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Alleviation of Unbalanced Rotor Loads by Single Blade Controllers

W.E. Leithead
University of Strathclyde
w.leithead@eee.strath.ac.uk
Glasgow, G1 1QE , UK

V. Neilson
University of Strathclyde
victoria.neilson@eee.strath.ac.uk
Glasgow, G1 1QE, UK

S. Dominguez
MLS
sdominguez@mls-control.com
Madrid 28037, Spain

Abstract:

A novel approach to reducing the unbalance rotor loads by pitch control is presented in this paper. Each blade has its own actuator, sensors and controller. These localised blade control systems operate in isolation without need of communication with each other. This single blade control approach to regulation of unbalanced rotor loads has several advantages including being straightforward to design and easy to tune. Furthermore, it does not affect the operation of the central controller and the latter need not be re-designed when used in conjunction with the single blade controllers. Their performance is assessed using BLADED simulations.

Keywords— Control, individual pitch, unbalanced rotor loads

1. Introduction

Largely driven by concerns over the environment, wind energy has developed rapidly in recent years. Wind turbine size has increased with the machines becoming more flexible and dynamic. This has lead to greater demands being placed on the control system including the reduction of structural loads. The focus of current developments is on the alleviation of asymmetric loads on the rotor.

As a wind turbine blade sweeps through the wind-field, it experiences loads caused by the rotational sampling of the wind-field. These $n\Omega_0$ loads are concentrated at integer multiples (*n*) of the rotor speed (Ω_0) and consist of both deterministic and stochastic components. The stochastic component largely arises from the turbulence of the wind. The deterministic loads largely arise from wind shear, tower shadow and blade imbalance [1].

Since the wind-field is continuously changing in time and over the swept area, each blade of the rotor experiences slightly different loads resulting in load imbalance across the rotor. These imbalances not only impact on the rotor but on the rest of the wind turbine structure and the drive-train. The most significant components of these unbalanced load are typically those at $1\Omega_0$ and $2\Omega_0$.

Individual blade pitch control has demonstrated great potential to alleviate rotor loads in above rated wind speed operation [2, 3, 4]. While they may differ in structure or implementation details, all aim to reduce the asymmetric loads by varying the pitch angle of each blade individually in response to some suitable measurement such as blade bending moments. Improvements in sensor technology are now making these individual pitch control algorithm a practical possibility [2]. In Bell *et al* [2] significant reductions in fatigue equivalent loads on the blade, the main shaft and the yaw bearing are reported.

In previously reported approaches to individual pitch control, individual pitch control is realised through the wind turbine central controller. The loads on each blade are measured, communicated to some controller which then determines the pitch angle demand for each blade using all the measurements. The direct-quadrature (d-q) axis transformation is central to this procedure. In this paper a novel approach to reducing the unbalanced rotor loads by individual pitch control is presented. Each blade has its own pitch control system operating in isolation from the wind turbine central controller. The objective for this SISO feedback loop is chosen so that only the contribution to rotor imbalance is regulated. An incremental adjustment to the pitch demand from the collective pitch demand from the central controller is determined for the blade using only the measurement of the load on that blade. The instrumentation required for each blade is bending moment sensors, typically optical fibre sensors, and linear and angular acceleration sensors to determine the tower motion.

The paper is structured as follows. In section 2, individual pitch control design based on the d-q axis transformation is discussed. In Section 3, the dynamic model for a single blade in an inertial reference frame is derived. The controller for the blade is designed using this simple model. However, all the blades of the rotor couple to the dynamics of the whole wind turbine. The required modification to the dynamics of the blade can be expressed in terms of fictitious forces dependent on measured accelerations only. The derivation of the fictitious forces is discussed in Section 4 and the dynamic model of the blade in a non-inertial frame moving with the wind turbine described in Section 5. The complete scheme based on single blade control is discussed in Section 6. and conclusions drawn in Section 7.

2. d-q Axis Transformation

The d-q transformation has its origins in three-phase electrical machine theory [3]. It is the co-ordinate system transformation from the three-vector, $[X_a \quad X_b \quad X_c]^T$, to the two-vector, $[X_d \quad X_q]^T$, such that

$$\begin{bmatrix} X_d \\ X_q \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{4\pi}{3}) \\ \sin(\theta) & \sin(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{4\pi}{3}) \end{bmatrix} \cdot \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix}$$

The inverse transformation is

$$\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \cos(\theta + \frac{4\pi}{3}) & \sin(\theta + \frac{4\pi}{3}) \end{bmatrix} \cdot \begin{bmatrix} X_d \\ X_q \end{bmatrix}$$

The application of this technique on a three bladed turbine is shown in Figure 1.

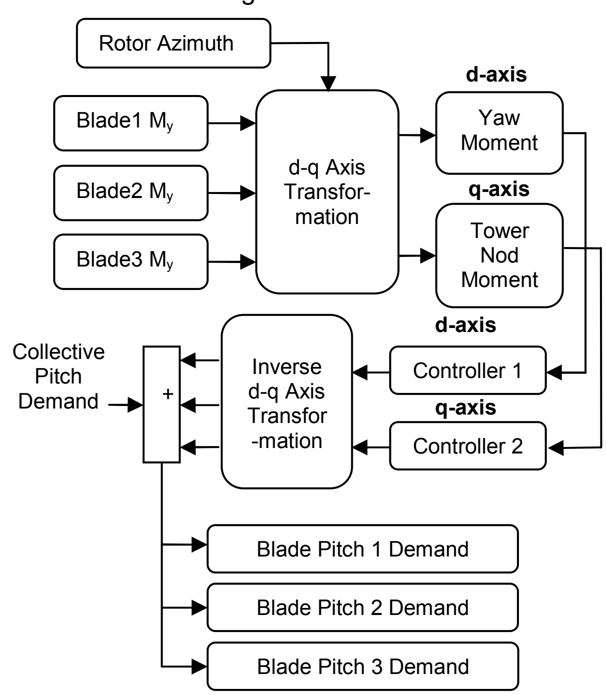

Figure 1: Individual Pitch Control Loop using d-q Axis Transformation [2]

The angle, θ, parameterising the transformation is the azimuth angle. The dynamics between the three blade bending moments, *blade 1* M_y, *blade 2* M_y and *blade 3* M_y, and the three pitch demands, *Blade Pitch 1 Demand*, *Blade Pitch 2 Demand*, *Blade Pitch 3 Demand*, consists of the complete wind turbine dynamics. The transformation achieves a degree of separation of design of the two controllers, *Controller 1* and *Controller 2*.

3. Single Blade Model

Consider the situation depicted in Figure 2. The root of a turbine blade is fixed at the origin of the stationary axes such that the blade initially lies along the *x*-axis in the *x-y* plane. The blade is pitched by angle, α, about the *x*-axis. The in-plane and out-plane angles of deflection of the blade are θ_R and ϕ_R as shown.

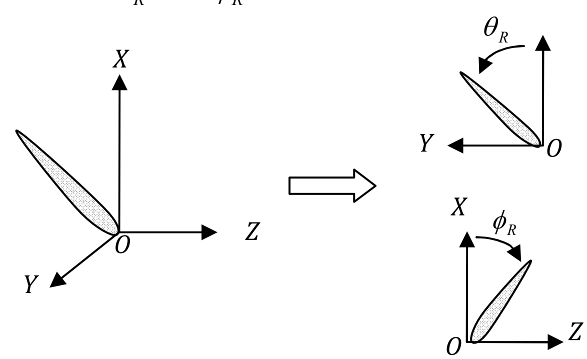

Figure 2: Single Blade Motion

The Lagrangian for the blade, ignoring external forces, is

$$L = T - V = \frac{1}{2}J\dot{\theta}_R^2 + \frac{1}{2}J\dot{\phi}_R^2 \\ -[\tfrac{1}{2}k_E(\theta_R c_\alpha - \phi_R s_\alpha)^2 + \tfrac{1}{2}k_F(\theta_R s_\alpha + \phi_R c_\alpha)^2]$$

where $k_E = J\omega_E^2$, $k_F = J\omega_F^2$, $s_\alpha = \sin(\alpha)$, $c_\alpha = \cos(\alpha)$, J is blade inertia and ω_E and ω_F are blade and flap frequencies, respectively. It follows that the equations of motion for the blade are

$$\begin{bmatrix} \ddot{\theta}_R \\ \ddot{\phi}_R \end{bmatrix} = -\begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -(\omega_E^2 - \omega_F^2)s_\alpha c_\alpha \\ -(\omega_E^2 - \omega_F^2)s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix} \\ + \frac{1}{J}\begin{bmatrix} M_{A\theta_R} \\ M_{A\phi_R} \end{bmatrix}$$

where $M_{A\theta_R}$ and $M_{A\phi_R}$ are the in-plane and out-of-plane aerodynamics moments. The in-plane and out-of-plane blade root bending moments, $M_{I/P}$ and $M_{O/P}$, are

$$\begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix} = J \cdot \begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -(\omega_E^2 - \omega_F^2) s_\alpha c_\alpha \\ -(\omega_E^2 - \omega_F^2) s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix}$$

However, the axes shown in Figure 2 are not stationary. The axes rotate with the hub and origin moves with the tower. Hence, the axes set depicted in Figure 2 are non-inertial. The modification to the dynamics required to account for the axes being non-inertial are the fictitious forces arising from the acceleration of the axes relative to some inertial reference frame such as earth axes.

4 Fictitious Forces

The linear and rotational acceleration of the tower are considered in separately. Four sets of coordinate systems are used to represent the dynamics of the single turbine blade. They are defined in Figure 3.

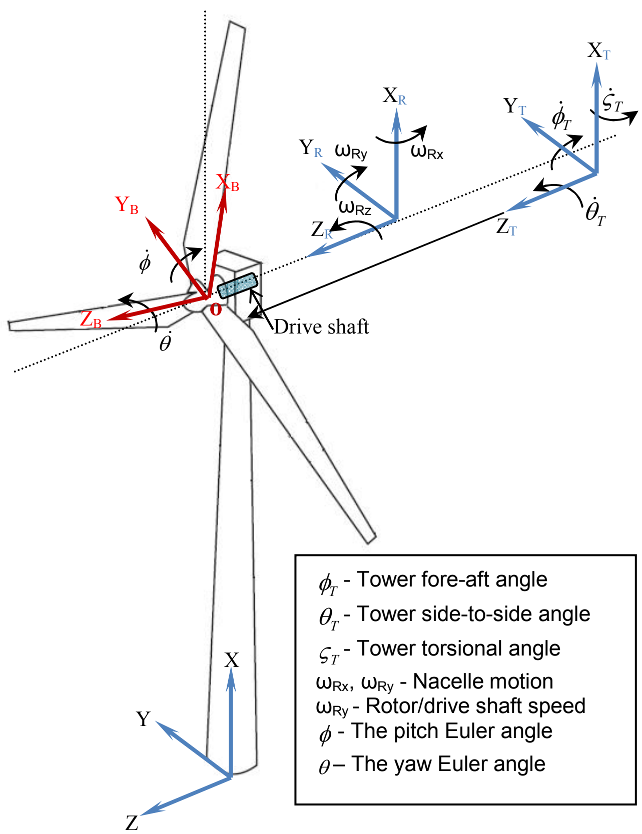

Figure 3: Axes sets

The axes sets are defined as follows.

1. Earth frame (X-Y-Z)
2. Tower frame (X_T-Y_T-Z_T): Parallel to the earth frame but with the origin coinciding with the hub and moving linearly with the top of the tower.
3. Rotor frame (X_R-Y_R-Z_R): Rotates with the drive shaft and its axis Z_B is aligned with the drive-shaft.
4. Blade body centred frame (X_B-Y_B-Z_B): The axis X_B is aligned with the blade and the origin coincides with the hub centre.

The rotations for all the axes sets are defined using Euler angles. The notations used in this Section are defined in Table 1.

Symbol	Description
$\tilde{\Omega}$	Angular velocity components of the body centred frame relative to the earth frame represented in the body centred frame.
$\tilde{\Omega}_R$	Angular velocity components of the rotor frame relative to the earth frame represented in the rotor frame
$\tilde{\Omega}'$	Angular velocity components of the rotor frame relative to the earth frame represented in the body centred frame
$\tilde{\omega}_T$	Angular velocity components of the tower frame relative to the earth frame represented in the tower frame
$\tilde{\omega}_R$	Angular velocity components of the rotor frame relative to the tower frame represented in the rotor frame
$\tilde{\omega}_B$	Angular velocity components of the body centred frame relative to the rotor frame represented in the body centred frame
$\tilde{r}_R$	Displacement from the origin of the blade centre of mass in the tower frame
$\ddot{\tilde{r}}_R$	Linear acceleration of the blade centre of mass in the tower frame
$\ddot{\tilde{r}}_{Ro}$	Linear acceleration of the tower frame in the earth frame
R_1	Euler angle (3-2-1) rotation from the tower frame to the rotor frame
R_2	Euler angle (3-2-1) rotation from the tower frame to the rotor frame. The yaw and pitch Euler angles are θ and ϕ
R_3	Euler angle (3-2-1) rotation from the rotor frame to the blade body centred frame. The yaw and pitch Euler angles are θ and ϕ

Table 1: Key Notations

4.1 Linear Fictitious Forces

The linear motion of the blade centre of mass has only two degrees of freedom due to the blade root being attached to the hub, see Figure 4. Hence, $\widetilde{F}_{Total}$, the total force exerted on the blade centre of mass in the rotor frame, is

$$\widetilde{F}_{Total} = \widetilde{F}_B + \widetilde{F}_T + 2\lambda \widetilde{r}_R$$

where $\widetilde{F}_B$ is the force on the blade with respect to the inertial reference frame instantaneously coinciding with the rotor frame, $\widetilde{F}_T$ is the fictitious force associated with the linear acceleration of the rotor frame and $2\lambda\widetilde{r}_R$ is the modification required to meet the constraint on the linear motion with λ a suitable Langrange multiplier. The linear acceleration of the rotor frame is due to the tower motion.

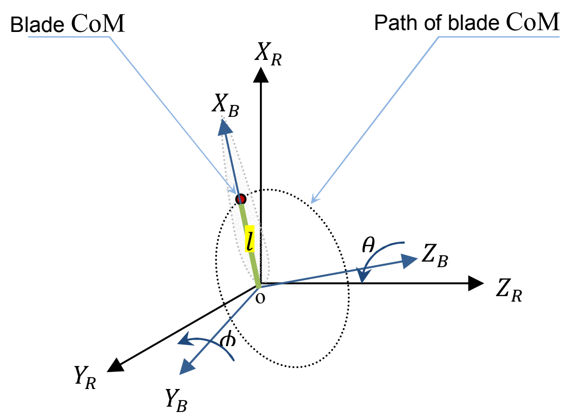

Figure 4: Motion of Blade's Centre of Mass

The fictitious force on the blade arising from the tower motion is determined by

$$\widetilde{F}_T + 2\lambda\widetilde{r}_R = \widetilde{F}_{Total} - \widetilde{F}_B = m_B(\ddot{\widetilde{r}}_{Ro} + \ddot{\widetilde{r}}_R) - m_B\ddot{\widetilde{r}}_R = m_B\ddot{\widetilde{r}}_{Ro}$$

where m_B is the blade mass. The torque, $\widetilde{M}_T$, in earth axes, corresponding to the fictitious force, is

$$\widetilde{M}_T = \widetilde{r}_R \times \widetilde{F}_T = \widetilde{r}_R \times \widetilde{F}_T + 2\lambda\widetilde{r}_R \times \widetilde{r}_R = m_B \widetilde{r}_R \times \ddot{\widetilde{r}}_{Ro}$$

which becomes in body centred axes

$$\widetilde{M}_{T_R} = R_2\widetilde{M}_T = R_2(m_b\widetilde{r}_R \times \ddot{\widetilde{r}}_{Ro})$$
$$= m_B l \begin{bmatrix} 0 \\ -c_\theta s_\phi \ddot{x}_{Ro} - s_\theta s_\phi \ddot{y}_{Ro} - c_\phi \ddot{z}_{Ro} \\ c_\theta \ddot{y}_{Ro} - s_\theta \ddot{x}_{Ro} \end{bmatrix}$$

with $\widetilde{r}_{Ro} = [x_{Ro} \quad y_{Ro} \quad z_{Ro}]^T$ and

$$\widetilde{r}_R = R_2^{-1}\begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_\theta c_\phi & -s_\theta & c_\theta s_\phi \\ s_\theta c_\phi & c_\theta & s_\theta s_\phi \\ -s_\phi & 0 & c_\phi \end{bmatrix}\begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix}$$

where l is the distance between the blade's centre of mass and the centre of rotation of the rotor.

These generalised forces have a simple interpretation. The linear acceleration of the tower frame in the body centred frame is

$$R_2 \cdot \ddot{\widetilde{r}}_{Ro} = \begin{bmatrix} a_{B1} \\ a_{B2} \\ a_{B3} \end{bmatrix} = \begin{bmatrix} c_\phi & 0 & -s_\phi \\ 0 & 1 & 0 \\ s_\phi & 0 & c_\phi \end{bmatrix} \cdot \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_{Ro} \\ \ddot{y}_{Ro} \\ \ddot{z}_{Ro} \end{bmatrix}$$
$$= \begin{bmatrix} c_\phi c_\theta \ddot{x}_{Ro} + c_\phi s_\theta \ddot{y}_{Ro} - s_\phi \ddot{z}_{Ro} \\ c_\theta \ddot{y}_{Ro} - s_\theta \ddot{x}_{Ro} \\ s_\phi c_\theta \ddot{x}_{Ro} + s_\phi s_\theta \ddot{y}_{Ro} + c_\phi \ddot{z}_{Ro} \end{bmatrix}$$

Hence,

$$\widetilde{M}_{T_R} = m_b l \cdot [0 \quad -a_{B3} \quad a_{B2}]^T$$

where a_{B2} is the acceleration of the centre of rotation of the blade perpendicular to the blade in the plane of rotation and a_{B3} is the acceleration of the centre of rotation of the blade perpendicular to the plane of rotation. It is straightforward to measure both a_{B2} and a_{B3} using appropriately aligned accelerometers. Note that a_{B2} also picks up gravity effects of the blade, hence gravity doesn't appear as part of the potential energy term in the Lagrangian of the single blade.

4.2 Rotational Fictitious Forces

The total torque $\widetilde{M}_{Total}$ on a rotating blade in the blade body centred frame is

$$\widetilde{M}_{Total} = \widetilde{M}_T + \widetilde{M}_B$$

where $\widetilde{M}_B$ is the torque on the blade with respect to the inertial reference frame instantaneously coinciding with the rotor frame, $\widetilde{M}_T$ is the fictitious rotational force associated with the angular acceleration of the rotor frame. The angular acceleration of the rotor frame is due to the tower motion.

The fictitious force on the blade arising from the tower motion is determined by

$$\tilde{M}_T = \tilde{M}_{Total} - \tilde{M}_B$$
$$= I_B\dot{\tilde{\Omega}} + \tilde{\Omega}\times\left(I_B\tilde{\Omega}\right) - \left[I_B\dot{\tilde{\omega}}_B + \tilde{\omega}_B \times \left(I_B\tilde{\omega}_B\right)\right]$$

where I_B is the inertia matrix of the blade in the blade body frame. Since

$$I_B \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J \end{bmatrix} = \begin{bmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Given that the rotational velocity components of the body centred frame can be expressed through the Euler's angles:

$$\tilde{\Omega} = R_3 \cdot \tilde{\Omega}_R + \tilde{\omega}_B$$
$$= \begin{bmatrix} c_\theta c_\phi & -s_\theta & c_\theta s_\phi \\ s_\theta c_\phi & c_\theta & s_\theta s_\phi \\ -s_\phi & 0 & c_\phi \end{bmatrix} \cdot \begin{bmatrix} \Omega_{xR} \\ \Omega_{yR} \\ \Omega_{zR} \end{bmatrix} + \begin{bmatrix} -\dot{\theta}s_\phi \\ \dot{\phi} \\ \dot{\theta}c_\phi \end{bmatrix} = \begin{bmatrix} \Omega_1' + \omega_1 \\ \Omega_2' + \omega_2 \\ \Omega_3' + \omega_3 \end{bmatrix}$$

$\tilde{M}_T$ can thus be expressed as:

$$\tilde{M}_T = \tilde{M}_{Total} - \tilde{M}_B$$
$$= I_B\dot{\tilde{\Omega}} + \tilde{\Omega}\times\left(I_B\tilde{\Omega}\right) - \left[I_B\dot{\tilde{\omega}}_B + \tilde{\omega}_B \times \left(I_B\tilde{\omega}_B\right)\right]$$
$$= I_B\left(\dot{R}_2\tilde{\Omega}_R + R_2\dot{\tilde{\Omega}}_R\right) - \begin{bmatrix} 0 \\ J\left(\Omega_1'\Omega_3' + \omega_1\Omega_3' + \Omega_1'\omega_3\right) \\ -J\left(\Omega_1'\Omega_2' + \omega_1\Omega_2' + \Omega_1'\omega_2\right) \end{bmatrix}$$

The two terms $J\Omega_1'\Omega_3'$ and $-J\Omega_1'\Omega_2'$ can be interpreted as centrifugal stiffening of the blade, see Appendix They can thus be ignored for the rest of the analysis. Furthermore, the term, $\tilde{\omega}_B \times \left(I_B\tilde{\omega}_B\right)$, can be ignored since each element is the square of small terms. It follows that

$$\tilde{M}_T = J\begin{bmatrix} 0 \\ -s_\theta\dot{\Omega}_{xR} + c_\theta\dot{\Omega}_{yR} \\ s_\phi c_\theta\dot{\Omega}_{xR} + s_\phi s_\theta\dot{\Omega}_{yR} + c_\phi\dot{\Omega}_{zR} \end{bmatrix}$$

Note that $-s_\theta\dot{\Omega}_{xR} + c_\theta\dot{\Omega}_{yR}$ and $s_\phi c_\theta\dot{\Omega}_{xR} + s_\phi s_\theta\dot{\Omega}_{yR} + c_\phi\dot{\Omega}_{zR}$ are the rotational acceleration components of the rotor frame relative to earth frame $\dot{\tilde{\Omega}}_R$, transformed into the body centred axes.

Thus the rotational fictitious torque is expressed in the rotor frame as followed.

$$\tilde{M}_{T_R} = J\begin{bmatrix} 0 & \dot{\Omega}_{zR} & \dot{\Omega}_{yR} \end{bmatrix}^T$$

where $\dot{\Omega}_{zR}$ and $\dot{\Omega}_{yR}$ are rotational accelerations measured at the origin of the rotor plane (hub).

5 Full Blade Model

The full non-linear model of the blade including the coupling to the rest of the wind turbine dynamics is:

$$\begin{bmatrix} \ddot{\theta}_R \\ \ddot{\phi}_R \end{bmatrix} = -\begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix}$$
$$+ \frac{1}{J}\begin{bmatrix} M_{A\theta} \\ M_{A\phi} \end{bmatrix} + \frac{1}{J}\begin{bmatrix} M_{T\theta_R} \\ M_{T\phi_R} \end{bmatrix}$$

$$\begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix} = J \cdot \begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix}$$

With the fictitious forces:

$$\begin{bmatrix} M_{T\theta_R} \\ M_{T\phi_R} \end{bmatrix} = m_b l\begin{bmatrix} a_{B2} \\ -a_{B3} \end{bmatrix} + J\begin{bmatrix} \dot{\Omega}_{zR} \\ \dot{\Omega}_{yR} \end{bmatrix}$$

More appropriately for the purpose intended here, this model can also be expressed in terms of the namely in-plane and out–of-plane moments:

$$\begin{bmatrix} \ddot{M}_{I/P} \\ \ddot{M}_{O/P} \end{bmatrix} = -A(\alpha)\begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix} + A(\alpha)\begin{bmatrix} M_{A\theta_R} + M_{T\theta_R} \\ M_{A\phi_R} + M_{T\phi_R} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_R \\ \dot{\phi}_R \end{bmatrix} = \frac{1}{J}A^{-1}(\alpha) \cdot \begin{bmatrix} \dot{M}_{I/P} \\ \dot{M}_{O/P} \end{bmatrix}$$

where

$$A(\alpha) = \begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix}$$

$$A^{-1}(\alpha) = \frac{1}{\omega_E^2\omega_F^2}\begin{bmatrix} \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 & \left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha \\ \left(\omega_E^2 - \omega_F^2\right)s_\alpha c_\alpha & \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 \end{bmatrix}$$

The derivation of this model is explained in Appendix B.

Now, the control system should be modified by subtracting the contribution of the fictitious forces from the measured bending moment as shown in Figure 5. The fictitious forces are derived directly from measured accelerations.

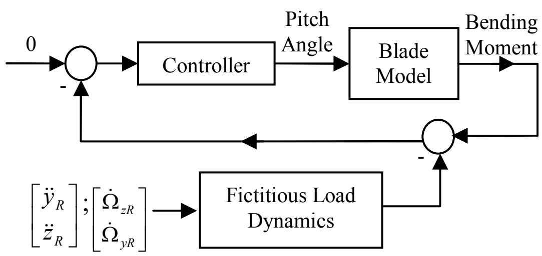

Figure 5: Feedback Control System for the Blade Model with Tower Dynamics Deducted

6 Control System Design

The controller for the blade is designed to achieve the following objectives:

1. The blade out-of-plane bending moment is regulated to follow a set point derived from the central controller pitch demand. The pitch of the blade is adjusted to compensate for the disturbance to the out-of-plane bending moment at $1\Omega_0$ and $2\Omega_0$.

2. The dynamics of the actuator must appear unchanged to the central controller.

3. Aerodynamic non-linearity is counteracted by global non-linear control.

4. Smooth switching between below-rated and above-rated must be achieved.

One possible controller structures is shown in Figure 6.

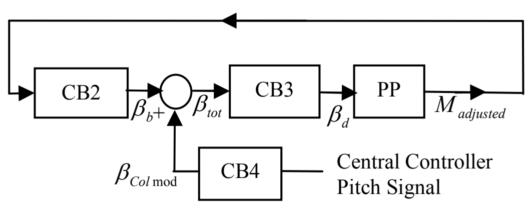

Figure 6: Overall Control Structure

PP represents the pitch actuator dynamics and the blade dynamics with the fictitious forces added to the measured out-of-plane bending moment to decouple their dynamics from the rest of the wind turbine dynamics.

CB2 is the pitch controller for the local feedback loop at the blade. If the sole objective of the blade controller is to reduce the unbalanced loads in the vicinity of $1\Omega_0$ then the pitch controller would be a form of band pass filter centred on $1\Omega_0$.

CB3 is the compensation for the aerodynamics non-linearity. Together with the switching position, velocity and acceleration of the actuator output are all constrained. Priority is given to the central controller demand. When the pitch position, velocity and the acceleration of the central controller demand approach their limits, the action of the local feedback loop is reduced accordingly.

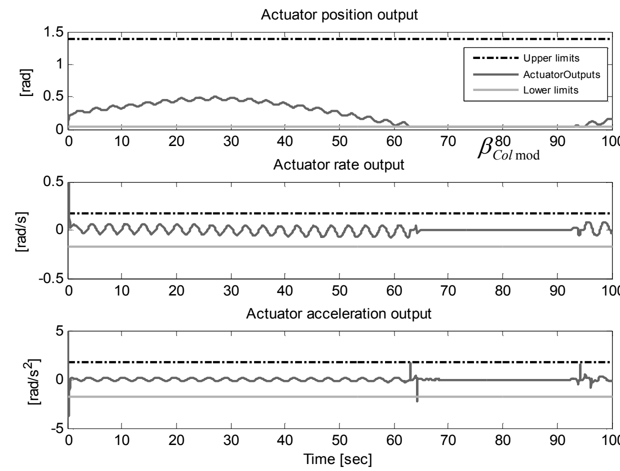

Figure 7: Actuator Position, Velocity and Acceleration Outputs

In Figure 7, a slowly varying pitch demand from the central controller is modified by rapidly varying increment due to the local feedback loop. Smooth switching in and out of the local feedback loop at 65 seconds and 93 seconds caused by the central control demand approaching and going below zero degrees. It can also be seen that the pitch velocity and acceleration limits are respected.

CB4 compensates the pitch demand from the central controller to counteract the change in the actuator dynamics caused by the local feedback loop. The local feedback loop is thereby made invisible to the central controller. The turbine speed controller sets the average demand for the pitch angles as required to control the speed; the blade controllers make incremental adjustments to this average. The former is an outer feedback loop; the latter are inner feedback loops. The actuator plus blade feedback loop can be considered to be a modified actuator. As long as it has 0dB at low frequency, the outer feedback loop continues to regulate the speed as required. The only consequence would be that the dynamics of this modified actuator might be different from the original actuator but this is compensated by block CB4.

Feedback loops as described above were applied to all three blades on a BLADED simulation of a large multi-MW wind turbine. To focus the assessment on the stochastic components rather than the deterministic components which can also be reduced by cyclic pitch control, the results are obtained with both tower shadow and wind shear turned off. The controller is active over a frequency range including $1\Omega_0$ and $2\Omega_0$ but with wash-out at low frequency and roll-off at high frequency. For a mean wind speed of 18m/s and turbulence intensity at 17%, the out-of-plane root bending moment is reduced by 12%. The tower torsional moment is reduced by 17%. The main bearing tilt moment is reduced by 23% and the main bearing yaw moment by 22%.

7. Conclusions

The role of the wind turbine controller has recently been extended to include the alleviation of structural loads. The alleviation of the rotor loads by pitch control has recently been investigated. By separately adjusting the angle of pitch of each blade, the unbalanced loads on the rotor could be reduced. A novel approach to reducing the unbalance rotor loads is presented in this paper. Each blade has its own actuator, sensors and controller. These localised blade control systems operate in isolation without need of communication with each other. The controller for a single blade is designed on the basis of the blade dynamics alone to determine the adjustment in pitch angle required to counteract the component of the blade bending moment contributing to unbalanced rotor loads. The following issues are discussed, the decoupling of the blade dynamics from the dynamics of the rest of the wind turbine, the dynamic model of the single blade, the nonlinear aspects of the controller design. This single blade control approach to regulation of unbalanced rotor loads has several advantages: there is no need to communicate with the central controller in the nacelle; the presence of the local blade controllers is invisible to the central controller; the controller, being dependent on the blade dynamics alone, is straightforward to design and easy to tune (indeed, re-tuning is not required if applied to a different wind turbine with the same blade). The performance of the single blade controllers is assessed using BLADED simulations.

8. Acknowledgement

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9. References

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10. Appendix A: centrifugal stiffening

Centrifugal stiffening is the restoring moment on the blade that opposes the blade flap or edge motions. This is caused by the rotational motion of the rotor.

The rotational tower fictitious load is expressed as followed.

$$\begin{aligned}\widetilde{M}_T &= \widetilde{M}_{Total} - \widetilde{M}_B \\ &= I_B\left(\dot{R}_2\widetilde{\Omega}_R + R_2\dot{\widetilde{\Omega}}_R\right) - \begin{bmatrix} 0 \\ J(\Omega_1'\Omega_3' + \omega_1\Omega_3' + \Omega_1'\omega_3) \\ -J(\Omega_1'\Omega_2' + \omega_1\Omega_2' + \Omega_1'\omega_2) \end{bmatrix}\end{aligned}$$

In which terms $J\Omega_1'\Omega_3'$ and $-J\Omega_1'\Omega_2'$ are the blade centrifugal stiffening. They are also the components of the following cross-products.

$$\begin{aligned}&\left(I_B\widetilde{\Omega}'\right)\times\widetilde{\Omega}' \\ &= J\cdot\begin{bmatrix}0\\ \Omega_2' \\ \Omega_3'\end{bmatrix}\times\begin{bmatrix}\Omega_1'\\ \Omega_2' \\ \Omega_3'\end{bmatrix} = J\cdot\begin{bmatrix}0\\ \Omega_1'\Omega_3' \\ \Omega_1'\Omega_2'\end{bmatrix}\end{aligned}$$

The first term of the cross product is the angular momentum components of the rotor frame represented in the body centred frame relative to earth frame. To show the effect of the resultant moment graphically, refer to the blade body centred frame as shown in Figure 8.

The cross product of $I_B\widetilde{\Omega}'$ and the component of $\widetilde{\Omega}'$ in the $Y_B - Z_B$ plane ($\Omega'_{2Z_B} + \Omega'_{3Y_B}$) is zero, hence the cross product $\left(I_B\widetilde{\Omega}'\right)\times\widetilde{\Omega}'$ is reduced down to the following cross products representing the moments in directions of Y_B and Z_B.

$$\begin{bmatrix} M_{cx} \\ M_{cy} \\ M_{cz} \end{bmatrix} = \begin{bmatrix} 0 \\ \left(J\widetilde{\Omega}'_2\right) \times \widetilde{\Omega}'_1 \\ \left(J\widetilde{\Omega}'_3\right) \times \widetilde{\Omega}'_1 \end{bmatrix} = \begin{bmatrix} 0 \\ J\Omega'_1\Omega'_3 \\ -J\Omega'_1\Omega'_2 \end{bmatrix}$$

Moments M_{cy} and M_{cz} are in opposite directions to the blade flap and edge motions. They are the restoring moments in the blade flap and edge directions, i.e. the blade centrifugal stiffening.

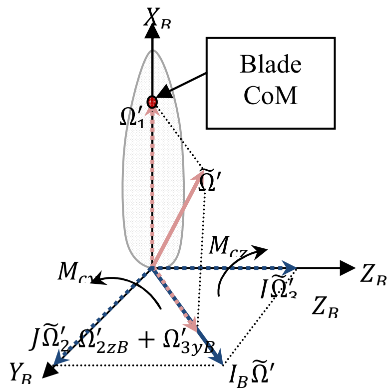

Figure 8: Graphical interpretation of the cross product between angular momentum and velocity components

This analysis shows that the centrifugal stiffness of a rotating blade can be calculated from multiplying the blade inertia by the products of the angular velocity components both along the blade and perpendicular to the blade.

8. Appendix B: derivation of the final model

Given the blade non-linear model:

$$\begin{bmatrix} \ddot{\theta}_R \\ \ddot{\phi}_R \end{bmatrix} = -\begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix} + \frac{1}{J}\begin{bmatrix} M_{A\theta} \\ M_{A\phi} \end{bmatrix} + \frac{1}{J}\begin{bmatrix} M_{T\theta_R} \\ M_{T\phi_R} \end{bmatrix}$$

$$\begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix} = J \cdot \begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix}$$

Now let:

$$A(\alpha) = \begin{bmatrix} \omega_E^2 c_\alpha^2 + \omega_F^2 s_\alpha^2 & -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha \\ -\left(\omega_E^2 - \omega_F^2\right) s_\alpha c_\alpha & \omega_E^2 s_\alpha^2 + \omega_F^2 c_\alpha^2 \end{bmatrix}$$
$$= \begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{bmatrix} \cdot \begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} \cdot \begin{bmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{bmatrix}$$
$$= T_\alpha^{-1} \begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} T_\alpha$$

The equation of motion can thus be re-written as followed.

$$\begin{bmatrix} \ddot{\theta}_R \\ \ddot{\phi}_R \end{bmatrix} = -T_\alpha^{-1} \begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} T_\alpha \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix} + \frac{1}{J}\begin{bmatrix} M_{A\theta_R} + M_{T\theta_R} \\ M_{A\phi_R} + M_{T\phi_R} \end{bmatrix}$$

Multiply both sides of the equation above by T_α gives the following.

$$T_\alpha \cdot \begin{bmatrix} \ddot{\theta}_R \\ \ddot{\phi}_R \end{bmatrix} = -T_\alpha \cdot T_\alpha^{-1} \begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} T_\alpha \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix} + T_\alpha \cdot \frac{1}{J}\begin{bmatrix} M_{A\theta_R} + M_{T\theta_R} \\ M_{A\phi_R} + M_{T\phi_R} \end{bmatrix}$$

let:

$$\begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix} = T_\alpha \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix}$$

$\ddot{\theta}_{R\alpha}$ and $\ddot{\phi}_{R\alpha}$ can also be expressed by the in-plane and out-plane moments ($M_{I/P}$ and $M_{O/P}$)as followed.

$$\begin{bmatrix} \ddot{\theta}_{R\alpha} \\ \ddot{\phi}_{R\alpha} \end{bmatrix} + \ddot{\alpha}\begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix}$$
$$= -\begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix} + T_\alpha \cdot \frac{1}{J}\begin{bmatrix} M_{A\theta_R} + M_{T\theta_R} \\ M_{A\phi_R} + M_{T\phi_R} \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \ddot{\theta}_{R\alpha} \\ \ddot{\phi}_{R\alpha} \end{bmatrix} = -\begin{bmatrix} \omega_E^2 + \ddot{\alpha} & 0 \\ 0 & \omega_F^2 + \ddot{\alpha} \end{bmatrix} \cdot \begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix} + T_\alpha \cdot \frac{1}{J}\begin{bmatrix} M_{A\theta_R} + M_{T\theta_R} \\ M_{A\phi_R} + M_{T\phi_R} \end{bmatrix}$$

$$\begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix} = J \cdot A(\alpha) \cdot \begin{bmatrix} \theta_R \\ \phi_R \end{bmatrix} = J \cdot T_\alpha^{-1} \cdot \begin{bmatrix} \omega_E^2 & 0 \\ 0 & \omega_F^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \theta_{R\alpha} \\ \phi_{R\alpha} \end{bmatrix} = \frac{1}{J}\begin{bmatrix} 1/\omega_E^2 & 0 \\ 0 & 1/\omega_F^2 \end{bmatrix} \cdot T_\alpha \cdot \begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \ddot{\theta}_{R\alpha} \\ \ddot{\phi}_{R\alpha} \end{bmatrix} = \frac{1}{J}\begin{bmatrix} 1/\omega_E^2 & 0 \\ 0 & 1/\omega_F^2 \end{bmatrix} \cdot \left(\frac{d}{dt}\left(T_\alpha \begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix}\right)\right)$$
$$\cong \frac{1}{J}\begin{bmatrix} 1/\omega_E^2 & 0 \\ 0 & 1/\omega_F^2 \end{bmatrix} \cdot \left(T_\alpha \begin{bmatrix} \ddot{M}_{I/P} \\ \ddot{M}_{O/P} \end{bmatrix} - \ddot{\alpha} T_\alpha \begin{bmatrix} M_{I/P} \\ M_{O/P} \end{bmatrix}\right)$$

Combined with the equation derived previously, the following expression for the in-plane and out-plane loads can be derived.

$$\frac{1}{J}\begin{bmatrix}1/\omega_E^2 & 0\\ 0 & 1/\omega_F^2\end{bmatrix}\cdot\left(T_\alpha\begin{bmatrix}\ddot{M}_{I/P}\\ \ddot{M}_{O/P}\end{bmatrix}-\ddot{\alpha}T_\alpha\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}\right)$$

$$=-\begin{bmatrix}\omega_E^2+\ddot{\alpha} & 0\\ 0 & \omega_F^2+\ddot{\alpha}\end{bmatrix}\cdot\frac{1}{J}\begin{bmatrix}1/\omega_E^2 & 0\\ 0 & 1/\omega_F^2\end{bmatrix}\cdot T_\alpha\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}$$

$$+T_\alpha\cdot\frac{1}{J}\begin{bmatrix}M_{A\theta_R}+M_{T\theta_R}\\ M_{A\phi_R}+M_{T\phi_R}\end{bmatrix}$$

$$\Rightarrow T_\alpha\begin{bmatrix}\ddot{M}_{I/P}\\ \ddot{M}_{O/P}\end{bmatrix}-\ddot{\alpha}T_\alpha\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}$$

$$=-\begin{bmatrix}\omega_E^2+\ddot{\alpha} & 0\\ 0 & \omega_F^2+\ddot{\alpha}\end{bmatrix}\cdot T_\alpha\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}$$

$$+\begin{bmatrix}\omega_E^2 & 0\\ 0 & \omega_F^2\end{bmatrix}\cdot T_\alpha\cdot\begin{bmatrix}M_{A\theta_R}+M_{T\theta_R}\\ M_{A\phi_R}+M_{T\phi_R}\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\ddot{M}_{I/P}\\ \ddot{M}_{O/P}\end{bmatrix}=-T_\alpha^{-1}\begin{bmatrix}\omega_E^2 & 0\\ 0 & \omega_F^2\end{bmatrix}\cdot T_\alpha\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}$$

$$+T_\alpha^{-1}\cdot\begin{bmatrix}\omega_E^2 & 0\\ 0 & \omega_F^2\end{bmatrix}\cdot T_\alpha\cdot\begin{bmatrix}M_{A\theta_R}+M_{T\theta_R}\\ M_{A\phi_R}+M_{T\phi_R}\end{bmatrix}$$

$$\therefore\begin{bmatrix}\ddot{M}_{I/P}\\ \ddot{M}_{O/P}\end{bmatrix}=-A(\alpha)\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}+A(\alpha)\begin{bmatrix}M_{A\theta_R}+M_{T\theta_R}\\ M_{A\phi_R}+M_{T\phi_R}\end{bmatrix}$$

The aerodynamic loadings are functions of wind speed, time derivatives of in-plane, out-plane angles and the pitch angle.

$$\begin{bmatrix}M_{A\theta}\\ M_{A\phi}\end{bmatrix}=\begin{bmatrix}M_{A\theta}\left(\left(w-L\dot{\phi}_R\right),\left(\Omega+\dot{\theta}_R\right),\alpha\right)\\ M_{A\phi}\left(\left(w-L\dot{\phi}_R\right),\left(\Omega+\dot{\theta}_R\right),\alpha\right)\end{bmatrix}$$

$$\cong\begin{bmatrix}M_{A\theta}(w,\Omega,\alpha)-L\dfrac{\partial M_{A\theta}}{\partial w}\dot{\phi}_R+\dfrac{\partial M_{A\theta}}{\partial\Omega}\dot{\theta}_R\\ M_{A\phi}(w,\Omega,\alpha)-L\dfrac{\partial M_{A\phi}}{\partial w}\dot{\phi}_R+\dfrac{\partial M_{A\phi}}{\partial\Omega}\dot{\theta}_R\end{bmatrix}$$

The derivatives of the in-plane and out-plane angles can be derived from the in-plane and out-plane moments as followed.

$$\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}=J\cdot A(\alpha)\cdot\begin{bmatrix}\theta_R\\ \phi_R\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\theta_R\\ \phi_R\end{bmatrix}=\frac{1}{J}\cdot A^{-1}(\alpha)\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\dot{\theta}_R\\ \dot{\phi}_R\end{bmatrix}=\frac{1}{J}\left(\frac{d}{dt}\left(A^{-1}(\alpha)\right)\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}+A^{-1}(\alpha)\cdot\begin{bmatrix}\dot{M}_{I/P}\\ \dot{M}_{O/P}\end{bmatrix}\right)$$

$$=\frac{1}{J\omega_E^2\omega_F^2}\cdot\left(\begin{bmatrix}\omega_E^2s_\alpha^2+\omega_F^2c_\alpha^2 & \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha\\ \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha & \omega_E^2c_\alpha^2+\omega_F^2s_\alpha^2\end{bmatrix}\cdot\begin{bmatrix}\dot{M}_{I/P}\\ \dot{M}_{O/P}\end{bmatrix}+\dot{\alpha}\left(\omega_E^2-\omega_F^2\right)\cdot\begin{bmatrix}2s_\alpha c_\alpha & c_\alpha^2-s_\alpha^2\\ c_\alpha^2-s_\alpha^2 & -2s_\alpha c_\alpha\end{bmatrix}\cdot\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}\right)$$

$$=\frac{1}{J\omega_E^2\omega_F^2}\cdot\begin{bmatrix}\omega_E^2s_\alpha^2+\omega_F^2c_\alpha^2 & \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha\\ \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha & \omega_E^2c_\alpha^2+\omega_F^2s_\alpha^2\end{bmatrix}\cdot\begin{bmatrix}\dot{M}_{I/P}\\ \dot{M}_{O/P}\end{bmatrix}$$

The non-linear single blade model can thus be expressed in a simpler form which makes linearisation easier.

$$\begin{bmatrix}\ddot{M}_{I/P}\\ \ddot{M}_{O/P}\end{bmatrix}=-A(\alpha)\begin{bmatrix}M_{I/P}\\ M_{O/P}\end{bmatrix}+A(\alpha)\begin{bmatrix}M_{A\theta_R}+M_{T\theta_R}\\ M_{A\phi_R}+M_{T\phi_R}\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_R\\ \dot{\phi}_R\end{bmatrix}=\frac{1}{J}A^{-1}(\alpha)\cdot\begin{bmatrix}\dot{M}_{I/P}\\ \dot{M}_{O/P}\end{bmatrix}$$

Where:

$$A(\alpha)=\begin{bmatrix}\omega_E^2c_\alpha^2+\omega_F^2s_\alpha^2 & -\left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha\\ -\left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha & \omega_E^2s_\alpha^2+\omega_F^2c_\alpha^2\end{bmatrix}$$

$$A^{-1}(\alpha)=\frac{1}{\omega_E^2\omega_F^2}\begin{bmatrix}\omega_E^2s_\alpha^2+\omega_F^2c_\alpha^2 & \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha\\ \left(\omega_E^2-\omega_F^2\right)s_\alpha c_\alpha & \omega_E^2c_\alpha^2+\omega_F^2s_\alpha^2\end{bmatrix}$$