Automatica 46 (2010) 405–412

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

SMC design for robust H_∞ control of uncertain stochastic delay systems☆

Lirong Huang*, Xuerong Mao

Department of Mathematics and Statistics, University of Strathclyde, Glasgow G1 1XH, UK

ARTICLE INFO

Article history:
Received 18 February 2009
Received in revised form
5 July 2009
Accepted 6 November 2009
Available online 24 November 2009

Keywords:
Sliding mode control
Stochastic systems
Time delay
Robust control
Finite-time stabilization

ABSTRACT

Recently, sliding mode control method has been extended to accommodate stochastic systems. However, the existing results employ an assumption that may be too restrictive for many stochastic systems. This paper aims to remove this assumption and present in terms of LMIs a sliding mode control design method for stochastic systems with state delay. In some cases, the proposed method provides a control scheme for finite-time stabilization of stochastic delay systems.

1. Introduction

Sliding mode control (SMC) has various attractive features such as fast response, good transient performance, order reduction and, particularly, robust with matched uncertainties, and is well known to be an effective way to handle many challenging problems of robust stabilization. Over the past decades, SMC has been one of the most popular control methods among the control community and has found wide applications to automotive systems, observers design, chemical processes, electrical motor control, aero-engineering and so on (see, e.g., Choi (2007), Gouaisbaut, Dambrine, and Richard (2002), Hu, Ge, and Su (2004), Hu, Ma, and Xie (2008), Jafarov (2005), Li and Decarlo (2003), Utkin (1992), Utkin, Guldner, and Shi (1999), Edwards, Akoachere, and Spurgeon (2001), Oucheriah (2003) and the references therein). Generally speaking, SMC uses a discontinuous control law (relays) to force and restrict the state trajectories to a predefined sliding surface on which the system has some desired properties such as stability, disturbance rejection capability and tracking (see Gouaisbaut et al. (2002), Li and Decarlo (2003) and Utkin (1992)).

In recent years, there has been a growing interest in extension of SMC to accommodate stochastic systems (see, e.g., Chang and Chang (1999), Chang and Wang (1999a,b), Huang and Deng (2008a), Niu, Ho, and Lam (2005) and Niu, Ho, and Wang (2007, 2008)) since stochastic modelling has come to play an important role in many branches of science and engineering (see, e.g., Feng and Liu (1995), Kolmanovskii and Myshkis (1999) and Mao (2007)). For example, Niu et al. (2005) studied the integral SMC for the stochastic delay system

$$\begin{aligned} dx(t) = {} & [(A+\Delta A(t))x(t) + (A_d+\Delta A_d(t))x(t-\tau(t)) \\ & + B(u(t)+f(x(t),t))]dt + D[(C+\Delta C(t))x(t) \\ & + (C_d+\Delta C_d(t))x(t-\tau(t))]\mathrm{d}w(t), \end{aligned} \tag{1}$$

where it is assumed that there is matrix $G \in R^{m\times n}$ such that

$$\det(GB) \neq 0 \quad \text{and} \quad GD = 0 \tag{2}$$

with $\det(\cdot)$ denoting the determinant of a matrix. However, these existing results employ assumptions such as (2) on the structure of the control system such that their controller design do not need to deal with stochastic perturbation and hence they can use the SMC design method for deterministic systems (see Remark 1 and 4 in Huang and Deng (2008a)). These existing results may be considered as studies of SMC with stochastic perturbation in sliding mode. But such an assumption may be too restrictive for stochastic systems in many practical situations.

The main purpose of this paper is to remove this assumption. Moreover, in some cases, our design method provides a control scheme for finite-time stabilization of stochastic delay systems (see Remark 2 and the Example). Problems of finite-time stabilization of stochastic systems (Yang, Li, & Chen, 2009) are relatively seldom studied while those of deterministic systems have received much attention (see Huang, Lin, and Yang (2005) and

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Emilia Fridman under the direction of Editor Ian R. Petersen.

* Corresponding address: Department of Mathematics and Statistics, University of Strathclyde, 26 Richmond Street, Glasgow, G1 1XH, UK. Tel.: +44 0 141 548 3809; fax: +44 0 141 552 2079.

E-mail address: lirong.huang@strath.ac.uk (L. Huang).

0005-1098/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2009.11.013

Hong, Wang, and Cheng (2006) and the references therein). Our proposed design method is presented in terms of LMIs (see Boyd, El Ghaoui, Feron, and Balakrishnan (1994)), which can be easily implemented.

2. Problem statement

Throughout the paper, unless otherwise specified, we will employ the following notation. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ and $\mathbb{E}[\cdot]$ be the expectation operator with respect to the probability measure. Let $W(t) = (W_1(t), \ldots, W_{r_w}(t))^T$ be an r_w-dimensional Brownian motion defined on the probability space. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, $P > 0\,(P < 0)$ means that P is a symmetric positive (negative) definite matrix of appropriate dimensions while $P \geq 0\ (P \leq 0)$ is a symmetric positive (negative) semidefinite matrix. I stands for the identity matrix of appropriate dimensions. Denote by $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ the maximum and minimum eigenvalue of a matrix respectively. Let $|\cdot|$ and $|\cdot|_1$ denote the Euclidean norm and 1-norm of a vector and their induced norms of a matrix respectively. Unless explicitly stated, matrices are assumed to have real entries and compatible dimensions. Let $h > 0$ and $C([-h, 0]; R^n)$ denote the family of all continuous R^n-valued functions φ on $[-h, 0]$ with the norm $\|\varphi\| = \sup\{|\varphi(\theta)| : -h \leq \theta \leq 0\}$. For $p > 0$, let $\tilde{L}^p_{\mathcal{F}_0}([-h, 0]; R^n)$ be the family of all $\mathcal{F}_0$-measurable $C([-h, 0]; R^n)$-valued random variables such that $\mathbb{E}\|\varphi\|^p < \infty$.

Let us consider an n-dimensional uncertain stochastic system with state delay

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[A_0(t)x(t) + A_1(t)x(t-h) + B(u(t) + \phi(t, x_t)) \\ & + B_v v(t)\big]\mathrm{d}t + g(t, x(t))\mathrm{d}W(t), \end{aligned} \tag{3}$$

$$z(t) = Cx(t) + Dv(t) \tag{4}$$

on $t \geq 0$ with initial data $x_0 = \{x(\theta) : -h \leq \theta \leq 0\} = \xi \in \tilde{L}^2_{\mathcal{F}_0}([-h, 0]; R^n)$, where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control input; $z(t) \in R^p$ is the controlled output; $v(t) \in R^q$ is the exogenous disturbance input belonging to $L_2[0, \infty)$; $h > 0$, time delay of the system, is a known number; B, B_v, C, D are constant matrices and B is of full column rank; $A_i(t), i = 0, 1$, are matrix functions with time-varying uncertainties described as $A_i(t) = A_i + \Delta A_i(t)$, where A_i, $i = 0, 1$, are known constant matrices while uncertainties $\Delta A_i(t)$ are assumed to be norm bounded, i.e.,

$$\Delta A_i(t) = L_i F_i(t) E_i, \quad i = 0, 1 \tag{5}$$

with known constant matrices L_i, E_i, and unknown matrix functions $F_i(t)$ having Lebesgue measurable elements and satisfying $F_i^T(t)F_i(t) \leq I$ for all $t \geq 0$; matched uncertainty $\phi(t, x_t)$ satisfies

$$|\phi(t, x_t)| \leq k_\phi(|x(t)| + |x(t-h)|), \quad \forall\, t \geq 0 \tag{6}$$

where k_ϕ is a nonnegative number; $g(t, x(t))$ may be not exactly known but there is a constant matrix G such that

$$\mathrm{trace}[g^T(t, x(t))g(t, x(t))] \leq |Gx(t)|^2 \tag{7}$$

for all $t \geq 0$ (see, e.g., Chen, Guan, and Lu (2005) and Yue and Han (2005)). It is also assumed that pair (A_0, B) is controllable, that is, there exists matrix $K_0 \in R^{m\times n}$ such that matrix $A_0 + BK_0$ is stable.

It is easy to verify that Eq. (3) with $u(t) = 0$ and $v(t) = 0$ has a unique solution (see, e.g., Mao (2002, 2007)). In this paper, we intend to design a sliding surface and a switching control law such that the state trajectories are drawn in finite time to the sliding surface with probability 1, on which system (3) and (4) is robustly mean-square exponentially stable with some prescribed disturbance attenuation γ (>0) (see Definition 2). It should be noted that, for simplicity only, we take a relatively simple model. The proposed method can be easily extended to many systems such as those of large scale, with Markovian switching and time-varying and multiple delays (see Chang and Chang (1999), Chang and Wang (1999a,b), Chen et al. (2005), Niu et al. (2005, 2007, 2008) and Huang and Deng (2008a)).

At the end of this section, let us introduce the following definitions.

Definition 1 (*Mao (2007)*). Uncertain stochastic delay system (3) with $u(t) = 0$ and $v(t) = 0$ is said to be robustly mean-square exponentially stable if there is a positive constant λ such that

$$\limsup_{t\to\infty} \frac{1}{t} \log \mathbb{E}|x(t; \xi)|^2 \leq -\lambda \tag{8}$$

for all admissible uncertainties (5).

Definition 2. Uncertain stochastic delay system (3) and (4) is said to be robustly mean-square exponentially stable with disturbance attenuation γ (>0) if system (3) with $v(t) = 0$ is robustly mean-square exponentially stable and moreover, under zero initial condition,

$$\mathbb{E}\int_0^\infty |z(t)|^2 \mathrm{d}t \leq \gamma^2 \int_0^\infty |v(t)|^2 \mathrm{d}t \tag{9}$$

for all nonzero $v \in L_2[0, \infty)$ and admissible uncertainties (5).

For definitions of mean-square stability with a given disturbance attenuation γ, please see, e.g., Berman and Shaked (2006), Niu et al. (2007) and Xu and Chen (2002). Moreover, let us present the definition of finite-time stability of stochastic systems, which is consistent with that of deterministic systems (see, e.g., Bhat and Bernstein (2000), Huang et al. (2005) and Hong et al. (2006)).

Definition 3. The equilibrium $x = 0$ of uncertain stochastic delay system (3) with $u(t) = 0$ and $v(t) = 0$ is said to be pth ($p > 0$) moment finite-time stable if system (3) with $u(t) = 0$ and $v(t) = 0$ is pth moment stable and if for every $\xi \in \tilde{L}^p_{\mathcal{F}_0}([-h, 0]; R^n)$, there exists (a settling time) $T = T(\xi) > 0$ such that $0 < \mathbb{E}|x(t; \xi)|^p < \infty$ for all $0 \leq t < T$, $\lim_{t\to T} \mathbb{E}|x(t; \xi)|^p = 0$ and $\mathbb{E}|x(t; \xi)|^p = 0$ for all $t > T$.

For definition of pth moment stability, please see, e.g., Huang and Deng (2008b). We also cite the following well-known results that are useful for the development of this paper (see, e.g., Xu (1997) and Lütkepohl (1996)).

Lemma 1. *For any constant matrix $M \in R^{q\times p}$, inequality*

$$2u^T M v \leq r u^T M G M^T u + \frac{1}{r} v^T G^{-1} v, \quad u \in R^q,\ v \in R^p$$

holds for any pair of symmetric positive definite matrix $G \in R^{p\times p}$ and positive number r.

Lemma 2. *For a pair of constant matrices $G \in R^{p\times p}$ and $M \in R^{p\times q}$, if $G \geq 0$, then*

$$\mathrm{trace}(M^T G M) \leq \lambda_M(G)\mathrm{trace}(M^T M).$$

3. Switching surface and control scheme design

This section is devoted to designing the sliding surface and the switching control law such that the task of this paper is fulfilled. We present the design method as follows.

Given constant $\gamma > 0$, assume that there exist matrices $X > 0$, $R > 0$, Y_0, Y_1 and positive numbers β_0, β_1, λ_g, ζ_g such that

$$\Theta = \begin{bmatrix} \Theta_1 & * & * & * & 0 & * & * \\ XA_1^T + Y_1^T B^T & -R & 0 & 0 & * & 0 & 0 \\ GX & 0 & -\lambda_g I & 0 & 0 & 0 & 0 \\ E_0 X & 0 & 0 & -\beta_0 I & 0 & 0 & 0 \\ 0 & E_1 X & 0 & 0 & -\beta_1 I & 0 & 0 \\ B_v^T + D^T CX & 0 & 0 & 0 & 0 & \Theta_6 & 0 \\ CX & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (10)$$

$$\lambda_g I \leq X, \quad (11)$$

and

$$\begin{bmatrix} -BB^T & XG^T \\ GX & -\zeta_g I \end{bmatrix} \leq 0, \quad (12)$$

where $\Theta_1 = A_0 X + XA_0^T + BY_0 + Y_0^T B^T + R + \beta_0 L_0 L_0^T + \beta_1 L_1 L_1^T$, $\Theta_6 = -\gamma^2 I + D^T D$ and entries denoted by $*$ can be readily inferred from symmetry of the matrix. Let $P = X^{-1}$. It is easy to find $\zeta_b > 0$ such that

$$PBB^T P \leq \zeta_b I. \quad (13)$$

And then let

$$\beta = \frac{1}{2}\zeta_b \zeta_g. \quad (14)$$

In this work, we choose the switching surface as a linear function of the current states

$$s(t) = s(t, x(t)) = B^T P x(t) = 0 \quad (15)$$

for all $t \geq 0$. Note that matrix B is of full column rank and matrix $P > 0$. It is easy to see that

$$B^T PB > 0. \quad (16)$$

Moreover, function sgn : $R^m \to R^m$ is defined by

$$\text{sgn}(u) = \begin{bmatrix} \text{sgn}(u_1) & \text{sgn}(u_2) & \cdots & \text{sgn}(u_m) \end{bmatrix}^T, \quad (17)$$

where

$$\text{sgn}(u_i) = \begin{cases} 1, & u_i > 0 \\ 0, & u_i = 0 \\ -1, & u_i < 0 \end{cases}$$

for $i = 1, 2, \ldots, m$. Since time delay $h > 0$ is known, the past state $x(t-h)$ can be used in the control law (see, e.g., Gouaisbaut et al. (2002), Li and Decarlo (2003), Niu et al. (2005) and Oucheriah (2003)). In this case, we design the switching control law as follows

$$u(t) = -(B^T PB)^{-1}\left[\beta s(t) + u_1(t) + u_2(t)\right] \quad (18)$$

for all $t \geq 0$, where $u_1(t) = B^T P(A_0 x(t) + A_1 x(t-h))$ and $u_2(t) = [\alpha + \rho(t)]\text{sgn}(s(t))$ with $\alpha > 0$ and

$$\rho(t) = |B^T PL_0|\,|E_0 x(t)| + |B^T PL_1|\,|E_1 x(t)| + k_\phi\,|B^T PB|\,(|x(t)| + |x(t-h)|) + |B^T P|\,|B_v v(t)|. \quad (19)$$

4. Reachability analysis

In this section, we consider reachability of the sliding surface (15).

Theorem 1. *The state trajectories of system (3) synthesized with switching control (18) are drawn to sliding surface (15) in finite time almost surely, or say, with probability* 1.

Proof. Without loss of generality, assume $|s(0)| > 0$. Define a stopping time

$$\tau_s = \inf\{t \geq 0 : s(t) = 0\}. \quad (20)$$

We need to prove that there exists $0 < t_r < \infty$ such that $\tau_s \leq t_r$ a.s., or say, $\mathbb{P}\{\tau_s \leq t_r\} = 1$.

Let us consider function $U(t) = s^T(t)s(t)$ for all $t \geq 0$. By Itô's formula, we have

$$\mathrm{d}U(t) = \mathcal{L}U(t)\mathrm{d}t + 2s^T(t)B^T Pg(t, x(t))\mathrm{d}W(t), \quad (21)$$

where

$$\begin{aligned} \mathcal{L}U(t) = {} & 2s^T(t)B^T P\big[A_0(t)x(t) + A_1(t)x(t-h) \\ & + B(u(t) + \phi(t, x_t)) + B_v v(t)\big] \\ & + \text{trace}\big[g^T(t, x(t))PBB^T Pg(t, x(t))\big]. \end{aligned} \quad (22)$$

Substitution of (18) into (22) yields

$$\begin{aligned} \mathcal{L}U(t) = {} & 2s^T(t)B^T P\big[\Delta A_0(t)x(t) + \Delta A_1(t)x(t-h) \\ & + B\phi(t, x_t) + B_v v(t)\big] - 2\rho(t)s^T(t)\,\text{sgn}(s(t)) \\ & - 2\alpha s^T(t)\,\text{sgn}(s(t)) - 2\beta s^T(t)s(t) \\ & + \text{trace}\big[g^T(t, x(t))PBB^T Pg(t, x(t))\big] \\ \leq {} & -2\alpha|s(t)| - 2\beta s^T(t)s(t) \\ & + \text{trace}\big[g^T(t, x(t))PBB^T Pg(t, x(t))\big]. \end{aligned} \quad (23)$$

Inequality $|s(t)|_1 \geq |s(t)|$ is used in the last step of inequality (23).

But LMI (12) implies

$$G^T G \leq \zeta_g PBB^T P. \quad (24)$$

Combination of Lemma 2 and inequalities (7), (13) and (24) gives

$$\begin{aligned} & \text{trace}\big[g^T(t, x(t))PBB^T Pg(t, x(t))\big] \\ & \leq \lambda_M(PBB^T P)\text{trace}\big[g^T(t, x(t))g(t, x(t))\big] \\ & \leq \zeta_b \text{trace}\big[g^T(t, x(t))g(t, x(t))\big] \\ & \leq x^T(t)(\zeta_b G^T G)x(t) \\ & \leq x^T(t)(\zeta_b \zeta_g PBB^T P)x(t) \\ & = 2\beta s^T(t)s(t). \end{aligned} \quad (25)$$

Inequalities (23) and (25) imply

$$\mathcal{L}U(t) \leq -2\alpha\sqrt{U(t)}, \quad \forall t \geq 0. \quad (26)$$

But, by Itô's formula, this yields

$$\mathcal{L}|s(t)| = \mathcal{L}\sqrt{U(t)} \leq -\alpha, \quad (27)$$

and hence

$$\mathbb{E}|s(t)| \leq \mathbb{E}|s(0)| - \alpha t, \quad (28)$$

which implies $\mathbb{E}|s(t)|$ converges to zero in finite time. Specifically, there is $t_r = r_0/\alpha$ such that $\mathbb{E}|s(t)| = 0$ for all $t \geq t_r$, where $r_0 = \mathbb{E}|s(0)| < \infty$. This implies $\mathbb{E}|s(t)| = 0$ and hence $|s(t)| = 0$ a.s., or say, $\mathbb{P}\{|s(t)| = 0\} = 1$ for all $t \geq t_r$. For any $\varepsilon_r > 0$, suppose that $\mathbb{P}\{\tau_s > t_r\} \geq \varepsilon_r$. Then $\mathbb{P}\{|s(t_r)| > 0\} \geq \varepsilon_r$, which leads to a contradiction. Therefore we have $\tau_s \leq t_r$ almost surely. The proof is complete. □

Remark 1. It is observed that we may choose $\beta = 0$ when assumption $B^T Pg(t, x(t)) = 0$ (see Chang and Chang (1999), Chang and Wang (1999a,b), Huang and Deng (2008a) and Niu et al. (2005), Niu et al. (2007, 2008)) holds.

Remark 2. In the case when $m = n$, the design method (18) proposes a control scheme for 1st moment finite-time stabilization of stochastic delay system (3) (see Definition 3).

5. Stability of sliding mode

Since it has been shown that the state trajectories of closed-loop system (3) and (18) are drawn to sliding surface (15) in finite time, we proceed to discuss stability of the sliding mode. Let us rewrite system (3) in the following form

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h) + B(u(t) + \bar{\phi}(t, x_t)) \\ & + B_v v(t)\big]\mathrm{d}t + g(t, x(t))\mathrm{d}W(t), \end{aligned} \tag{29}$$

where $\bar{A}_0(t) = \bar{A}_0 + \Delta A_0(t) = (A_0 + BK_0) + \Delta A_0(t)$, $\bar{A}_1(t) = \bar{A}_1 + \Delta A_1(t) = (A_1 + BK_1) + \Delta A_1(t)$, $\bar{\phi}(t, x_t) = \phi(t, x_t) - K_0x(t) - K_1x(t-h)$ and matrices K_i, $i = 0, 1$, are to be determined.

Remark 3. System (3) may also be rewritten in the form of (see, e.g., Li and Decarlo (2003))

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h) + B(\bar{u}(t) + \phi(t, x_t)) \\ & + B_v v(t)\big]\mathrm{d}t + g(t, x(t))\mathrm{d}W(t), \end{aligned} \tag{30}$$

where $\bar{u}(t) = u(t) - BK_0x(t) - BK_1x(t-h)$. But (30) may be somewhat misleading that control law (18) is changed. In fact, control commands are always input as the scheme (18). Note that control scheme (18) is different from that in Li and Decarlo (2003) even in the case when $\beta = 0$ (see Remark 1). At this point, system (29) is clear to show that part of the system dynamics is treated as perturbation (but not counteracted by control input). This may also help highlight the advantage of SMC that sliding mode dynamics is insensitive to matched uncertainties.

Remark 4. It should be stressed that, unlike many cases in references, matrices K_0 and K_1 are not feedback gain matrices. As a matter of fact, there is neither K_0 nor K_1 in control scheme (18). Matrices BK_0 and BK_1 are introduced into the stability analysis of sliding mode because the sliding surface and the Lyapunov–Krasovskii functional are chosen as (15) and (31) respectively, by which we take advantage of their relationship on the sliding surface $s(t) = B^TPx(t) = 0$.

In this section, we consider stability of dynamics of the sliding mode, that is, system (29) restricted on sliding surface (15).

Theorem 2. *Given constant $\gamma > 0$, sliding mode dynamics of system (29) and (4) on sliding surface (15) is robustly mean-square exponentially stable with disturbance attenuation γ provided that LMIs (10) and (11) are satisfied.*

Proof. First, let us consider stability of system (29) with $v(t) = 0$ restricted on sliding manifold (15). Choose a Lyapunov–Krasovskii functional candidate as

$$V(t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(\tau)Qx(\tau)\mathrm{d}\tau \tag{31}$$

for all $t \geq t_0 = t_r + h$, where $P = X^{-1}$ and $Q = PRP$ while matrices $X > 0$ and $R > 0$ are determined by LMIs (10) and (11). By Itô's formula, we have

$$\mathrm{d}V(t) = \mathcal{L}V(t)\mathrm{d}t + 2x^T(t)Pg(t, x(t))\mathrm{d}W(t), \tag{32}$$

where

$$\begin{aligned} \mathcal{L}V(t) = {} & 2x^T(t)P\big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h)\big] \\ & + 2x^T(t)PB\big[u(t) + \bar{\phi}(t, x_t)\big] \\ & + \mathrm{trace}\big[g^T(t, x(t))Pg(t, x(t))\big] \\ & + x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \\ = {} & 2x^T(t)P\big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h)\big] \\ & + \mathrm{trace}\big[g^T(t, x(t))Pg(t, x(t))\big] \\ & + x^T(t)Qx(t) - x^T(t-h)Qx(t-h), \end{aligned} \tag{33}$$

since system (29) is restricted on sliding surface (15). By Lemma 2, we obtain

$$\begin{aligned} \mathcal{L}V(t) \leq {} & 2x^T(t)P\big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h)\big] \\ & + \lambda_M(P)\mathrm{trace}\big[g^T(t, x(t))g(t, x(t))\big] \\ & + x^T(t)Qx(t) - x^T(t-h)Qx(t-h). \end{aligned} \tag{34}$$

Moreover, LMI (11) implies

$$P \leq \lambda_g^{-1}I. \tag{35}$$

Substitution of (7) and (35) into (34) yields

$$\begin{aligned} \mathcal{L}V(t) \leq {} & 2x^T(t)P\big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x(t-h)\big] \\ & + x^T(t)(\lambda_g^{-1}G^TG)x(t) + x^T(t)Qx(t) \\ & - x^T(t-h)Qx(t-h) \\ = {} & x^T(t)\big(P\bar{A}_0 + \bar{A}_0^TP + \lambda_g^{-1}G^TG + Q\big)x(t) \\ & + 2x^T(t)P\bar{A}_1x(t-h) - x^T(t-h)Qx(t-h) \\ & + 2x^T(t)P\Delta A_0(t)x(t) \\ & + 2x^T(t)P\Delta A_1(t)x(t-h). \end{aligned} \tag{36}$$

But, by Lemma 1, we see

$$\begin{aligned} 2x^T(t)P\Delta A_0(t)x(t) \leq {} & x^T(t)\beta_0PL_0L_0^TPx(t) \\ & + x^T(t)\beta_0^{-1}E_0^TE_0x(t), \end{aligned} \tag{37}$$

$$\begin{aligned} 2x^T(t)P\Delta A_1(t)x(t-h) \leq {} & x^T(t)\beta_1PL_1L_1^TPx(t) \\ & + x^T(t-h)\beta_1^{-1}E_1^TE_1x(t-h). \end{aligned} \tag{38}$$

This implies

$$\begin{aligned} \mathcal{L}V(t) \leq {} & x^T(t)\big(P\bar{A}_0 + \bar{A}_0^TP + \lambda_g^{-1}G^TG + Q \\ & + \beta_0PL_0L_0^TP + \beta_0^{-1}E_0^TE_0 + \beta_1PL_1L_1^TP\big)x(t) \\ & + 2x^T(t)P\bar{A}_1x(t-h) \\ & + x^T(t-h)\big(-Q + \beta_1^{-1}E_1^TE_1\big)x(t-h) \\ = {} & \big[x^T(t) \quad x^T(t-h)\big]\,\Omega\,\big[x^T(t) \quad x^T(t-h)\big]^T, \end{aligned} \tag{39}$$

where

$$\Omega = \begin{bmatrix} \Omega_1 & P(A_1 + BK_1) \\ (K_1^TB^T + A_1^T)P & -Q + \beta_1^{-1}E_1^TE_1 \end{bmatrix} \tag{40}$$

with $\Omega_1 = P(A_0 + BK_0) + (A_0 + BK_0)^TP + \lambda_g^{-1}G^TG + Q + \beta_0PL_0L_0^TP + \beta_0^{-1}E_0^TE_0 + \beta_1PL_1L_1^TP$.

Let us look at matrix Γ given as follows

$$\Gamma = \begin{bmatrix} \Gamma_1 & * & * & * & 0 \\ XA_1^T + Y_1^TB^T & -R & 0 & 0 & * \\ GX & 0 & -\lambda_gI & 0 & 0 \\ E_0X & 0 & 0 & -\beta_0I & 0 \\ 0 & E_1X & 0 & 0 & -\beta_1I \end{bmatrix}, \tag{41}$$

where $\Gamma_1 = A_0X + XA_0^T + BY_0 + Y_0^TB^T + R + \beta_0L_0L_0^T + \beta_1L_1L_1^T$. Observe that Γ is a principal submatrix of matrix Θ given in (10). By Schur complement lemma, LMI (10) implies $\Gamma < 0$. But, also by Schur complement lemma, this implies $\tilde{\Omega} < 0$, where $\tilde{\Omega}$ is given as

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_1 & A_1X + BY_1 \\ Y_1^TB^T + XA_1^T & -R + \beta_1^{-1}XE_1^TE_1X \end{bmatrix} \tag{42}$$

with $\tilde{\Omega}_1 = A_0X + XA_0^T + BY_0 + Y_0^TB^TP + \lambda_g^{-1}XG^TGX + R + \beta_0L_0L_0^T + \beta_0^{-1}XE_0^TE_0X + \beta_1L_1L_1^T$. Let $K_i = Y_iP$, $i = 0, 1$, then it is observed $\Omega = D_p\tilde{\Omega}D_p$, where $D_p = \mathrm{diag}\{P, P\}$. This implies $\Omega < 0$ and hence

$$\mathcal{L}V(t) \le -\lambda_0|x(t)|^2, \tag{43}$$

where $\lambda_0 = \lambda_m(-\Omega) > 0$.

According to (31), we have

$$\alpha_0|x(t)|^2 \le V(t) \le \alpha_1|x(t)|^2 + \alpha_2\int_{t-h}^t |x(\tau)|^2\mathrm{d}\tau \tag{44}$$

for all $t \ge t_0$, where $\alpha_0 = \lambda_m(P)$, $\alpha_1 = \lambda_M(P)$ and $\alpha_2 = \lambda_M(Q)$. Choose $\varepsilon_0 > 0$ such that

$$\varepsilon_0(\alpha_1 + \alpha_2he^{h\varepsilon_0}) \le \lambda_0. \tag{45}$$

By Itô's formula, we have

$$\begin{aligned} \mathrm{d}\,[\mathrm{e}^{\varepsilon_0\tau}V(\tau)] &= \mathrm{e}^{\varepsilon_0\tau}\,[\varepsilon_0V(\tau) + \mathcal{L}V(\tau)]\,\mathrm{d}s \\ &\quad + 2\mathrm{e}^{\varepsilon_0\tau}x^T(t)Pg(\tau, x(\tau))\mathrm{d}W(\tau). \end{aligned} \tag{46}$$

Integrating from t_0 to t and taking expectation on both sides of (46) yield

$$\begin{aligned} &\mathbb{E}\left[\mathrm{e}^{\varepsilon_0t}V(t)\right] - \mathbb{E}\left[e^{\varepsilon_0t_0}V(t_0)\right] = \mathbb{E}\int_{t_0}^t \mathrm{e}^{\varepsilon_0\tau}\,[\varepsilon_0V(\tau) + \mathcal{L}V(\tau)]\,\mathrm{d}\tau \\ &\quad\le \int_{t_0}^t \mathrm{e}^{\varepsilon_0\tau}\left\{\mathbb{E}\left[\varepsilon_0\alpha_1|x(\tau)|^2 + \varepsilon_0\alpha_2\int_{\tau-h}^{\tau}|x(v)|^2\mathrm{d}v\right]\right. \\ &\quad\quad \left. -\,\lambda_0\mathbb{E}|x(\tau)|^2\right\}\mathrm{d}\tau. \end{aligned} \tag{47}$$

Since

$$\begin{aligned} &\int_{t_0}^t \mathrm{e}^{\varepsilon_0\tau}\mathrm{d}\tau\int_{\tau-h}^{\tau}|x(v)|^2\mathrm{d}v \le \int_{t_0-h}^t |x(v)|^2\mathrm{d}v\int_v^{v+h}\mathrm{e}^{\varepsilon_0\tau}\mathrm{d}\tau \\ &\quad\le he^{h\varepsilon_0}\int_{t_0-h}^t |x(\tau)|^2\mathrm{e}^{\varepsilon_0\tau}\mathrm{d}\tau \\ &\quad\le he^{h\varepsilon_0}\int_{t_0}^t |x(\tau)|^2\mathrm{e}^{\varepsilon_0\tau}\mathrm{d}\tau + he^{h\varepsilon_0}\int_{t_0-h}^{t_0}|x(\tau)|^2\mathrm{d}\tau, \end{aligned}$$

it follows

$$\begin{aligned} \mathrm{e}^{\varepsilon_0t}\mathbb{E}V(t) &\le \mathrm{e}^{\varepsilon_0t_0}\mathbb{E}V(t_0) \\ &\quad + \int_{t_0}^t \mathrm{e}^{\varepsilon_0\tau}\left[\varepsilon_0(\alpha_1 + \alpha_2he^{h\varepsilon_0}) - \lambda_0\right]\mathbb{E}|x(\tau)|^2\mathrm{d}\tau \\ &\quad + h\varepsilon_0\alpha_2\mathrm{e}^{h\varepsilon_0}\int_{t_0-h}^{t_0}\mathbb{E}|x(\tau)|^2\mathrm{d}\tau \\ &\le C_{t_0}, \end{aligned} \tag{48}$$

where

$$C_{t_0} = [\alpha_1\mathrm{e}^{\varepsilon_0t_0} + \alpha_2h(\mathrm{e}^{\varepsilon_0t_0} + \varepsilon_0he^{h\varepsilon_0})]\sup_{t_r\le\theta\le t_0}\mathbb{E}|x(\theta)|^2.$$

So we have

$$\alpha_0|x(t)|^2 \le \mathbb{E}V(t) \le C_{t_0}\mathrm{e}^{-\varepsilon_0t}, \quad \forall t \ge 0 \tag{49}$$

or

$$\limsup_{t\to\infty}\frac{1}{t}\log\mathbb{E}|x(t;\xi)|^2 \le -\varepsilon_0. \tag{50}$$

The mean-square exponential stability of the sliding mode dynamics has been proved. In fact, by Theorem 6.2, p. 175, Mao (2007) or Theorem 2.2, Mao (1996), (50) also implies almost sure exponential stability. We proceed to show

$$\mathbb{E}\int_{t_0}^{\infty}|z(t)|^2\mathrm{d}t \le \gamma^2\int_{t_0}^{\infty}|v(t)|^2\mathrm{d}t \tag{51}$$

for all nonzero $v \in L_2[t_0, \infty)$ and admissible uncertainties (5) under zero initial condition $x(\theta) = 0$ for all $\theta \in [t_0 - h, t_0]$.

For prescribed constant $\gamma > 0$, define the performance index function

$$J(t) = \int_{t_0}^t [z^T(\tau)z(\tau) - \gamma^2v^T(\tau)v(\tau)]\mathrm{d}\tau \tag{52}$$

for all $t > t_0$. Let

$$Y(t) = J(t) + V(t), \qquad \bar{J}(t) = \mathbb{E}J(t), \qquad \bar{Y}(t) = \mathbb{E}Y(t). \tag{53}$$

Obviously, $Y(t) \ge J(t)$ and $\bar{Y}(t) \ge \bar{J}(t)$ for all $t > t_0$. Since $x(\theta) = 0$ for all $\theta \in [t_0 - h, t_0]$, by Dynkin's formula (see, e.g., Corollary 6.3.2, p. 142, Klebaner (1998)), we have

$$\mathbb{E}V(t) = \mathbb{E}\int_{t_0}^t \mathcal{L}V(\tau)\mathrm{d}\tau, \quad \forall t > t_0 \tag{54}$$

and therefore

$$\begin{aligned} \bar{Y}(t) &= \mathbb{E}\int_{t_0}^t [z^T(\tau)z(\tau) - \gamma^2v^T(\tau)v(\tau) + \mathcal{L}V(\tau)]\mathrm{d}\tau \\ &= \mathbb{E}\int_{t_0}^t \left[x^T(\tau) \quad x^T(t-h) \quad v^T(\tau)\right]\Omega_v \\ &\quad \times \left[x^T(\tau) \quad x^T(t-h) \quad v^T(\tau)\right]^T\mathrm{d}\tau, \end{aligned} \tag{55}$$

where

$$\Omega_v = \begin{bmatrix} \Omega_{v1} & P(A_1 + BK_1) & PB_v + C^TD \\ (K_1^TB^T + A_1^T)P & -Q + \beta_1^{-1}E_1^TE_1 & 0 \\ B_v^TP + D^TC & 0 & -\gamma^2I + D^TD \end{bmatrix}$$

with $\Omega_{v1} = P(A_0+BK_0)+(A_0+BK_0)^TP+\lambda_g^{-1}G^TG+Q+\beta_0PL_0L_0^TP+\beta_0^{-1}E_0^TE_0 + \beta_1PL_1L_1^TP + C^TC$.

Using similar techniques as above, we find

$$\Theta < 0 \Rightarrow \Omega_v < 0. \tag{56}$$

But this implies

$$\bar{J}(t) \le \bar{Y}(t) \le -\lambda_v\int_{t_0}^t |v(\tau)|^2\mathrm{d}\tau \quad \forall t \ge t_0 \tag{57}$$

with $\lambda_v = \lambda_m(-\Omega_v) > 0$, which completes the proof. □

6. Example

Let us consider a water-quality dynamic model subject to environmental noise (see Example 4.2, p. 157, Mahmoud (2000))

$$\begin{aligned} \mathrm{d}x(t) &= \left[A_0(t)x(t) + A_1(t)x(t-h) + Bu(t) + B_vv(t)\right]\mathrm{d}t \\ &\quad + g(t, x(t))\mathrm{d}W(t), \end{aligned} \tag{58}$$

$$z(t) = Cx(t) \tag{59}$$

with

$$A_0 = \begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix}, \quad L_0 = \begin{bmatrix} 0.6 & -0.2 \\ 0 & 0.8 \end{bmatrix}, \quad E_0 = \begin{bmatrix} 0 & 0.4 \\ 0.2 & 0.2 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & -0.1 \\ 0.5 & 1 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0.4 \\ 0.2 & 0.2 \end{bmatrix}, \quad F_0(t) = \begin{bmatrix} 0.7\sin(t) & 0 \\ 0 & 0.3\sin(3t) \end{bmatrix},$$

$$F_1(t) = \begin{bmatrix} 0.5\sin(2t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4\sin(t) \end{bmatrix}^T,$$

and

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad B_v = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$G = \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Obviously, in this case, we have $m = n = 2$ (see Remark 2). For $\gamma = 1$, solving inequalities (10)–(14) yields

$$P = \begin{bmatrix} 2.2138 & 0.0002 \\ 0.0002 & 0.3918 \end{bmatrix}, \quad \beta = 15.5091.$$

By Theorem 1, closed-loop system (58) and (18) designed with the parameters above converges to sliding surface

$$s(t) = B^T P x(t) = \begin{bmatrix} 2.2138 & 0.0002 \\ 0.0001 & 0.1959 \end{bmatrix} x(t) = 0 \tag{60}$$

in finite time. Since $\text{rank}(B^T P) = \text{rank}(B) = m = n = 2$, $s(t) = 0 \Rightarrow x(t) = 0$. That is, system (58) and (18) arrives at the equilibrium in finite time and stays there afterwards. By Definition 3 and Remark 2, we see that our method provides a control scheme for 1st moment finite-time stabilization of system (58). This control strategy is more desired than the control method in Mahmoud (2000) in a case when the states of the water quality are required to reach the equilibrium in finite time and stay at the point.

In the following, let us consider system (58) and (59) with single input, to which SMC strategy may be applied. To illustrate the effectiveness of the result proposed in this paper, we consider the case of

$$B = \begin{bmatrix} 2 & 2 \end{bmatrix}^T, \tag{61}$$

where it is easy to see that the assumption (2) in the existing results (see Chang and Chang (1999), Chang and Wang (1999a,b), Huang and Deng (2008a) and Niu et al. (2005, 2007, 2008)) is not satisfied and hence those results cannot be applied in this case. For prescribed $\gamma = 1$, inequalities (10)–(14) give

$$P = \begin{bmatrix} 0.0729 & -0.0726 \\ -0.0726 & 0.0726 \end{bmatrix}, \quad \beta = 31.6466.$$

Hence, sliding surface (15) and control scheme (18) can be designed with these given parameters. In this case, the state trajectories of system (58) and (59) converge to sliding manifold

$$s(t) = B^T P x(t) = 10^{-3} \times \begin{bmatrix} 0.4970 & 0 \end{bmatrix} x(t) = 0 \tag{62}$$

in finite time, on which the sliding mode is robustly mean-square exponentially stable with disturbance attenuation $\gamma = 1$. The curves given in Figs. 1–5 are the result of a simulation with diffusion $g(t, x(t)) = Gx(t)$, design parameter $\alpha = 10^{-4}$, initial condition $x(\theta) = [10\ 10]^T, \theta \in [-h, 0]$, and time delay $h = 10^3\text{Dt}$, where step size $\text{dt} = 0.5 \times 10^{-4}$ and $\text{Dt} = R\text{dt} = 10^{-4}$ (see Higham (2001)). The curve of mean square of 1000 samples are given in Figs. 6 and 7.

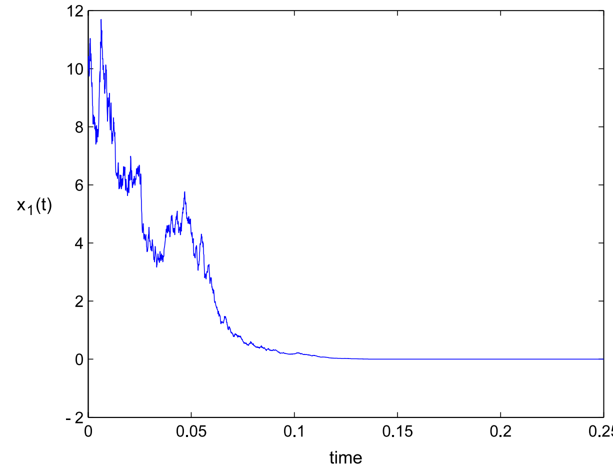

Fig. 1. The curve of $x_1(t)$.

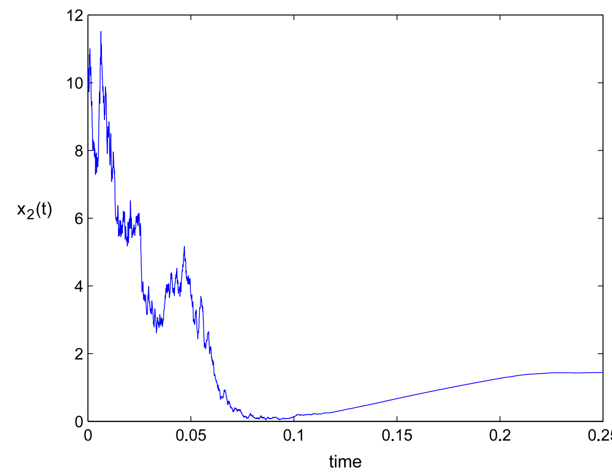

Fig. 2. The curve of $x_2(t)$ before entering the sliding mode.

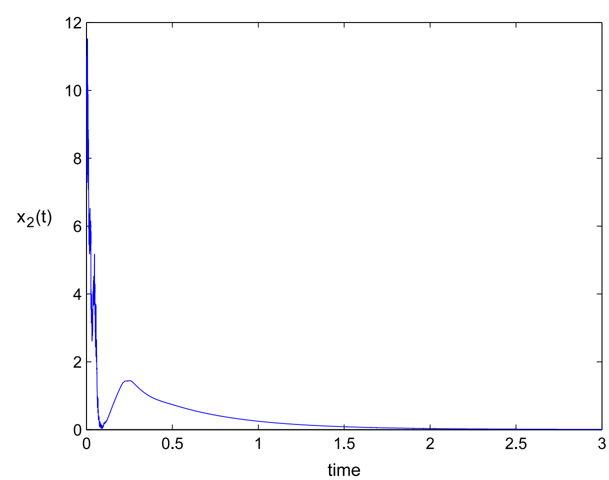

Fig. 3. The curve of $x_2(t)$.

7. Conclusion

This paper proposes a SMC design for robust H_∞ control for uncertain stochastic delay systems. The proposed method removes a restriction in the existing results. The idea in this paper may also be applied in an alternative way to linear stochastic delay systems with $m < n$. Since pair (A_0, B) is controllable and matrix B is of full column rank, system (3) can be transformed to a variant

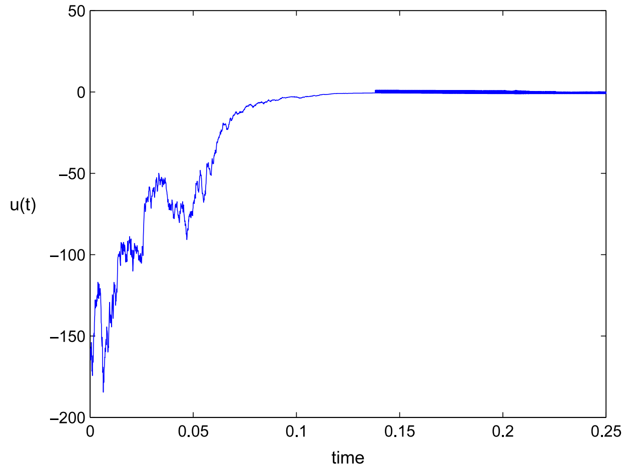

Fig. 4. The curve of $u(t)$ before entering the sliding mode.

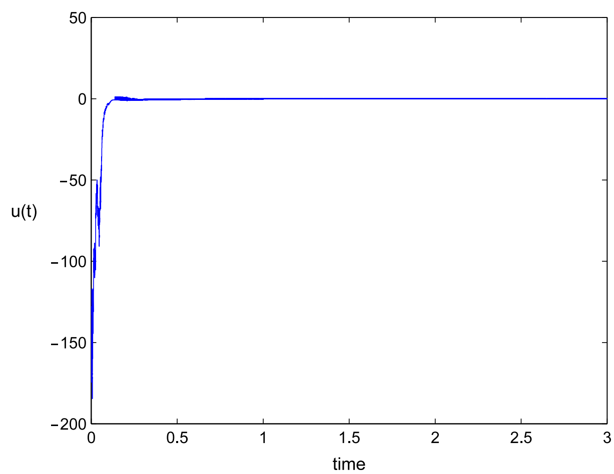

Fig. 5. The curve of $u(t)$.

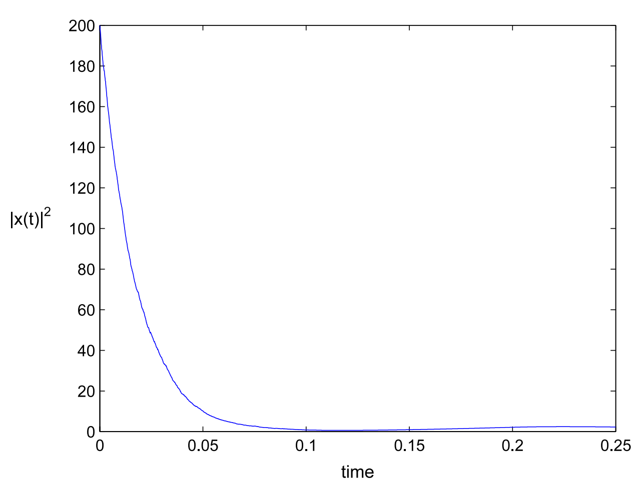

Fig. 6. The curve of mean square of 1000 paths before entering the sliding mode.

of canonical controller-type form (see Remark 4 Huang and Deng (2008a)). This may be considered as a decomposition into two interconnected subsystems, one of which, denoted by subsystem $y_2(t) \in R^m$, includes control input $u(t)$ and the other denoted by subsystem $y_1(t) \in R^{n-m}$ is free of input. If the sliding mode is chosen as $s(t) = Sy_2(t) = 0$, where S is a nonsingular matrix, then the condition for reachability of sliding mode can be figured out from the subsystem $y_2(t)$ while the condition for stability of

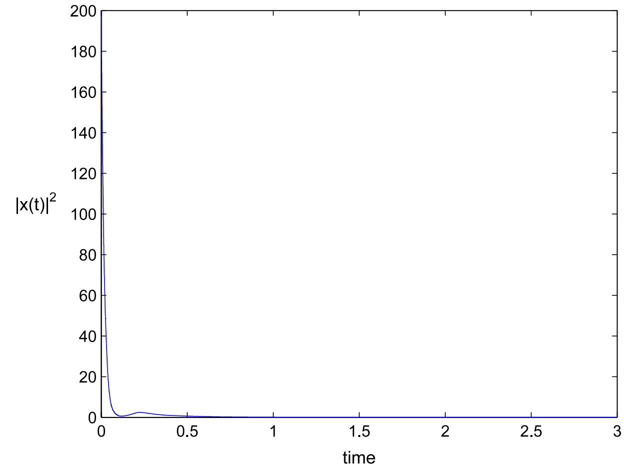

Fig. 7. The curve of mean square of 1000 paths.

sliding mode is indeed that for stability of subsystem $y_1(t)$ with $y_2(t) = 0$.

It is noted that the case when state delay appears in diffusion is not considered in this paper. In that case, (23) involves a positive definite function with respect to the delay states such that control law (18) may not guarantee that the state trajectories will be drawn onto sliding surface (15) in finite time. This is one of the problems of SMC for stochastic delay systems that are to be studied.

Acknowledgments

The authors gratefully acknowledge the reviewers' comments and suggestions. The authors would like to thank the financial support from UK ORSAS and University of Strathclyde.

References

Bhat, S. P., & Bernstein, D. S. (2000). Stabilization of uncertain nonholonomic systems via time-varying sliding mode control. *SIAM J. Control and Optimization*, *38*, 751–766.
Berman, N., & Shaked, U. (2006). H_∞-like control for nonlinear stochastic systems. *Systems & Control Letters*, *55*, 247–257.
Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in systems and control*. Philadelphia: SIAM.
Chang, K.-Y., & Chang, W.-J. (1999). Variable structure controller design with H_∞ norm and variance constraints for stochastic model reference systems. *IEE Proc.-Control Theory and Applications*, *146*, 511–516.
Chang, K.-Y., & Wang, W.-J. (1999). H_∞ norm constraint and variance control for stochastic uncertain large-scale systems via the sliding mode concept. *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, *46*, 1275–1280.
Chang, K.-Y., & Wang, W.-J. (1999). Robust covariance control for perturbed stochastic multivariable system via variable structure control. *Systems & Control Letters*, *37*, 323–328.
Chen, W.-H., Guan, Z.-H., & Lu, X. (2005). Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: An LMI approach. *Systems & Control Letters*, *54*, 547–555.
Choi, H. H. (2007). LMI-based sliding surface design for integral sliding mode control of mismatched uncertain systems. *IEEE Transactions on Automatic Control*, *52*, 736–742.
Edwards, C., Akoachere, A., & Spurgeon, S. K. (2001). Sliding-mode output feedback controller design using linear matrix inequalities. *IEEE Transactions on Automatic Control*, *46*, 115–119.
Feng, Z., & Liu, Y. (1995). *Stability analysis and stabilization synthesis of stochastic large scale systems*. Beijing: Science Press.
Gouaisbaut, F., Dambrine, M., & Richard, J. R. (2002). Robust control of delay systems: A sliding mode control design via LMI. *Systems & Control Letters*, *46*, 219–230.
Higham, D. (2001). An algorithmic introduction to numerical simulation of stochastic differential equations. *SIAM Review*, *43*, 525–546.
Hong, Y., Wang, J., & Cheng, D. (2006). Adaptive finite-time control of nonlinear systems with parametric uncertainty. *IEEE Transactions on Automatic Control*, *51*, 858–862.
Hu, Q., Ma, G., & Xie, L. (2008). Robust and adaptive variable structure output feedback control of uncertain systems with input nonlinearity. *Automatica*, *44*, 552–559.

Hu, Y., Ge, S. S., & Su, C.-Y. (2004). Stabilization of uncertain nonholonomic systems via time-varying sliding mode control. *IEEE Transactions on Automatic Control*, *49*, 757–763.

Huang, X., Lin, W., & Yang, B. (2005). Global finite-time stabilization of a class of uncertain nonlinear systems. *Automatica*, *41*, 881–888.

Huang, L., & Deng, F. (2008a). Exponential stabilization of stochastic delay systems using sliding mode controllers. *Dynamics of Continuous, Discreet, Impulsive Systems-Series B*, *15*, 873–890.

Huang, L., & Deng, F. (2008b). Razumikhin-type theorems on stability of neutral stochastic functional differential equations. *IEEE Transactions on Automatic Control*, *53*, 1718–1723.

Jafarov, E. M. (2005). Robust sliding mode controllers design techniques for stabilization of multivariable time-delay systems with parameter perturbations and external disturbances. *International Journal of Systems Science*, *36*, 433–444.

Klebaner, F. C. (1998). *Introduction to stochastic calculus with applications*. London: Imperial College Press.

Kolmanovskii, V. B., & Myshkis, A. (1999). *Introduction to the theory and applications of functional differential equations*. Dordrecht: Kluwer Academic Publishers.

Li, X., & Decarlo, R. A. (2003). Robust sliding mode control of uncertain time delay systems. *International Journal of Control*, *76*, 1296–1305.

Lütkepohl, H. (1996). *Handbook of matrices*. Chichester: John Wiley & Sons.

Mahmoud, M. S. (2000). *Robust control and filtering for time-delay systems*. New York: Marcel Dekker, Inc.

Mao, X. (1996). Razumikhin-type theorems on exponential stability of stochastic functional differential equations. *Stochastic Processes and their Applications*, *65*, 233–250.

Mao, X. (2002). A note on the LaSalle-type theorems for stochastic differential delay equations. *Journal of Mathematical Analysis and Applications*, *268*, 125–142.

Mao, X. (2007). *Stochastic differential equations and applications* (2nd ed.). Chichester: Horwood Publishing.

Niu, Y., Ho, D. W. C., & Lam, J. (2005). Robust sliding mode control for uncertain stochastic systems with time-varying delay. *Automatica*, *41*, 837–880.

Niu, Y., Ho, D. W. C., & Wang, X. (2007). Sliding mode control for Itô stochastic systems with Markovian switching. *Automatica*, *43*, 1784–1790.

Niu, Y., Ho, D. W. C., & Wang, X. (2008). Robust H_∞ control for nonlinear stochastic systems: A sliding-mode approach. *IEEE Transactions on Automatic Control*, *53*, 1695–1701.

Oucheriah, S. (2003). Exponential stabilization of linear delayed systems using sliding-mode controllers. *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, *50*, 826–830.

Utkin, V. I. (1992). *Sliding modes in control and optimization*. Berlin: Springer-Verlag.

Utkin, V., Guldner, J., & Shi, J. (1999). *Sliding mode control in electromechanical systems*. London: Taylor & Francis.

Xu, B. (1997). Stability robustness bounds for linear systems with multiple time-varying delayed perturbations. *International Journal of Systems Science*, *28*, 1311–1317.

Xu, S., & Chen, T. (2002). Robust H_∞ control for uncertain stochastic systems with state delay. *IEEE Transactions on Automatic Control*, *47*, 2089–2094.

Yang, Y., Li, J., & Chen, G. (2009). Finite-time stability and stabilization of nonlinear stochastic hybrid systems. *Journal of Mathematical Analysis and Applications*, *356*, 338–345.

Yue, D., & Han, Q.-L. (2005). Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching. *IEEE Transactions on Automatic Control*, *50*, 217–222.

Lirong Huang received the B.E. and M.E. degrees from South China University of Technology, Guangzhou and MSc degree from King's College London. He is currently working toward a Ph.D. degree at the University of Strathclyde, Glasgow, UK.

His research is concerned with stability and stabilization of stochastic delay systems.

Xuerong Mao received the Ph.D. degree from Warwick University, England in 1989 and was then SERC (Science and Engineering Research Council, UK) Post-Doctoral Research Fellow 1989–92. Moving to Scotland, he joined the Department of Statistics and Modelling Science, University of Strathclyde, Glasgow as a lecturer in 1992, was promoted to Reader in 1995, and was made Professor in 1998 which post he still holds. He has authored 5 books and over 180 research papers. His main research interests lie in the field of stochastic analysis including stochastic stability, stabilization, control, numerical solutions. He is a Fellow of the Royal Society of Edinburgh (FRSE). He is also a member of the editorial boards of several international journals including Journal of Stochastic Analysis and Applications and Proceedings of the Royal Society of Edinburgh, Section A: Mathematics.