Garey and Johnson, 1979)).

Static scheduling algorithms can be used in such a system for several different requirements (Braun et al., 2001). The first, and most common, is for planning an efficient schedule for some set of jobs that are to be run at some time in the future, and to work out if sufficient time or computational resources are available to complete the run *a priori*. Static scheduling may also be useful for analysis of heterogeneous computing systems, to work out the effect that losing (or gaining) a particular piece of hardware, or some sub-network of a grid for example, will have. Static scheduling techniques can also be used to evaluate the performance of a dynamic scheduling system after it has run, to check how effectively the system is using the resources available.

The ant colony optimisation (ACO) meta-heuristic was first described by Dorigo (Dorigo, 1992) as a technique to solve the travelling salesman problem, and was inspired by the ability of real ant colonies to efficiently organise the foraging behaviour of the colony using external chemical *pheromone* trails as a means of communication. ACO algorithms have since been widely employed on many other combinatorial optimisation problems (see (Dorigo and Stützle, 2002) for a review), including several domains related to the problem in hand, such as bin packing (Levine

and Ducatelle, 2003) and job shop scheduling (van der Zwaan and Marques, 1999), but ACO has not previously been applied to finding good job schedules in an HC environment.

Simulation Model

Real-world HC systems, such as a computational grid, are complex combinations of hardware, software and network components and so it is often hard to make fair comparisons of the different techniques that are being used on various different systems. To address this problem (Braun et al., 2001) describes a benchmark simulation model for comparison of static scheduling algorithms for HC environments. They define the notion of a *metatask* as a collection of independent jobs with no inter-job dependencies, and the goal of a scheduling algorithm is to minimise the total execution time of the metatask. As the scheduling is performed statically all necessary information about the jobs in the metatask and processors in the system is assumed to be available *a priori*. Essentially, the expected running time of each individual job on each processor must be known, and this information can be stored in an ‘expected time to compute’ (ETC) matrix. Of course, in a real scheduler, there may be some difference between the expected time to compute and the real time taken to complete a job, but for this paper we assume that the value in the ETC matrix is the running time of the job and we leave it to further work to test the scheduler in a more realistic environment. A row in an ETC matrix contains the ETC for a single job on each of the available processors, and so any ETC matrix will have $n \times m$ entries, where n is the number of jobs and m is the number of processors. A simple example ETC matrix with details for 4 jobs and 2 processors is given in table 1.

In any real heterogeneous computing system the running time of a particular job is not the only factor that must be taken into consideration when allocating jobs, the time that it takes to move the executables and data associated with each job should also be considered. To resolve this the entries in the ETC matrix are assumed to include such overheads. Also, if a job is not executable on a particular processor (for whatever reason) then the entry in the ETC matrix is set to infinity.

In order to simulate various possible heterogeneous scheduling problems as realistically as possible (Braun et al.,

	processor 1	processor 2
job 1	2	3
job 2	3	4
job 3	4	5
job 4	5	6

Table 1: An example ETC matrix. The figures indicate the time that processor m is expected to take to execute job n . This is a *consistent* ETC matrix, as processor 1 is consistently faster than processor 2.

2001) define different types of ETC matrix according to three metrics: *task heterogeneity*, *machine heterogeneity* and *consistency*. The task heterogeneity is defined as the amount of variance possible among the execution times of the jobs, two possible values were defined: *high* and *low*. Machine heterogeneity, on the other hand, represents the possible variation of the running time of a particular job across all the processors, and again has two values: *high* and *low*. In order to try to capture some other possible features of real scheduling problems, three different ETC consistencies were used: *consistent*, *inconsistent* and *semi-consistent*. An ETC matrix is said to be consistent if whenever a processor p_j executes a job j_i faster than another processor p_k , then p_j will execute all other jobs faster than p_k . A consistent ETC matrix can therefore be seen as modelling a heterogeneous system in which the processors differ only in their processing speed. In an inconsistent ETC a processor p_j may execute some jobs faster than p_k and some slower. An inconsistent ETC matrix could therefore simulate a network in which there are different types of machine available, e.g. a UNIX machine may perform jobs that involve a lot of symbolic computation faster than a Windows machine, but will perform jobs that involve a lot of floating point arithmetic slower. A semi-consistent ETC matrix is an inconsistent matrix which has a consistent sub-matrix of a predefined size, and so could simulate, for example, a computational grid which incorporates a sub-network of similar UNIX machines (but with different processor speeds), but also includes an array of different computational devices.

These different considerations combine to leave us with 12 distinct types of possible ETC matrix (e.g. high task, low machine heterogeneity in an inconsistent matrix, etc.) which simulate a range of different possible heterogeneous systems. The matrices used in the comparison study of (Braun et al., 2001) were randomly generated with various constraints to attempt to simulate each of the matrix types described above as realistically as possible. The methods used to generate the matrices are briefly described here. Initially a $m \times 1$ ‘baseline’ vector B is generated by repeatedly selecting m uniform random floating point values from between 1 and ϕ_b , the upper bound on values in B . Then the ETC matrix is constructed by taking each value $B(i)$ in B and multiplying it by a uniform random number $x_r^{i,k}$ which has an upper bound of ϕ_r . $x_r^{i,k}$ is known as a *row multiplier*. Each row in the ETC matrix is then given by $ETC(j_i, p_k) = B(i) \times x_r^{i,k}$ for $0 \leq k \leq n$. The vec-

tor B is not used in the actual matrix. This process is repeated for each row until the $m \times n$ matrix is full. Each of the different task and machine heterogeneities described above is modelled by using different baseline values: high task heterogeneity was represented by setting $\phi_b=3000$ and low task heterogeneity used $\phi_b=100$. High machine heterogeneity was represented by setting $\phi_r=1000$, and low machine heterogeneity was modelled using $\phi_r=10$. To model a consistent matrix each row in the matrix was sorted independently, with processor p_1 always being the fastest, and p_m being the slowest. Inconsistent matrices were not sorted at all and are left in the random state in which they are generated. Semi-consistent matrices are generated by extracting the row elements $\{0, 2, 4, \dots\}$ of each row i , sorting them and then replacing them in order, while the elements $\{1, 3, 5, \dots\}$ are left in their original order, this means that the even columns are consistent while the odd columns are (generally) inconsistent.

For their study 100 matrices were generated of each of the 12 possible types, modelling 16 processors and 512 jobs for all matrices. Exactly the same matrices used in their study were used in the experiments described below.

Current techniques

(Braun et al., 2001) provides a comparison of 11 static heuristics for scheduling in HC environments, and the reader is referred there for details of the various schemes that are used. A range of simple greedy construction heuristic approaches are compared and some of these are briefly described below.

Opportunistic Load Balancing (OLB) assigns each job in arbitrary order to the processor with the shortest schedule, irrespective of the ETC on that processor. OLB is intended to try to balance the processors, but because it does not take execution times into account it finds rather poor solutions.

Minimum Execution Time (MET) assigns each job in arbitrary order to the processor on which it is expected to be executed fastest, regardless of the current load on that processor. MET tries to find good job-processor pairings, but because it does not consider the current load on a processor it will often cause load imbalance between the processors.

Minimum Completion Time (MCT) assigns each job in arbitrary order to the processor with the minimum expected *completion time* for the job. The completion time of a job j on a processor p is simply the ETC of j on p added to p ’s current schedule length. This is a much more successful heuristic as both execution times and processor loads are considered.

Min-min establishes the minimum completion time for every unscheduled job (in the same way as MCT), and then assigns the job with the *minimum* minimum completion time (hence Min-min) to the processor which offers it this time.

Min-min uses the same intuition as MCT, but because it considers the minimum completion time for all jobs at each iteration it can schedule the job that will increase the overall makespan the least, which helps to balance the processors better than MCT.

Max-min is very similar to Min-min. Again the minimum completion time for each job is established, but the job with the *maximum* minimum completion time is assigned to the corresponding processor. Max-min is based on the intuition that it is good to schedule larger jobs earlier on so they won't 'stick out' at the end causing a load imbalance. However experimentation shows that Max-min cannot beat Min-min on any of the test problems used here.

The best solution technique found in (Braun et al., 2001)'s comparison was a genetic algorithm (GA). The GA described works on *chromosomes* which represent a complete solution to the problem. Each chromosome is simply a array of n elements, in which position i represents job i , and each entry in the array is a value between 1 and m which represents the processor to which the corresponding job is allocated. The main steps of the algorithm are described below.

1. Generate an initial population of 200 chromosomes. Two policies were used; either use 200 randomly generated chromosomes, or use 199 randomly generated ones, plus the Min-min solution (known as *seeding* the population).
2. Evaluate the 'fitness' of each individual. The fitness is defined simply as the makespan of the solution encoded by a chromosome, a lower fitness is therefore preferable.
3. Create the next generation using:
 - Selection of the fitter individuals. A rank-based roulette wheel scheme was used that duplicated individuals with a probability according to their fitness. An *elitist* strategy was also employed which guarantees that the fittest individual is always duplicated in the next generation.
 - Crossover between random pairs of individuals. Single point crossover was used and each chromosome was considered for crossover with a probability of 60%.
 - Random mutation of individuals. A chromosome is randomly selected, then a random task in the chromosome is randomly assigned to a new processor. Every chromosome is considered for mutation with a probability of 40%.
4. While the stopping criteria are not met, repeat from step 2. The GA stops when either 1000 iterations have been completed, there has been no change in the elite chromosome for 150 iterations, or all chromosomes have converged to the same solution.

This GA finds the best or equal best solutions to all the ETC matrix types tested in (Braun et al., 2001), although it does takes significantly longer than Min-min which was the second best technique for most problems (around 60 seconds compared to under a second for Min-Min).

Applying ACO to the problem

As noted before, ACO has been shown to be an effective strategy for several problems closely related to scheduling jobs in an HC environment, and so it seems that it may be an effective strategy in this domain. In this section we describe an ACO approach to the problem, largely following the ACO algorithm design described in (Dorigo and Stützle, 2002).

Defining the pheromone trail

In any ACO algorithm we must first determine what information we will encode in the pheromone trail, which will allow the ants to share useful information about good solutions. The fact that jobs will run at different speeds on different processors suggests that it would be useful to store information about good processors for each job. The pheromone value $\tau(i, j)$ was therefore selected to represent the favourability of scheduling a particular job i onto a particular processor j . The pheromone matrix will thus have a single (real-valued) entry for each job-processor pair in the problem, allowing the ants to share information about good processors for particular jobs.

The heuristic

The ants build their solution using both information encoded in the pheromone trail and also problem-specific information in the form of a heuristic. As discussed earlier, (Braun et al., 2001) show that the Min-min heuristic is a relatively simple but effective algorithm for this problem (it was only consistently beaten by the GA approach). Min-min suggests that the heuristic value of particular job j should be proportional to the MCT of j , that is the time j can be expected to finish on its 'best' processor p_{best}^j . p_{best}^j is established for each job j according to equation 1. In this equation $t(p_i)$ is the current total running time of a processor p_i , and $ETC(j, p)$ is the ETC matrix entry representing the ETC of j on p .

$$\min_{1 \leq i \leq m} (t(p_i) + ETC(j, p_i)) \quad (1)$$

The completion time ($ct()$) of job j on p_{best}^j (i.e. j 's MCT) is then used for the heuristic function, a lower value is preferable and so the inverse is used. The resulting $\eta(j)$ function used by the ants is defined in equation 2.

$$\eta(j) = \frac{1}{ct(j, p_{best}^j)} \quad (2)$$

If the MCT of j is large, as it often is when dealing with the values in the ETC matrices, $\eta(j)$ will be a very small value. To allow this value to be effectively controlled with the β parameter (which determines the extent to which heuristic information is used by the ants), it is necessary to 'scale' the heuristic value up. Therefore in the implementation of this function all the $\eta(j)$ values are computed for each job and then the job list is sorted into descending order of these values. The value then used in equation 5 below is the jobs position in the sorted list, which will be in the range 1 - n . A job with a lower $\eta(j)$ value will have thus have a relatively larger integer heuristic value.

The fitness function

The goal of the fitness function is essentially to help the algorithm discern between high and low quality solutions built by the ants. Clearly the makespan of a solution is a sensible metric, and as in this problem the chances of two different solutions having equal makespans is very low, it was decided that the ‘raw’ makespan could be used. As a lower makespan is preferred the inverse is used, as shown in equation 3.

$$f(s) = \frac{1}{ms(s)} \quad (3)$$

Updating the pheromone trail

To allow the ants to share information about good solutions a policy for updating the pheromone trail must be established. Dorigo’s original *Ant System* followed the biological analogy closely and allowed all the ants to leave pheromone, but Stützle & Hoos have shown with their *Max-Min Ant System* (MMAS, described in detail in (Stützle and Hoos, 2000)) that allowing only the best ant, s_{best} , to leave pheromone after each iteration makes the search much more aggressive and significantly improves the performance of ACO algorithms. This was therefore the policy chosen. Also following Stützle & Hoos’ example, the best ant s_{best} can be defined as either the iteration best ant s_{ib} , or the global best ant s_{gb} (the best ant solution found so far), a parameter γ is used to define how often s_{ib} is used instead of s_{gb} when updating the pheromone trail. Increasing the value of γ allows the search to be less aggressive, and encourages the ants to explore more of the solution space. As mentioned above the pheromone matrix will have an entry for each job-processor pair in the problem, and each such pair in s_{best} ’s solution is reinforced in proportion to the relative fitness value of s_{best} compared to the global best solution s_{gb} . If γ is 0 and so only s_{gb} is used for updates this proportion will always be 1. In order to allow the ants to ‘forget’ poor information, each pheromone value is also decayed at this stage, this is implemented with a parameter ρ which takes a value between 0 and 1, if ρ is set to 1 then no decay will take place, if ρ is 0 then each pheromone value will be ‘wiped’ at each iteration and the pheromone trail is effectively switched off. Equation 4 defines the policy.

$$\tau(i, j) = \begin{cases} \rho \cdot \tau(i, j) + \frac{f(s_{best})}{f(s_{gb})} & \text{if job } i \text{ is allocated to} \\ & \text{processor } j \text{ in } s_{best} \\ \rho \cdot \tau(i, j) & \text{otherwise} \end{cases} \quad (4)$$

Building a solution

With all our ACO policies in place we can now describe how the ants actually build a solution, using both the information stored in the pheromone trail and the heuristic function. The ant solution building technique is an attempt to follow the concept of the best heuristic method, Min-min. Each

ant starts with an empty schedule and the processor p_{best}^j which will complete each unscheduled job $j_1 \dots j_n$ earliest is established (following the process described in section above). A job j is then probabilistically chosen to schedule next based on the pheromone value between j and its best processor p_{best}^j , and j ’s heuristic value (as determined by the process in section above). The probability of selecting job j to schedule next is given by equation 5. In this equation α is parameter which defines the relative weighting given to the pheromone information, and β defines the relative weighting given to the heuristic information. If α is set to 0 only heuristic information is used and the ants effectively perform a probabilistic Min-min search. If β is set to 0 then only pheromone information is used.

$$prob(j) = \frac{[\tau(j, p_{best}^j)]^\alpha \cdot [\eta(j)]^\beta}{\sum_{i=1}^n [\tau(i, p_{best}^i)]^\alpha \cdot [\eta(i)]^\beta} \quad (5)$$

The chosen job j_c is then allocated to $p_{best}^{j_c}$. This process is repeated until all jobs have been scheduled and a complete solution has been built. Each ant in the colony (the size of which is defined by a parameter, *num.Ants*) builds a solution in this manner in each iteration. Once all the ants have built a solution the pheromone trail update procedure is performed as described above.

It was observed in test runs that the ants often take some time to start building good solutions because it takes a few iterations before the pheromone trail is populated with good job-processor pairings. To attempt to resolve this a pheromone *seeding* strategy was used which initially sets the global best solution s_{gb} to be the Min-min solution after local and tabu searches (described below), and a pheromone update is performed once before the ants start building solutions. This seems to work well as the ants start producing solutions better than or near the global best almost immediately.

Adding local search

Other researchers (e.g. (Levine and Ducatelle, 2003), (Dorigo and Stützle, 2002)) have demonstrated that ACO algorithms can often effectively be improved by combining them with local search (LS) techniques. (Ritchie and Levine, 2003) describe a local search for the problem in hand which can quickly and effectively improve the solutions found by many solution building techniques. The reader is referred there for details of the algorithm and detailed results, but briefly the local search works as follows. Any solution s will have (at least) one processor with a schedule length equal to the makespan of the solution, we call this the ‘problem’ processor (if there is more than one problem processor one is picked arbitrarily). The neighbourhood N of s is defined as all solutions which differ by a single transfer of a job currently allocated to the problem processor to any other processor, or by a single swap of a job currently allocated to the problem processor with a job allocated to other processor. The local search procedure exhaustively analyses this neighbourhood and selects the swap or transfer which reduces the maximum schedule length of the two processors

involved the most. This process is repeated until no further improvement is possible.

This local search is applied to each of the solutions built by the ants before the pheromone update stage to take the ant solution to its local optimum in the search space.

Adding tabu search

Tabu search (TS) (Glover and Laguna, 1997) is essentially a more sophisticated local search strategy which tries to avoid entrapment in local minima by using a *tabu* list of previously visited regions of the search space and disallowing moves which would result in a solution that is contained in the list, i.e. one that has been seen before. As a ‘smarter’ local search strategy, tabu search also seems like it might be a useful addition to an ACO algorithm. (Ritchie, 2003) describes a fairly standard tabu search procedure for this problem (based on the approach described in (Thesen, 1998)) which uses the same notion of a solution neighbourhood as used for the local search described above. The reader is referred to (Ritchie, 2003) for implementation details (although it should be noted that the tabu list size was set to 10 for all experiments discussed here).

When used in conjunction with the ACO algorithm the tabu search is simply used for $nTrials$ (a parameter) iterations to try and improve the solution of the iteration best ant (which will already have had local search applied to it). The tabu search is not applied to every ant solution (as for the local search) due to the longer running time. As can be seen in the results below it cannot always improve on the locally optimised ant solution, but it can sometimes ‘break through’ local optima, and adds significantly to the performance of the algorithm as a whole.

Setting parameter values

This algorithm has many parameters, which seem to interact in a fairly complex way, both with each other and with the specific problem class under investigation. Due to the time taken for a decent length run of the whole ACO algorithm, and also to the stochasticity of the approach, finding the optimal values for these parameters is a complex and time-consuming task, for the purposes of brevity only a brief rationale for each value used is given below.

- α determines the extent to which pheromone information is used as the ants build their solution. Pheromone is critical for the success of this algorithm, and having experimented with values between 1-50, it seems this algorithm works best with a relatively high value of 10 for all problems.
- β determines the extent to which heuristic information is used by the ants. Again, values between 1-50 were tested, and a value of 10 worked well for most problem types, and to allow fair comparison this was the value used for all results presented here. Further tests on longer runs show that lower values produce better long term results for the inconsistent matrices. As a longer run progresses, the ants’ solutions become significantly better than that produced by the Min-min heuristic, and it was observed that in long runs the ants could not reproduce solutions as

close to the global best as well as they could earlier on in the run. It was therefore hypothesised that the high β value which is necessary to get good solutions to begin with might actually be limiting the ants later in the run. A β decay mechanism was therefore implemented to allow this value to gradually decrease as the run progresses. Tests showed, however, that as the β value decays the ants start producing worse solutions, and so this feature was not used.

- γ is used to indicate how many times the iteration best ant is used in place of the global best ant in pheromone updates. Initially it seemed that a value of 0 (i.e. only the global best is ever used) worked best, and for comparison this was the value used in the results shown below. However, again, longer test runs show that higher values could work well for some problem instances. Specifically, the very best result found for the problem *u-c-hihi.0* used a γ value of 1.
- *numAnts* defines the number of ants to use in the colony. A value of 10 seems to be a good compromise between amount of search per iteration and speed of execution.
- τ_{min} defines the minimum pheromone value that can be returned between any job and any processor. A value of 0.01 worked well, balancing exploration and avoiding bad job-processor pairings.
- τ_0 is the value to which the pheromone matrix values are initialised, it was set to τ_{min} for all test runs.
- ρ is the pheromone evaporation parameter, a value of 0.75 (as used in other ACO algorithms) gives good results.
- *nTrials* is the number of trials performed in the tabu search phase. As the results below show, a value of around 1000 allows the tabu search to help improve solutions enough, while a longer run would slow execution time without providing significantly better results.

The values used work well enough, as the results below show, but there is undoubtedly room for improvement. The β decay mechanism did not prove to be helpful, but it may be that changing the values of other parameters, γ in particular, over the course of a run would improve results. There is a lot of scope for future work to experiment with changing these values for various considerations, such as which work best for particular problem types, or perhaps different values might work well depending on the length of run desired: a high β value may provide good solutions quickly, but a lower value may provide better results after a longer period of time.

Experimental results

In the original study by (Braun et al., 2001) 100 instances of each of the 12 problem classes are used to test the approaches, leaving us with a total of 1200 instances. The ACO approach as a whole takes a comparatively long time to build solutions, approximately 12 seconds per iteration (when 10 ants are used), and so it would be unfeasible to run even a fair-sized test on all 1200 problems. Results are therefore only provided for the first problem (where $n=0$)

in each class of ETC matrix. The actual makespans found are provided in table 2, along with the results from (Braun et al., 2001) for Min-min and the GA, and results for local and tabu searches described here applied to the Min-min solution. Tests were carried out on 1.6 GHz machines running Linux, and all programs were written in Java.

For these tests the ACO algorithm was allowed to run for 1000 iterations which took an average of 12792 seconds (just over 3.5 hours). The ACO algorithm is allowed to run for so long because this allows it reasonable time to build up a useful pheromone trail. The ants need a decent length run to find solutions which significantly improve on the other solutions. To allow fair comparison the tabu searches were run for 1,000,000 iterations which took 12967 seconds for the Min-min+Tabu search. The Min-min search took an average of 0.19 seconds, the Min-min+LS algorithm ran for an average of 0.37 seconds, and the GA took an average of 65.16 seconds. It would perhaps have been fairer to allow the GA to run for an equivalent amount of time to the ACO algorithm but the program was not available for testing. However, (Braun et al., 2001) note that the GA was usually stopped because the elite chromosome (best solution) had not changed for 150 iterations, so it may be that GA would not have found better solutions with much more time anyway.

In the results the different problem instances are identified according to the following scheme: $w-x-yyzz.n$, where:

- w denotes the probability distribution used; only uniform distributions were used so this is u for all files.
- x denotes the type of consistency, one of:
 - c : consistent matrix
 - i : inconsistent matrix
 - s : semi-consistent
- yy denotes the task heterogeneity, one of:
 - hi : high heterogeneity
 - lo : low heterogeneity
- zz denotes machine heterogeneity, one of:
 - hi : high heterogeneity
 - lo : low heterogeneity
- n is the test case number, numbered from 0 to 99.

From these results it is clear that the ACO approach can consistently find shorter makespans than any of the other approaches for all classes of ETC matrix tested, and these preliminary results suggest that it would beat all the other approaches for the full problem suite (although more extensive tests should be carried out to confirm this, and also to check how consistently the ACO approach works). The ACO approach does, however, take significantly longer than any other approach, at around 3.5 hours it takes approximately 200 times longer than the GA. For some applications, such as an online grid scheduler, this amount of time may not be acceptable, and so we would suggest simply using Min-min with a local search (as described in (Ritchie and Levine, 2003)). However, for applications where the makespan is of

critical importance, and when there is plenty of time available to search for good schedules, such as perhaps advance scheduling of jobs to be performed on a satellite, the approach described here could be used. We think that the suite of techniques used here (LS, TS and ACO) provide a toolkit of possible approaches to be used on scheduling problems with different constraints.

As this is a hybrid algorithm it is interesting to compare the results of the algorithm used with and without certain components, such as the local and tabu searches and pheromone seeding. A decent comparison would, unfortunately, take too long to describe, but the interested reader is referred to (Ritchie, 2003) for such a discussion, along with an analysis of an example run of the algorithm.

Conclusions and Future Work

Statically scheduling independent jobs in heterogeneous computing environments is useful for several different considerations in domains such as grid computing. The hybrid ACO algorithm described here can consistently find better schedules for several benchmark problems than other techniques found in the literature, and it seems a promising approach to scheduling in HC environments. There is, however, much room for further investigation. More work could be carried out with the algorithm described here, for example investigating different parameter settings, ant solution building techniques, or different local search strategies, and also testing it in a more realistic environment. In broader terms we feel that investigating the use of ACO strategies in different forms of HC scheduling, such as scheduling jobs with precedence constraints or in dynamic environments might also be fruitful. The techniques used may have to diverge somewhat from those described here, but we hope that the results presented here suggest that there is considerable scope for future research in this area.

Acknowledgments

We would like to thank Tracy Braun and Howard Siegel for sharing their test data and detailed results with us.

References

- Braun, T. D., Siegel, H. J., Beck, N., Bölöni, L. L., Maheswaran, M., Reuther, A. I., Robertson, J. P., Theys, M. D., Yao, B., Hensgen, D., and Freund, R. F. (2001). A comparison of eleven static heuristics for mapping a class of independent tasks onto heterogeneous distributed computing systems. *Journal of Parallel and Distributed Computing*, 61(6):810–837.
- Dorigo, M. (1992). *Optimization, Learning and Natural Algorithms*. PhD thesis, DEI, Politecnico di Milano, Milan, Italy. (in Italian).
- Dorigo, M. and Stützle, T. (2002). The ant colony optimization metaheuristic: Algorithms, applications, and advances. In Glover, F. and Kochenberger, G., editors, *Handbook of Metaheuristics*, volume 57 of *International Series in Operations Research and Management Science*, pages 251–285. Kluwer Academic Publishers.

problem	Min-min	GA	Min-min+LS	Min-min+Tabu	ACO
u-c-hihi.0	8460675.00	8050844.50	7711037.16	7568871.83	7497200.85
u-c-hilo.0	164022.44	156249.20	154873.05	154644.48	154234.63
u-c-lohi.0	275837.34	258756.77	251434.50	245981.55	244097.28
u-c-lolo.0	5546.26	5272.25	5231.13	5202.51	5178.44
u-i-hihi.0	3513919.25	3104762.50	3021155.10	3021155.10	2947754.12
u-i-hilo.0	80755.68	75816.13	74400.68	74400.68	73776.24
u-i-lohi.0	120517.71	107500.72	104309.12	104309.12	102445.82
u-i-lolo.0	2779.09	2614.39	2580.62	2580.62	2553.54
u-s-hihi.0	5160343.00	4566206.00	4256736.40	4248200.21	4162547.92
u-s-hilo.0	104540.73	98519.40	97711.72	97711.72	96762.00
u-s-lohi.0	140284.48	130616.53	126117.51	126115.39	123922.03
u-s-lolo.0	3867.49	3583.44	3505.69	3505.69	3455.22

Table 2: Results for the first problem in each ETC matrix class, comparing the ACO approach with the other approaches. The ACO algorithm was allowed to run for 1000 iterations and took an average of 12792 seconds (just over 3.5 hours). The Min-min+Tabu search ran for an average of 12967 seconds. The best result is indicated in bold.

Garey, M. R. and Johnson, D. (1979). *Computers and Intractability: A Guide to the theory of NP-Completeness*. Freeman and Company, San Francisco.

Glover, F. and Laguna, M. (1997). *Tabu Search*. Kluwer Academic publishers, Boston.

Levine, J. and Ducatelle, F. (2003). Ant colony optimisation and local search for bin packing and cutting stock problems. *Journal of the Operational Research Society*. (forthcoming).

Ritchie, G. (2003). Static multi-processor scheduling with ant colony optimisation and local search. Master's thesis, University of Edinburgh. available at: <http://www.inf.ed.ac.uk/publications/thesis/msc.html>.

Ritchie, G. and Levine, J. (2003). A fast, effective local search for scheduling independent jobs in heterogeneous computing environments. In Porteous, J., editor, *Proceedings of the 22nd Workshop of the UK Planning and Scheduling Special Interest Group (PLANSIG 2003)*, pages 178–183.

Stützle, T. and Hoos, H. (2000). Max-min ant system. *Future Generation Computer Systems*, 16(8):889–914.

Thesen, A. (1998). Design and evaluation of tabu search algorithms for multiprocessor scheduling. *Journal of Heuristics*, 4:141–160.

van der Zwaan, S. and Marques, C. (1999). Ant colony optimisation for job shop scheduling. In *Proceedings of the Third Workshop on Genetic Algorithms and Artificial Life (GAAL 99)*.