

# PRIVATIZATION IN OLIGOPOLY: THE IMPACT OF THE SHADOW COST OF PUBLIC FUNDS

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The aim of this paper is to investigate the welfare effect of privatization in oligopoly when the government takes into account the distortionary effect of raising funds by taxation (shadow cost of public funds). We analyze the impact of the change in ownership not only on the objective function of the firms, but also on the timing of competition by endogenizing the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games. We show that, absent efficiency gains, privatization never increases welfare. Moreover, even when large efficiency gains are realized, an inefficient public firm may be preferred.

*Keywords:* Mixed oligopoly; Privatization; Endogenous Timing; Distortionary taxes.

*JEL Classification:* H2; H42; L13; L32; L33

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# 1 Introduction

In the last decades of the XX century a process of liberalization and/or privatization occurred in most of the industrialized countries and nowadays public utilities are generally no longer run by public monopolies. The motivations for this program were essentially linked to the general perception of poor performance of public monopolies and to the idea that entry of private subjects could enhance efficiency. For example, during the nineties, in Italy, in France and in UK, as well as in many EU countries, the public incumbent faced the entry of private competitors in many communication services. The same occurred in the production of electricity, in gas retailing and more recently in some postal services. In the same years, national (public) airlines started competing with private or foreign ones in the domestic markets. Moreover, examples of public monopolies that became mixed oligopoly can be found in a broad range of industries including railways, steel and overnight-delivery industries, as well as services including banking, home loans, health care, life insurance, hospitals, broadcasting, and education. In these cases, instead of regulating a privatized monopoly, governments decided to enforce a facility-based competitions in order to achieve a so-called dynamic efficiency.<sup>1</sup>

Our investigation starts downward the liberalization process of a public monopoly, and the aim of the present work is to build a theoretical model for the analysis of the welfare effect of privatization in oligopoly. Following the literature on industrial organization, a market where a public welfare-maximizing firm competes against private profit-maximizing firms is denoted by *mixed oligopoly*. By *privatization* we mean a transfer of property rights from the government to domestic private investors. This translates into a change in the objective function of the firm from welfare to profit maximization and a transfer of money from the investors to the government. Since we mainly refer to public utility markets open to competition, we consider a mixed duopoly in which production is characterized by increasing returns to scale (with fixed and constant marginal costs) and we assume that the public firm is *typically* less efficient than its private competitor.<sup>2</sup> We analyze the effect of privatization on welfare comparing the equilibrium outcomes before and after privatization. Even though no dynamic effect is explicitly analyzed, we consider both the case in which the privatized firm remains productively inefficient, and the case where privatization leads to a full recovery of the efficiency.

The first novel contribution of this paper is represented by the introduction of the shadow cost of public funds in the objective function of the public firm. That is, we assume

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<sup>1</sup> For deeper viewpoints on the role played by facility-based competition in EU and US Telecommunications liberalization and regulation processes see Taschdjian (1997) and Stehmann and Borthwick (1994).

<sup>2</sup> Differently from Cremer et al. (1989), the higher production cost of the public firm is not a neutral transfer from firm to workers belonging to the same economy but, as an *X*-inefficiency, it reduces any utilitarian measure of welfare.

that the public firm is required to take into account the distortionary effect of the taxes that are needed to cover its deficit and, in general, public expenditures. In fact, absent lump-sum tax instruments, if government raises 1 Euro from taxation, society pays  $(1 + \lambda)$  Euros. Coherently, public profits, when positive, avoid an equivalent public transfer, reducing distortionary taxes.<sup>3</sup> As initially analyzed in Meade (1944) and exploited in Laffont and Tirole (1986, 1993), this approach has been used to characterize public monopolies running a deficit and, more generally, regulated markets. Here we apply the same analysis to a public firm competing in a duopoly and to the effects of privatization, given that *getting money for reducing public debt or distortionary taxes* is often a complementary target of privatization. The main consequence is that, taking into account the shadow cost of public funds, the public firm puts more weight on its own profits and, at least partially, it mimics the behavior of a private firm.

The second contribution of this work is that we consider the effect of privatization on the timing of competition by endogenizing the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games. That is, the structure of the game is not assumed *a priori*, but it is the result of preplay independent and simultaneous decisions by the players. From the standard analysis of the second-best theory (Lipsey and Lancaster, 1956; Rees, 1984; Bös, 1986), the assumption of public leadership has long been considered rather natural in mixed markets. In fact, claiming the sub-optimality of the marginal-cost pricing, the public firm can optimally depart from this rule by taking into account the reaction of the private firms when maximizing social welfare. This *natural* assumption has been criticized by Cremer et al. (1989) who instead supported the general plausibility of the (Cournot-) Nash equilibrium in the analysis of oligopolistic markets. Beato and Mas-Colell (1984) shared the idea of a dominant position of the public firm, but questioned the optimality of public leadership. If private leadership is preferred to public leadership equilibrium, the public firm can commit itself to behaving as follower by announcing the marginal-cost pricing rule instead of announcing its output. However, in all the cases it is assumed that the public firm can choose the type of the game independently of the behaviour of private firms.

In the present work, in order to endogenize the timing of the game, we apply the model developed by Hamilton and Slutsky (1990) to the mixed oligopoly framework. In their insightful paper, the authors build an endogenous timing game by adding to the basic quantity game a preplay stage at which players simultaneously and independently decide whether to move early or late in the basic game. Therefore, the type of competition endogenously emerges in the subgame-perfect equilibrium (SPE) of this extended game. Amir and Grilo (1999) apply this model to a private duopoly showing that, in a quantity setting with strategic substitutes, Cournot competition always occurs in the SPE of the endogenous timing game. Pal (1998) uses the same game structure to analyse a mixed oligopoly and shows that

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<sup>3</sup> Since profit or deficit of the public firm are not neutral transfers among agents of the same economy, they ought not to be weighted as private profits or consumer net surplus in the utilitarian measure of welfare, but they should be weighted  $(1 + \lambda)$ .

sequential play always emerges in the SPE of the endogenous timing game. However, the public firm generally does not produce in equilibrium and public production is just a disciplinary device that forces the private firms to produce more. Even though after Pal (1998) other authors analyzed the endogenous timing in mixed oligopolies, there is no work, at our best knowledge, that extends this line of research to the welfare evaluation of privatization.<sup>4</sup>

We believe that by endogenizing the timing of the game we can gain insights and provide a theoretic foundation to the idea that strategic interaction in mixed oligopolies may be different from the one in private oligopolies, and we can also explain the emergence of different types of competition in mixed oligopolies.

This is consistent with stylized facts and empirical findings in mixed markets. For example, the state-owned Norwegian supplier of electricity, Statkraft, behaves as it was the residual producer in the case of dramatic price changes (Magnus and Midttun, 2000). We can interpret this behavior as the announcement by the state-owned firm that it is committed to sustaining a certain price level on the market, that is to behave as follower with respect to private competitors that can choose the optimal level of production knowing the reaction of Statkraft. In industries such as the energy markets, capacity investments can easily work as credible commitments by private firms to behave as leader in a Stackelberg game.

In general, we can think that whenever public firms commit themselves to pursuing market goals such as total quantity, price level or universal service obligations, they are announcing to “move late”, using the simple endogenous timing terminology.

In health care markets the public provider in countries like France and Germany is usually considered as a leader when competing with private firms.<sup>5</sup> The definition of performance objectives can be understood as a credible commitment to “move early” in order to act as a leader that takes into account what will be produced by the private competitors.

Furthermore, we think that this approach is especially relevant for the analysis of privatization, given that the results and the policy prescriptions emerged in the literature crucially rely on the type of competition that is *assumed*. For example, in de Fraja and Delbono (1989) it is shown that a shift from welfare to profit maximization may improve welfare under Cournot competition even without efficiency gains; while, if a Stackelberg game with public leadership is exogenously assumed, this cannot occur.<sup>6</sup>

The main results of our analysis can be summed up as follows.

With respect to the determination of the endogenous timing in mixed oligopoly, our

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<sup>4</sup> Matsumura (2003), Cornes and Sepahvand (2003), and Sepahvand (2004) apply the same model to international mixed oligopolies finding that public leadership may emerge as the unique SPE of the endogenous timing game.

<sup>5</sup> See for example Barros (1995).

<sup>6</sup> Our definition of privatization is in fact very similar to the change in objective function defined in de Fraja and Delbono (1989). The only difference is that the proceedings from privatization are not a neutral transfer from domestic residents to the government since they are weighted by  $(1 + \lambda)$  in the latter case.

results differ from Pal (1998) in that either Nash, or private leadership, or both Stackelberg outcomes can result as subgame-perfect equilibria (SPE) of the endogenous timing game. Moreover, following the intuition of Beato and Mas-Colell (1984), we show that when both Stackelberg games are SPE of the endogenous timing game, private leadership is preferred by the public firm and is indeed selected when risk-dominance is used as equilibrium selection criterion. In addition, differently from Pal (1998), the public firm is generally active on the market by producing positive quantity, except when too inefficient.

The identification of the equilibrium is crucial to the evaluation of the welfare impact of privatization. Indeed we show that, absent efficiency gains, privatization never increases welfare. Furthermore, even when large efficiency gains are realized by privatization, an inefficient public firm may still be preferred. These results in the welfare comparisons rely on sequential outcomes being supported as SPE in a mixed oligopoly, while in a private oligopoly only simultaneous equilibria are implemented.

It is worth noting that our results are obtained in a framework of complete information and we do not take into account incentive problems. That is, we assume that the owners of the private firm as well as the government have direct control over the behaviour of managers of the private firm and the public one, respectively. So, both the choice of the timing and the strategies in the quantity competition maximize the objective function of the owners.<sup>7</sup>

We also assume that government has the full bargaining power in the privatization process. This has important implications on the amount of the privatization proceeds; in such a case the price paid by the new private owners for the former public firm is equal to its profit in the new (Cournot) equilibrium. This assumption drives the results in favour of privatization, since it overweights the revenue from privatization by  $\lambda$  in any welfare comparison.

In what follows, the next Section sets up the model. Section 3 is focused on the issue of endogenous timing in mixed oligopoly, while Section 4 is devoted to the analysis of privatization. Our conclusions are delegated to Section 5. The proofs are collected in the Appendix.

## 2 The basic setting

In a static, partial equilibrium analysis, we consider the simplest setting of a mixed duopoly, where a domestic private firm and a public one, labeled  $p$  and  $g$  respectively, produce a commodity and compete in a quantity game. Demand preferences are described by a linear function where intercept and slope are normalized to one:

$$p(q_g + q_p) = 1 - q_p - q_g.$$

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<sup>7</sup> See Barros (1995) for the analysis of competition in a mixed duopoly in the context of asymmetry of information between owners and managers.

Both firms are characterized by constant marginal costs and fixed costs corresponding to irreversible investments.<sup>8</sup> More precisely, we assume that the public firm is already in the market and its fixed cost  $K_g$  is sunk. Conversely, the fixed cost of the private firm,  $K_p$ , is borne only in the case of producing. Moreover, in order to simplify the analysis, the marginal cost of the private firm is normalized to zero, while the one of the public firm is positive and equal to  $c \geq 0$ . Therefore  $c$  can be considered as an index of the inefficiency of the public firm.

The private firm maximizes its profit:

$$\Pi_p(q_g, q_p) = (1 - q_g - q_p)q_p - K_p.$$

The public firm maximizes a utilitarian measure of welfare taking into account the shadow cost of public funds,  $\lambda > 0$ . This parameter is a measure of the dead-weight loss due to distortionary taxation. In particular, let  $S(q_p + q_g)$  denote the consumer gross surplus. We assume that the government can choose the output level of the public firm  $q_g$  and can make a monetary transfer  $T$  to it. Therefore, in the presence of the shadow cost of public funds, the maximization problem of the government is:

$$\begin{aligned} \max_{T, q_g} \quad & W(q_p, q_g) = S(q_p + q_g) - cq_g - K_g - K_p - \lambda T \\ \text{such that} \quad & \tilde{\Pi}_g = p(q_p + q_g)q_g - cq_g - K_g + T \geq 0 \end{aligned} \quad (1)$$

where  $\tilde{\Pi}_g$  is the budget of the public firm including the (positive or negative) transfer  $T$ . Notice that the constraint (1) is not a hard budget balance constraint but it is compatible with operative losses when  $T$  is positive. Since welfare is decreasing in  $T$  when  $\lambda$  is positive, it is optimal to set  $T$  such that  $\tilde{\Pi}_g = 0$ . Then, from (1) we get

$$T = -[p(q_p + q_g)q_g - cq_g - K_g]$$

and substituting  $T$  in the objective function we obtain:

$$\max_{q_g} \quad W = S(q_p + q_g) - cq_g - K_g - K_p + \lambda [p(q_p + q_g)q_g - cq_g - K_g].$$

Defining the consumer net surplus as

$$CS(q_p + q_g) = S(q_p + q_g) - p(q_p + q_g)(q_p + q_g), \quad (2)$$

and the operative profit of the public firm as

$$\Pi_g(q_g, q_p) = p(q_p + q_g)q_g - cq_g - K_g, \quad (3)$$

the maximization problem of the government is reduced to:

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<sup>8</sup> We consider that the assumption of increasing returns to scale is consistent with the presence of a public incumbent, former monopolist, in a liberalized public utility industry. Nevertheless, the assumption decreasing return to scale is popular in the literature. For papers considering constant marginal costs, see Cremer et al. (1989), Pal (1998), and Martin (2004); while for papers adopting increasing marginal costs, see Beato and Mas-Colell (1984), de Fraja and Delbono (1989), Fjell and Pal (1996), Pal and White (1998), Matsumura (1998).

$$\max_{q_g} W(q_g, q_p) = CS(q_p + q_g) + \Pi_p(q_g, q_p) + (1 + \lambda)\Pi_g(q_g, q_p). \quad (4)$$

So, the objective defined in equation (4) implies that a transfer occurs in order to guarantee the budget balance. This transfer is positive (negative) when the profits of the public firm are negative (positive).

The objective function (4) can be also interpreted as a weighted average of welfare – defined as the net surplus generated in the market – and profit, where the former is weighted by  $1/(1 + \lambda)$ , while the latter by  $\lambda/(1 + \lambda)$ .

$$W(q_g, q_p) = V(q_g, q_p) + \lambda\Pi_g(q_g, q_p) \quad (5)$$

$$\propto \frac{1}{1 + \lambda}V(q_g, q_p) + \frac{\lambda}{1 + \lambda}\Pi_g(q_g, q_p)$$

$$\text{where } V(q_g, q_p) = CS(q_g + q_p) + \Pi_g(q_g, q_p) + \Pi_p(q_g, q_p).$$

Other works on mixed oligopoly define the objective function of the public firm as a weighted average of welfare and profit. In fact, if we assumed a hard budget balance constraint without public transfer, as in Cremer et al. (1989), the weight given to the profit of the public firm would be endogenous and equal to the associate Lagrangian multiplier. Alternatively, as in Hindriks and Claude (2006), the weight could be positively related to the endogenously chosen private share of a partially public firm, while a negative relation may occur in equilibrium if incentive problems are taken into account (Matsumura, 1998). In the present paper the weight associated to the profit of the public firm is exogenously correlated to the shadow costs of public funds. In our analysis, introducing  $\lambda$  extends the contract theory approach of public monopoly regulation to the case of (mixed) oligopoly.

As usual, the best-reply (or reaction) function of the private firm is derived from the first order condition:

$$\frac{\partial \Pi_p(q_g, q_p)}{\partial q_p} = p(q_g + q_p) + p'(q_g + q_p)q_p = 0$$

In the presence of fixed costs, the reaction function of the private firm ought to be truncated in the point it crosses the zero-isoprofit curve and on-the-boundary solutions can occur in equilibrium. Given the model setting, it can be explicitly written in the following way:

$$r_p(q_g) = \begin{cases} \frac{1}{2}(1 - q_g) & \text{if } q_g < \bar{q}_g \\ 0 & \text{if } q_g \geq \bar{q}_g \end{cases} \quad (6)$$

$$\text{where } \bar{q}_g = q_g : \Pi_p(r_p(q_g), q_g) = 0.$$

The first order condition for the public firm can be derived from the objective (5):

$$\begin{aligned}\frac{\partial W(q_g, q_p)}{\partial q_g} &= \frac{\partial V(q_g, q_p)}{\partial q_g} + \lambda \frac{\partial \Pi_g(q_g, q_p)}{\partial q_g} \\ &= [p(q_g + q_p) - c] + \lambda [p(q_g + q_p) - c + p'(q_g + q_p)q_g] = 0.\end{aligned}$$

Notice that when  $\lambda = 0$  the output decision of the public firm follows the marginal cost pricing rule, and the first term in square brackets measures its effect on total surplus (*allocative* effect). The second term is the effect on its own profit that prevents the government from using distortionary taxation to raise money (we call it the *distortionary* effect). When  $\lambda \rightarrow +\infty$ , the public firm plays as a private (Cournot) competitor.

Since there is no hard budget balance constraint and its fixed cost  $K_g$  is sunk, the reaction function of the the public firm is not truncated and it can be explicitly derived:

$$r_g(q_p) = \max\left\{\frac{1 + \lambda}{1 + 2\lambda}(1 - c - q_p); 0\right\}. \quad (7)$$

However, we want to focus on the case in which both firms produce strictly positive quantities when they play simultaneously; so, we provide some assumptions on the admissible set in the space of parameters.

**Assumption 1** *The parameters  $c$  and  $\lambda$  belong to the subspace*

$$A \subset \mathbb{R} \times \mathbb{R} = \left\{ (c, \lambda) \mid c \in \left(0, \frac{1}{2}\right) \vee \lambda \in [0, \bar{\lambda}] \right\}$$

where  $\bar{\lambda}$  is a finite, reasonable value of the shadow cost of public funds.<sup>9</sup>

**Assumption 2**  *$K_p$ , the fixed cost of the private firm, belongs to the subspace  $B \subset \mathbb{R} = [0, \bar{K}_p]$ , where  $\bar{K}_p$  is smaller than the producer surplus of the private firm in any (simultaneous or sequential) equilibrium.*

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<sup>9</sup> According to the proof of Lemma 5,  $\lambda$  is supposed to be lower than 5.37. Notice that any reasonable estimation of the shadow cost of public funds is never higher than 1.

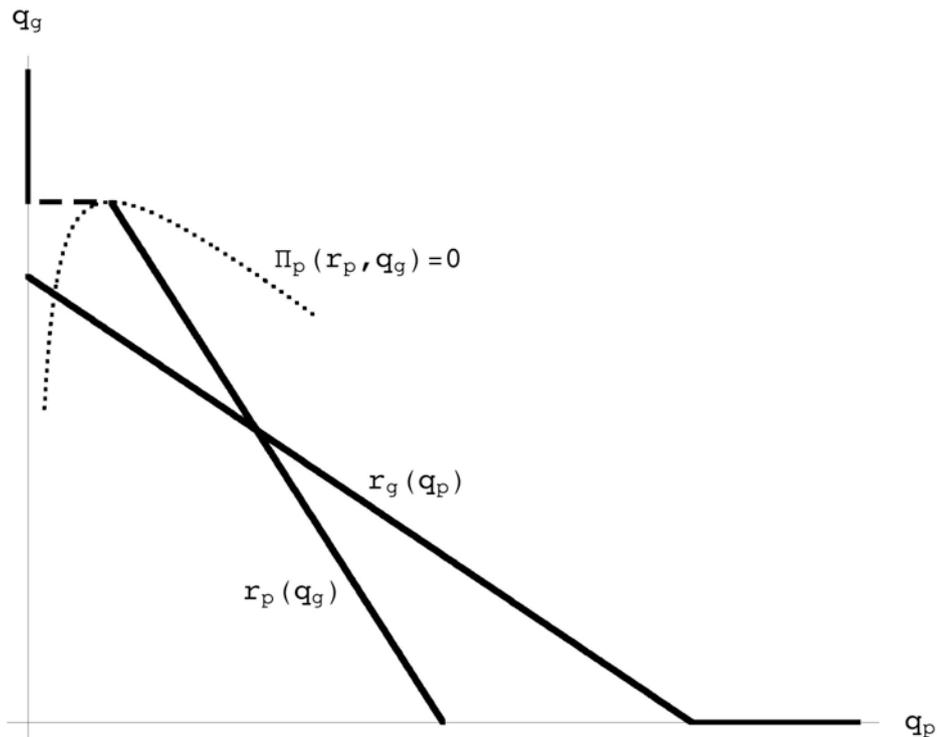


Figure 1: The reaction functions of the public and private firms.

In Figure 1, the reaction functions are depicted. Coherently with the Assumptions 1 and 2, the intersection occurs in the interior of the parameters space where both firms produce strictly positive quantities.

An increase in  $\lambda$  has the effect of reducing both intercept and slope of the reaction function of the public firm. When  $\lambda$  tends to infinite, the public firm plays as a profit maximizer.

### 3 Endogenous timing in a mixed duopoly

In this Section we investigate how the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games is the result of preplay independent and simultaneous decisions by the players.<sup>10</sup>

In many works on mixed oligopoly and privatization, the timing of the competition (simultaneous or sequential) is generally assumed, and simultaneous playing is mostly adopted.<sup>11</sup> Of course, this assumption is not neutral and, as discussed in the Introduction, the welfare impact of privatization crucially depends on the assumed timing.

<sup>10</sup>Notice that in formal game-theoretical terms, Stackelberg's proposal is not to be understood as a new solution concept for one-shot games, but rather as a subgame-perfect equilibrium of a two-stage game of perfect information with exogenously given first and second movers.

<sup>11</sup>In Cremer et al. (1989, p. 284), the reason for using a simultaneous timing is summarized as follows: "The common use of the Nash equilibrium in industrial organization [...] suggests that this is at least as plausible as the leader-follower situations [...]."

More recently, other works introduced the idea that the order of play should result from the timing decisions of the players. In particular, Hamilton and Slutsky (1990) and Amir and Grilo (1999) demonstrate that simultaneous play emerges as the unique equilibrium of the endogenous game in a private duopoly with strategic substitutability. Conversely, in a mixed duopoly Pal (1998) shows that sequential play always occurs in equilibrium.

Coherently with this approach, in order to endogenize the timing of the play, we use the game with observable delay defined by Hamilton and Slutsky (1990). In the first stage firms simultaneously and independently choose the timing of action (whether to move *early* or *late*) and then, after observing the decision of the other player, they play the basic quantity game. The extensive form of the game is represented in Figure 2.

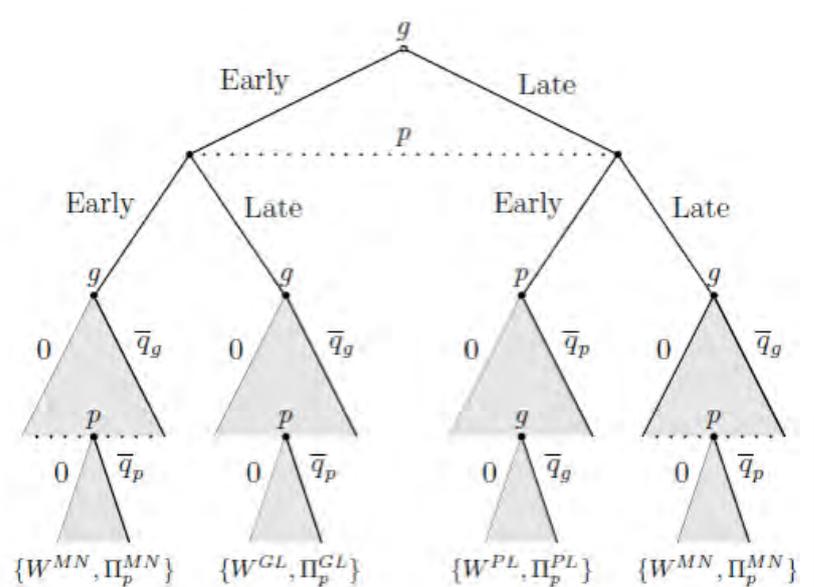


Figure 2: The extensive form of the extended game with observable delay defined by Hamilton and Slutsky (1990) applied to a mixed duopoly. MN, PL and GL stay for Mixed duopoly Nash, Private Leadership and Public Leadership equilibria respectively.

The relevant equilibrium concept is the subgame-perfect equilibrium (SPE) and each player decides the timing of action according to the outcomes in the second stage. Of course, none of the firms can choose the type of competition by itself, but it can only eliminate some outcome. For example, if firm  $i$  decides to move early two outcomes are possible according to the decision of the other player; only the Stackelberg outcome where firm  $i$  is follower is ruled out by its decision.

Assuming existence and uniqueness of equilibria in each basic game, the following Lemma summarizes the results obtained in Hamilton and Slutsky (1990) for any two-player game.

**Lemma 3** Consider a two-player game for which the Nash and the two Stackelberg equilibria exist. The set of (pure strategy) SPE of the endogenous timing game is defined in

the following way:

- i) the Stackelberg follower payoff is lower than the Nash payoff for each firm, then the unique SPE of the endogenous timing game is the Nash equilibrium where both firms decide to move early;
- ii) if the Stackelberg follower payoff is strictly larger than the Nash payoff for each firm, then both Stackelberg equilibria are SPE of the endogenous timing game;
- iii) if the payoff of firm  $i$  when Stackelberg follower is strictly larger than its Nash payoff, and if firm  $j$  prefers to play simultaneously rather than as Stackelberg follower, the unique SPE of the endogenous timing game is the Stackelberg equilibrium with firm  $j$  being the leader.

**Proof.** The proof of this Lemma follows from Theorems II, III and IV in Hamilton and Slutsky (1990). ■

The intuition behind these results is the following. Both firms prefer to be leader rather than playing simultaneously. This is true whenever the reaction correspondences are continuous, strictly decreasing function, and equilibria are interior, as in the present model. So, we need to compare the simultaneous-play payoff with the follower payoff in order to see whether firms have dominant strategies or not. If the Nash payoff is higher than the follower payoff, then any firm has a dominant strategy to move early. But if one firm prefers its follower payoff to the Nash payoff, there is no dominant strategy: when the other player moves early it prefers to move late and *vice versa*. This explains the three possible outcomes listed in Lemma 3. Existence and uniqueness of the equilibria in each basic game are assured by Assumptions 1 and 2. The reduced form of the endogenous timing game for the mixed duopoly is represented in Table 1.

	Private Firm	
Public Firm	Early	Late
Early	$W^{MN}(\cdot), \Pi_p^{MN}(\cdot)$	$W^{GL}(\cdot), \Pi_p^{GL}(\cdot)$
Late	$W^{PL}(\cdot), \Pi_p^{PL}(\cdot)$	$W^{MN}(\cdot), \Pi_p^{MN}(\cdot)$

Table 1: The reduced form of the endogenous timing game.

In order to solve the game we need to compare the equilibrium payoffs in each basic game. In what follows the simultaneous and sequential equilibria are derived.

### 3.1 Simultaneous equilibrium

When firms play simultaneously, the equilibrium output levels solve the system of the best-reply functions (6) and (7). We refer to this equilibrium as a *mixed duopoly Nash*

*equilibrium* and the equilibrium values are labeled by  $MN$ . The output levels and the price in equilibrium are:

$$\begin{aligned} q_g^{MN} &= (1 + \lambda) \frac{(1 - 2c)}{(3\lambda + 1)}; & q_p^{MN} &= c + \lambda \frac{(1 - 2c)}{(3\lambda + 1)}; \\ Q^{MN} &= (1 - c) - \lambda \frac{(1 - 2c)}{(3\lambda + 1)}; & p^{MN} &= c + \lambda \frac{(1 - 2c)}{(3\lambda + 1)}. \end{aligned}$$

Notice that when  $\lambda = 0$ , the equilibrium price is equal to the marginal cost of the public firm. Indeed the public firm adjusts its own output in order to have a total quantity equal to the one implemented by a welfare maximizer (but inefficient) monopoly; but now the welfare is higher.<sup>12</sup> Moreover, when the public firm is as efficient as the private one, the first best solution is implemented.

As  $\lambda$  increases, the public firm equilibrium output  $q_g^{MN}$  decreases and  $q_p^{MN}$  increases; then, the industry total cost decreases enhancing *productive efficiency*. This is because the concern for public transfers serves as a credible commitment for the public firm to decrease its own output. Moreover, since the best-reply functions are contractions, the total output level  $Q^{MN}$  decreases and the market price  $p^{MN}$  increases. It is obvious that the effect on consumer surplus is negative, raising an *allocative inefficiency*. There exists a clear trade-off between technical and allocative efficiency, and the net effect on total surplus is ambiguous and depends on the parameters. The equilibrium values of profits and welfare are summarized in the Appendix, Table 2.

### 3.2 Sequential equilibria

A Stackelberg equilibrium of this game corresponds to the SPE of a two stage game of perfect information in which the second mover (follower) chooses an action after having observed the action of the first mover (leader). Then, the Stackelberg equilibrium imposes that: (i) the strategy of the second mover is a selection from its own reaction function; and (ii) the first mover chooses an action that maximizes its objective given the anticipation of the reaction by the other player. In what follows we first analyze the case of public leadership and then the private leadership equilibrium.

*Public leadership (GL).*

When the public firm moves before its private competitor, the equilibrium quantities solve the following equation system:

$$\begin{aligned} q_g^{GL} &= \operatorname{argmax} W(q_g, r_p(q_g)) \\ q_p^{GL} &= r_p(q_g^{GL}). \end{aligned}$$

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<sup>12</sup>This is because the same total output is partially produced by the more efficient private competitor.

The solution is:

$$q_g^{GL} = \max \left\{ \frac{(1+2\lambda) - 4c(1+\lambda)}{(1+4\lambda)}, 0 \right\}$$

$$q_p^{GL} = \frac{1}{2}(1 - q_g^{GL}).$$

We have to distinguish two cases since there exists a threshold value of the marginal cost of the public firm such that  $\forall c \in \left[ 0, \frac{1+2\lambda}{4(1+\lambda)} \right)$  the public firm produces a positive quantity in

equilibrium. When  $c \in \left[ \frac{1+2\lambda}{4(1+\lambda)}, \frac{1}{2} \right)$  the public firm prefers not to produce and the private

firm acts as a monopolist: its quantity, market price, and welfare are the same as in a private monopoly.

Since the threshold value  $\frac{1+2\lambda}{4(1+\lambda)}$  is increasing, as  $\lambda$  increases an higher level of

inefficiency is compatible with positive production by the public firm. Table 3 in the Appendix summarizes quantities, profits and welfare in the public leadership equilibrium.

*Private leadership (PL).*

Assume that the private firm moves before its public competitor, that is, it behaves as a leader in the Stackelberg game. The equilibrium quantities solve the following equation system:

$$q_p^{PL} = \operatorname{argmax} \Pi_p(r_g(q_p), q_p)$$

$$q_g^{PL} = r_g(q_p^{PL}).$$

The solution is:

$$q_p^{PL} = \min \left\{ \frac{1}{2} \frac{(c + \lambda + c\lambda)}{\lambda}, 1 - c \right\}$$

$$q_g^{PL} = \frac{1 + \lambda}{1 + 2\lambda} (1 - c - q_p^{PL}).$$

As before, we have two different cases depending on the value of  $c$ .  $\forall c \in \left[ 0, \frac{\lambda}{3\lambda + 1} \right)$  the public firm produces a positive quantity in equilibrium; more precisely, it is optimal for the private leader to choose a quantity such that the best response of the public firm is positive. When  $c \in \left[ \frac{\lambda}{3\lambda + 1}, \frac{1}{2} \right)$  the public firm does not produce in equilibrium and the private firm

produces the same quantity as an inefficient public monopolist: its quantity, as a limit level, is such that the market price is equal to the marginal cost of the public firm.<sup>13</sup> Of course total surplus is higher because the private competitor produces more efficiently.

Moreover, as  $\lambda$  increases, a larger inefficiency is compatible with a positive production by the public firm. Quantities, profits and welfare in the private leadership equilibrium are summarized in the Appendix, Table 4.

### 3.3 Endogenous timing equilibria

In this section we derive the endogenous timing equilibrium of the mixed duopoly game. In order to apply Lemma 3 we need to rank the payoffs of both firms in the different equilibria. More precisely, we need only to compare the payoff of the follower to the simultaneous-play payoff since, by the standard properties of Stackelberg equilibrium, both players always prefer to be leader rather than simultaneous-move player. Furthermore, the comparison between the payoff of the leader and the one if the follower is useless since no firm can unilaterally switch from one sequential equilibrium to the other.

In Lemma 4 we perform the comparison on the profit of the private firm, while in Lemma 5 we compare the welfare under private leadership with the one in the Nash equilibrium.

**Lemma 4** *There exists a subspace  $F_1 = (c, \lambda) \subseteq A$ , such that the private firm strictly prefers the public leadership equilibrium to the mixed duopoly Nash equilibrium. In the subspace  $\hat{F}_1 = A - F_1$  the reverse is true.*

**Lemma 5** *There exists a subspace  $F_2 = (c, \lambda) \subseteq A$ , such that the public firm strictly prefers the private leadership equilibrium to the mixed duopoly Nash equilibrium. In the subspace  $\hat{F}_2 = A - F_2$  the reverse is true.*

Theorem 6 provides the results for the endogenous timing equilibrium in the mixed duopoly.

**Theorem 6** *Consider a mixed duopoly game in which the order of moves is endogenous. The SPE of the endogenous timing game are defined in the following way:*

- i) When  $(c, \lambda) \in \hat{F}_2$ , the mixed duopoly Nash equilibrium is the unique SPE of the game;
- ii) When  $(c, \lambda) \in \hat{F}_1 \cap F_2$ , the unique SPE of the game is the Stackelberg equilibrium with the private firm acting as leader;

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<sup>13</sup>This is the standard case when  $\lambda = 0$ : the public follower can always produce the quantity needed to achieve this target and, anticipating this strategy, the best action for the private firm is to produce that quantity.

iii) When  $(c, \lambda) \in F_1$ , both Stackelberg outcomes are the (pure strategy) SPE of the game.

The results may be better understood when compared to those obtained in a private duopoly.<sup>14</sup> In the latter case, moving early is a dominant strategy for both private firms since the payoff of the leader is larger than the simultaneous-play payoff, which is in turn larger than the payoff of the follower. This result is driven by two properties of the objective functions of private firms:

- a) the profit of any private firm is strictly decreasing in the output of the rival;
- b) reaction functions are downward sloping.

So, when leader, any private firm increases its own production with respect to Cournot output in order to induce a reduction in the output of the rival. As a result, the follower is always worse off and strictly prefers to be simultaneous player rather than follower. Therefore, the conditions of point i) of Lemma 3 apply.

In the mixed duopoly described in the present paper the objective function of the private firm has the same properties highlighted above. However, the one of the public firm has different features:

- a) welfare function (4) may be either decreasing or increasing in the output of the private firm;
- b) the reaction function of the public firm is downward sloping.

When welfare is *increasing* in the output of the private firm, the public leader decreases its own output with respect to Cournot equilibrium in order to induce an increase in private production. The private follower is then better off with respect to Cournot equilibrium. When the private firm is the leader, it always increases its own output in order to induce a contraction in the output of the public firm. However, welfare will be positively affected and the public firm will prefer to be follower than simultaneous player. As a result, the conditions of point ii) of Lemma 3 apply and both Stackelberg equilibria are SPE of the endogenous timing game, as stated at point iii) of Theorem 6.

It is worth noting that under public leadership welfare increases with respect to Cournot equilibrium despite a reduction in total quantity.<sup>15</sup> This result occurs when the increase in productive efficiency due to the shift of some production to the private firms outweighs the negative allocative efficiency effect due to the reduction in total quantity.

Under private leadership, on the contrary, both private production and total output increase; therefore allocative and productive efficiency rise. Then, we would expect that the space of parameters such that the public firm prefers to be follower rather than simultaneous

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<sup>14</sup>See Amir and Grilo (1999).

<sup>15</sup>The public leader reduces its quantity and the private follower increases it, but by a smaller amount since the reaction functions are contraction.

player is larger than the one where the private firm does. This explains the result at point ii) of Theorem 6 where private leadership is the unique SPE of the endogenous timing game.

Finally, when the welfare function (4) is decreasing in the output of the private firm, the same equilibrium as a private duopoly is selected; this explains the result at point i) of Theorem 6.

The introduction of the shadow cost of public funds is the key determinant of the difference of our results with respect to Pal (1998). In his work simultaneous play is never an equilibrium because the objective function of the public firm is always increasing in the output of the rival. The novel contribution of our analysis is to enlarge the set of possible outcomes. Figure 3 depicts the three possible outcomes of the endogenous timing game in the space  $(c, \lambda)$ .

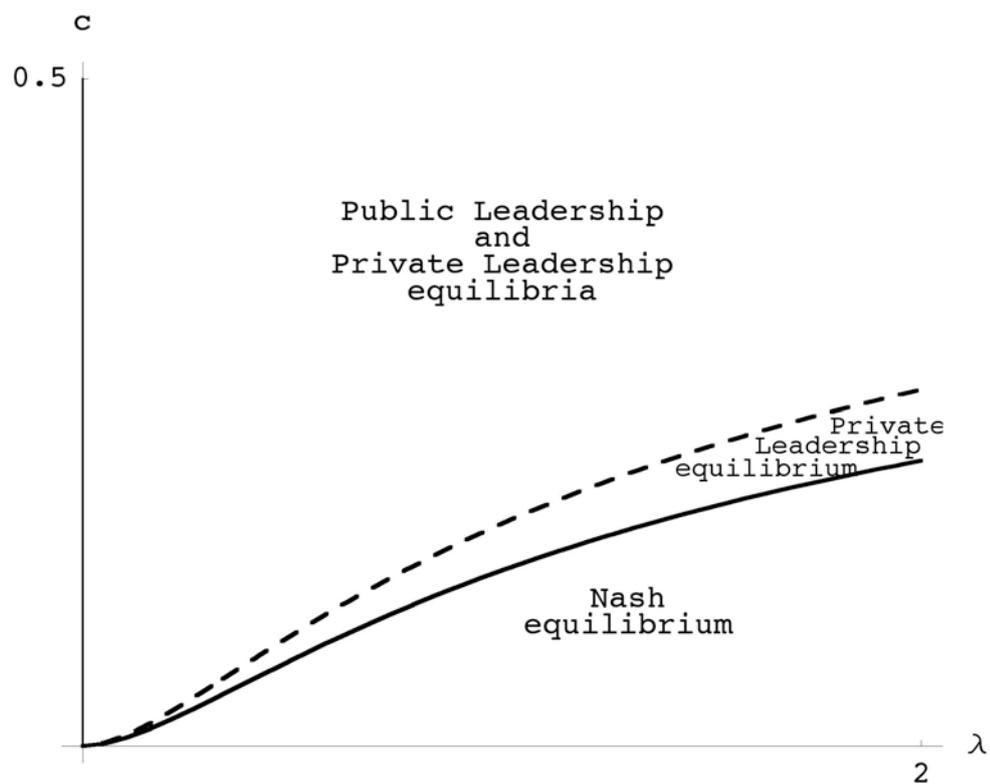


Figure 3: SPE of the endogenous timing game in the space of parameters  $(c, \lambda)$ .

Moreover, the public firm in Pal (1998) generally does not produce in equilibrium and essentially acts as a disciplinary device that forces the private firms to produce the limit quantity in order to avoid entry.<sup>16</sup> In our framework, taking into account the shadow cost of public funds, the public firm usually produces a positive quantity in equilibrium. This result enhances the realism of our approach where the public firm represents not only a threat of producing, but it has an active role in the industry. The following corollary define the conditions under which the public firm is active on the market.

<sup>16</sup>In particular, in the private leadership case, the author shows that the public firm never produces.

**Corollary 7** *In a mixed duopoly, when the public firm maximizes welfare taking into account the shadow costs of public funds  $\lambda > 0$ :*

- i) whenever the mixed duopoly Nash equilibrium is the unique SPE of the endogenous timing game the public firm produces a positive quantity;
- ii) when private leadership arises in the SPE of the endogenous timing game, the public firm produces positive quantities when  $c < \frac{\lambda}{3\lambda + 1}$ ;
- iii) when public leadership arises in the SPE of the endogenous timing game, the public firm produces positive quantities when  $c < \frac{1 + 2\lambda}{4(1 + \lambda)}$

**Proof.** See Tables 2, 3, and 4 in the Appendix. ■

### 3.4 Equilibrium selection

In the subspace  $F_1$  the endogenous timing game of the mixed duopoly has two pure-strategy Nash equilibria. Then, this is a standard coordination game with two pure-strategy Nash equilibria and one mixed-strategy equilibrium. We now analyze the pure-strategy equilibrium selection problem according to the risk dominance criterion developed by Harsanyi and Selten (1988).<sup>17</sup> Applied to coordination games with two pure-strategy equilibria, this procedure picks the equilibrium that has the largest basin of attraction in the initial beliefs of each player on the behavior of the other player. In other words, it minimizes the risk of a coordination failure due to the strategic uncertainty that players face in a coordination game (Amir and Stepanova, 2006). This criterion proved to be a powerful selection concept in experiments of coordination games (Cooper et al., 1990; Van-Huyck et al., 1990) and in evolutionary games characterized by experimentation and myopic learning (Ellison, 1993; Kandori et al., 1993). It is shown that the risk dominant equilibrium is often selected even when it is Pareto dominated by another equilibrium. One equilibrium risk-dominates the other if it is associated with the larger product of deviation losses. In our framework this means that private leadership is selected using the risk-dominance criterion if

$$(W^{PL} - W^{MN})(\Pi_p^{PL} - \Pi_p^{MN}) > (W^{GL} - W^{MN})(W^{GL} - \Pi_p^{MN}). \quad (8)$$

**Theorem 8** *The private leadership equilibrium risk-dominates the public leadership equilibrium  $\forall (c, \lambda) \in F_1$ .*

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<sup>17</sup>In the present setting it is possible to show that the standard refinements of Nash equilibrium for normal form games - perfection, properness and strategic stability - cannot be invoked to rule out one of the pure strategy SPE. Furthermore, Pareto dominance selects the private leadership equilibrium only in a subspace of  $F_1$ .

The risk dominance criterion selects in the whole set  $F_1$  the same equilibrium that the Pareto dominance criterion is able to select only in a subspace of  $F_1$ . It is important to highlight that the risk-dominance criterion is applied to the reduced game, and not to the entire two-stage game of endogenous timing, and the two options are a priori entirely different. However, since each subgame has a unique Nash equilibrium and given the use of subgame perfection in this framework, our application of the risk-dominance criterion on the reduced game seems to us rather natural.<sup>18</sup> Amir and Stepanova (2006) suggest the following interpretation: the private leadership equilibrium is chosen by firms that wish to minimize the risk of coordination failure in their timing decisions.

The preference for the private leadership equilibrium is the main contribution in Beato and Mas-Colell (1984), where it is *assumed* that the public firm is committed to a decision rule (in their case the marginal-cost pricing rule), and the private firm maximizes its own profit given the decision rule of the public competitor. In the present setting, using the game with observable delay of Hamilton and Slutsky (1990) coupled with risk dominance as a selection criterion, we show that the private leadership equilibrium emerges as the *endogenous* equilibrium in the mixed duopoly  $\forall (c, \lambda) \in F_2$ .

## 4 Welfare effect of privatization

In this Section we perform a comparative statics exercise in order to analyze the effects of privatization on welfare taking into account the result of the previous Section on the endogenous timing equilibrium.

By privatization we mean that the public firm is sold by the government and the management is instructed by the new owners to maximize profits:<sup>19</sup>

$$\Pi_g(q_g, q_p) = [p(q_g, q_p) - c]q_g.$$

This change in ownership might have in principle the effect of enhancing the productive efficiency of the former public firm. We consider the two extreme cases in which either no efficiency gain or full efficiency are achieved. In the first case, the privatized firm retains the same technology as before; in the latter, it is able to produce at the same marginal cost of its competitor, here normalized to zero. After privatization, the new reaction function of firm  $g$  is:

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<sup>18</sup>See van Damme and Hurkens (2004) and Amir and Stepanova (2006) for the application of the risk-dominance criterion on the reduced game of endogenous timing models in price game duopolies.

<sup>19</sup>As in the mixed duopoly case, we assume that the fixed cost is sunk and already paid by the government. So, it is included in the welfare, weighted  $(1 + \lambda)$ , but not in the profit of the privatized firm. Moreover, For simplicity, we keep the same subscripts as in the mixed oligopoly framework. From now on,  $g$  stands for the privatized firm.

$$r_g(q_p) = \max\left\{\frac{1}{2}(1-c-q_p), 0\right\} \quad (9)$$

with  $c = 0$  in the case of full efficiency gains. Comparing the reaction function before and after privatization, it is easy to see that it becomes steeper. Indeed:

$$\frac{1+\lambda}{1+2\lambda} > \frac{1}{2} \quad \forall \lambda \in (0, \bar{\lambda})$$

and only when  $\lambda \rightarrow \infty$  the slope of the reaction function (7) of the public firm converges to the slope of the reaction function (6) of the private firm.

Absent efficiency gains, also the intercept is reduced after privatization. With full efficiency gains the intercept increases only when  $c > \frac{1}{2(1+\lambda)}$ .

The change in the reaction function is not the sole effect of privatization. In fact, we have to consider the possible change in the endogenous timing equilibrium. In order to derive the SPE of the game after privatization, we can rely on the work of Amir and Grilo (1999) who apply the same endogenous timing structure to a private duopoly. The following Theorem summarizes the results in the framework of the present paper.

**Theorem 9** *Consider a private duopoly quantity game with strategic substitutes. When the values of the parameters are in the admissible set  $A$ , the unique SPE of the endogenous timing game is the Cournot-Nash equilibrium where both firms decide to move early.*

**Proof of Theorem.** Under Assumptions 1 and 2 no Nash equilibrium lies on the boundary, i.e. no firm produces zero output. In this case we can apply Theorem 2.2 in Amir and Grilo (1999) that proves that both firms prefer always to be simultaneous player rather than Stackelberg follower. So, according to point i) of Lemma 3, the unique SPE of the endogenous timing game is the Cournot-Nash equilibrium. ■

The intuition for this result is clear. Since the profit is strictly decreasing in the output of the rival, a private leader always increases its own quantity in comparison with the Cournot-Nash quantity. By the same reason, a private follower is always strictly worse off with respect to the Cournot-Nash equilibrium. Then, sequential play is only sustainable in a mixed duopoly.

The Cournot-Nash equilibrium is the solution of the system of equations (6) and (9). Quantities and price are:<sup>20</sup>

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<sup>20</sup>Superscript CN denotes the Cournot-Nash equilibrium.

$$q_g^{CN} = \frac{1-2c}{3}; \quad q_p^{CN} = \frac{1+c}{3}$$

$$Q^{CN} = \frac{2-c}{3}; \quad p^{CN} = \frac{1+c}{3}.$$

In our analysis the new owners are always national. The (domestic) total surplus and the profit of the privatized firm are:

$$V^{CN} = \frac{8-8c+11c^2}{18} - K_g - K_p; \quad \Pi_g^{CN} = \left(\frac{1-2c}{3}\right)^2$$

Recall that in the case of full efficient privatization  $c = 0$ .

In order to compare welfare before and after privatization, the price paid to the government for the privatized firm matters. Since we are taking into account the shadow cost of public funds, it is not indifferent whether profits are public or private, and if the government is able to raise enough money from privatization. Given the equilibrium after privatization, the more money the government is able to raise by selling the public firm, the higher the welfare after the privatization. In the first instance, we give full bargaining power to the government; i.e., we assume that it is able to extract the whole profit from the privatized firm. In such a case, total welfare in the Cournot-Nash equilibrium is:

$$W^{CN} = \frac{1}{18} \left( 2\lambda + 11c^2 + 8 - 8c(1 + \lambda - c\lambda) \right) - (1 + \lambda)K_g - K_p. \quad (10)$$

The following Theorem states the result of the comparison when no efficiency gain occurs after privatization.

**Theorem 10** *Consider a mixed duopoly game in which the order of moves is endogenous. In addition, assume that by privatization the firm does not achieve any efficiency gain. Then, privatization always reduces welfare even when all the profits are extracted by the government.*

This result crucially relies on the endogeneity of the timing of competition before and after privatization. In fact, when the public firm is highly inefficient, the endogenous timing equilibrium is characterized by private leadership (see Figure 3 and Theorem 8). Therefore, the effect of privatization consists not only in a change of the objective function of the former public firm, but also in a shift from sequential to simultaneous play. Since private leadership implements a welfare superior allocation, privatization can never improve total welfare absent efficiency gains. This reasoning does not apply when  $\lambda$  is high and Cournot-Nash equilibrium occurs both before and after privatization. In such a case, however, the public firm is already mimicking the behaviour of a private firm and privatization does not improve

the situation.<sup>21</sup>

Our result is in sharp contrast with those obtained *assuming* simultaneous playing. For example, de Fraja and Delbono (1989) show that, assuming Cournot competition, a change in the objective function of the private firm from welfare to profit maximization may enhance welfare absent efficiency gains.<sup>22</sup> The same result holds in the framework of the present paper. Disregarding the endogenous timing game, and comparing  $W^{MN}$  from Table 2 and  $W^{CN}$  from equation (10)  $\forall (c, \lambda) \in A$ , privatization may increase welfare. More precisely, it occurs when

$$c > \frac{4\lambda + 6\lambda^2 + 1}{26\lambda + 12\lambda^2 + 8}.$$

The latter result is true even when the shadow cost of public funds is disregarded (i.e.,  $\lambda = 0$ ) and therefore crucially relies on the assumption of simultaneous move game.<sup>23</sup> It is also important to notice that the assumption of linear cost is not crucial for the result. From Theorem 6 it is clear that the endogenous timing equilibrium is essentially determined by the objective function of the public firm being either increasing or decreasing in the production of the private firm. The latter condition is not affected by the shape of the cost function.

Now, we move to the analysis of the other extreme case: full efficient privatization. The following Theorem formalizes the result.

**Theorem 11** *Consider a mixed duopoly game in which the order of moves is endogenous. In addition, assume that by privatization the firm achieves full efficiency and all the profits are extracted by the government. Then, there exists a subset of the parameter  $J \subset A$ , such that the privatization reduces welfare.*

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<sup>21</sup>Actually, in such cases privatization may worsen the profitability of the firm. Under simultaneous-move quantity competition it is easy to show that the profits of a firm that maximizes a weighted average of welfare and profits may be larger than those of a firm maximizing profits only.

<sup>22</sup>This result is obtained in a different setting with symmetric firms and increasing marginal costs.

<sup>23</sup>A difference between our model and the one in de Fraja and Delbono (1989) is that in the latter the number of private firms matters for the equilibrium, and a shift toward profit maximization is welfare improving in a more competitive market. In the present model, the level of competition affects the results in two opposite directions. On one side, a more competitive environment reduces the allocative negative effect of a reduction in the production of the privatized firm. On the other side, following Pal (1998), a larger number of private competitors allows the selection of the private leadership equilibrium that is generally welfare superior to all the other possible outcomes. More work is needed to assess the relative importance of the two effects.

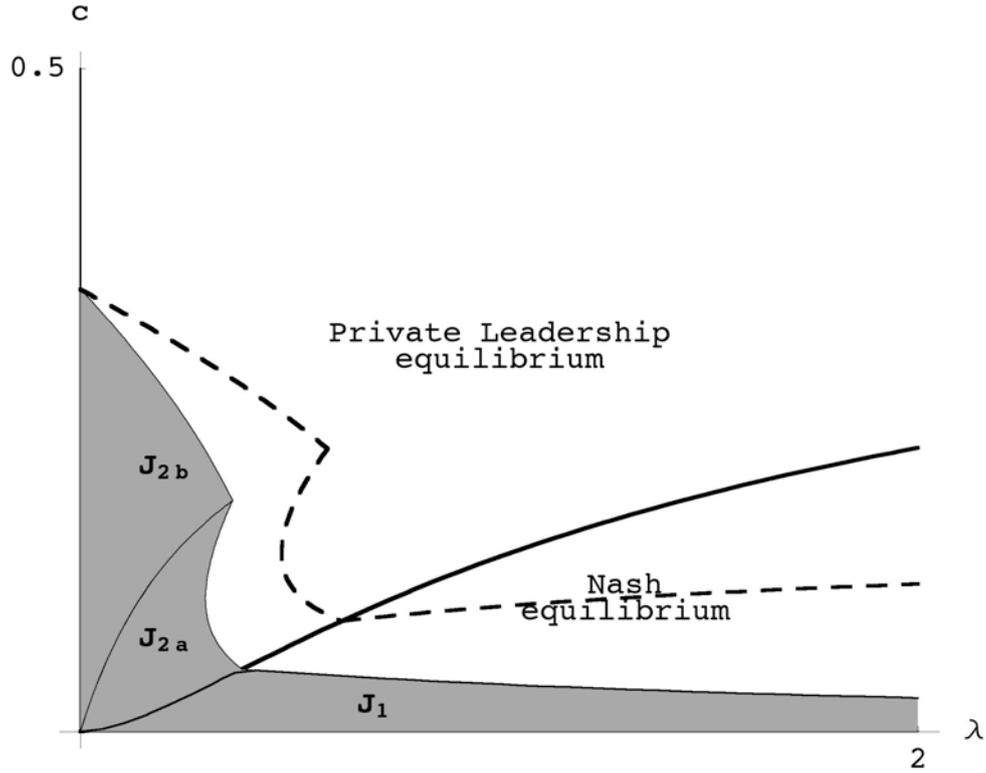


Figure 4: The welfare effect of a full efficient privatization: in gray the space of the parameters in  $(c, \lambda)$  where privatization reduces welfare when the government extracts all the profits from the privatized firm.

In Figure 4 we graph the set  $J$  in the space of parameters where a fully efficient privatization reduces welfare. Endogenizing the timing of competition, before and after privatization, enlarges this space with respect to the simultaneous case. In fact, it is easy to show that assuming simultaneous competition privatization reduces welfare if

$$c < \frac{3(1+2\lambda)^2 - (1+3\lambda)\sqrt{2(3+8\lambda(1+\lambda))}}{3(1+\lambda)(3+8\lambda)}.$$

It is interesting to notice that the level of  $c$  such that public ownership is the dominant solution in terms of welfare is decreasing in  $\lambda$ . Thus, the more the public firm behaves as a profit maximizer, the better is to privatize it.

This result is obtained assuming that the government is able to extract the whole profit from the new owners of the privatized firm. Suppose now that the government is able to take just half of the profit. In Figure 4 we can see how the space of the parameters such that the privatization reduces welfare is enlarged. The dashed line delimits the space of a welfare-reducing full efficient privatization when the government sells the public firm at a price equal to half of the future profits.

An extreme result occurs when the firm is sold for free. In this latter case, a full efficient privatization always lowers welfare.

## 5 Conclusions

The aim of the present work is to characterize the equilibrium and analyze the effect of privatization in a mixed duopoly where an inefficient welfare-maximizing public firm competes in the quantities with a domestic private one.

We do not assume the timing of competition *a priori*. Rather, we endogenize the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games by applying the model of Hamilton and Slutsky (1990) to this mixed duopoly framework.

Since we mainly refer to public utility markets open to competition, we assume that the production technology is characterized by increasing return to scale, with fixed cost and constant marginal cost. In this framework, we assume that the management of the public firm is instructed to maximize welfare taking into account the shadow cost of public funds. As the following citation suggests, this approach has been generally used to characterize public monopolies running a deficit.

*[M]any public enterprises are natural monopolies, i.e. firms that exhibit increasing returns to scale. Once it has been proved desirable to run such an enterprise at all, its product should be priced at marginal cost provided the resulting deficit can be financed through lump-sum taxes. If there are not lump-sum, discrepancies between consumer and producer taxes will result in inefficiencies in the rest of the economy. (...) This has been taken as an argument for requiring the public enterprise to cover, by its own means, at least part of its deficit. (Marchand et al., 1984)*

We believe that extending this approach to the mixed duopoly framework is rather natural and fills, at least partially, some gaps of the previous literature. Indeed, discussing the results of their paper, Beato and Mas-Colell (1984, p. 82) state:

*Finally, the limitations of this paper and the need for further work should be clear. We have, for example, ruled out both fixed cost and the general equilibrium effects of distortions in other markets. We do not know if reasonable versions of the main results of this paper [...] are available in these richer settings.*

The extensive process of privatization started in the eighties of the last century and still in place nowadays is essentially driven by the belief that private discipline and profit motivation can enhance efficiency. Moreover, privatization is also considered as a powerful instrument to raise money to reduce distortionary taxation. In this work we contrast the general extent of these ideas. We show that, absent efficiency gains, privatization never

increases welfare, and that an inefficient public firm may be preferred even when large efficiency gains could be realized by privatization. These results are obtained assuming that both the profit of the public firm and the privatization proceeds are substitute for distortionary taxation. The endogenous timing model applied to the mixed oligopoly framework is not less important for our results. While after privatization only the simultaneous (Nash-Cournot) equilibrium can be implemented, with a public firm sequential equilibria – that are always welfare superior – may be sustained as SPE of the endogenous timing game. Therefore, privatization changes not only the ownership and the objective function of the public firm, but also the type of competition in the market.

Finally, the assumption of larger marginal cost for the public firm deserves a last comment. In our model we follow the general presumption that public ownership is relatively inefficient when compared to private ownership. This has been justified by the theory of incentives that has been used to demonstrate that agency problems in state-owned enterprises can cause larger inefficiencies than in private-owned firms. But we are aware that from an empirical point of view the picture is quite mixed and the variance of the results substantial (Cuervo and Villalonga, 2000).<sup>24</sup> However, any relaxation of our assumption obviously strengthen the results obtained in this paper.

## Appendix

The Appendix contains all the proofs of the paper and the Tables showing the equilibrium values in the different basic games analyzed.

**Proof of Lemma 4.** Comparing the equilibrium profits  $\Pi_p^{GL}$  in Table 3 with  $\Pi_p^{MN}$  in Table 2, it easy to check that  $\forall \lambda \geq 0$ :

$$i) \forall c \in \left(0, \frac{1+2\lambda}{4(1+\lambda)}\right)$$

$$\Pi_p^{GL} - \Pi_p^{MN} > 0 \quad \forall c > \bar{c}(\lambda)$$

where  $\bar{c}(\lambda) = \frac{\lambda^2}{2\lambda^2 + 3\lambda + 1}$  with  $\frac{\partial \bar{c}(\lambda)}{\partial \lambda} > 0$ .

$$ii) \forall c \in \left[\frac{1+2\lambda}{4(1+\lambda)}, \frac{1}{2}\right), \Pi_p^{GL} - \Pi_p^{MN} > 0 \text{ always.}$$

Thus, we define the subspace  $F_1$  and  $\hat{F}_1$  as follows:

$$F_1 = \{(c, \lambda) \subseteq A \mid c > \bar{c}(\lambda)\} \quad \text{and} \quad \hat{F}_1 = \{(c, \lambda) \subseteq A \mid c \leq \bar{c}(\lambda)\}. \quad (11)$$

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<sup>24</sup>See for example the reviews of Megginson and Netter (2001) and Willner (2001) that report the results of hundreds of empirical papers on privatization and on the comparison of private and public ownership, and Newbery (2000, chapter 3) that summarizes empirical findings on the technical and economic efficiency of private and public firms in utility markets.

■

**Proof of Lemma 5 .** Comparing the welfare level  $W^{PL}$  in Table 4 with  $W^{MN}$  Table 2, it easy to check that  $\forall \lambda \in (\bar{\lambda})$ :<sup>25</sup>

$$\text{i) } \forall c \in \left(0, \frac{\lambda}{3\lambda+1}\right)$$

$$W^{PL} - W^{MN} > 0 \quad \forall c > \underline{c}(\lambda)$$

where

$$\underline{c}(\lambda) = \frac{3\lambda^2 + 7\lambda^3}{21\lambda + 34\lambda^2 + 17\lambda^3 + 4} \quad \text{with } \frac{\partial \underline{c}(\lambda)}{\partial \lambda} > 0.$$

$$\text{ii) } \forall c \in \left[\frac{\lambda}{3\lambda+1}, \frac{1}{2}\right),$$

$$W^{PL} - W^{MN} > 0 \quad \forall \lambda \in (0, \bar{\lambda}).$$

Thus, we define the subspace  $F_2$  and  $\hat{F}_2$  as follows:

$$F_2 = \{(c, \lambda) \subseteq A \mid c > \underline{c}(\lambda)\} \quad \text{and} \quad \hat{F}_2 = \{(c, \lambda) \subseteq A \mid c \leq \underline{c}(\lambda)\} \quad (12)$$

■

**Proof of Theorem 6.**

i) When  $(c, \lambda) \in \hat{F}_1$  the private firm prefers the mixed duopoly Nash equilibrium to the public leadership equilibrium. When  $(c, \lambda) \in \hat{F}_2$  the public firm is better off in the Nash equilibrium than in the private leadership equilibrium. Therefore, in the intersection space  $\hat{F}_1 \cap \hat{F}_2$ , no firm wants to be follower. Since  $\forall \lambda \in (0, \bar{\lambda})$ ,  $\underline{c}(\lambda) < \bar{c}(\lambda)$ , it follows that  $\hat{F}_2 \subset \hat{F}_1$ ; then  $\hat{F}_1 \cap \hat{F}_2$  coincides with  $\hat{F}_2$ . Given that each player always prefers to be the Stackelberg leader rather than a simultaneous player, point i) of Lemma 3 applies and the mixed duopoly Nash equilibrium is the unique SPE of the endogenous timing game.

ii) When  $(c, \lambda) \in \hat{F}_1$  the private firm prefers the mixed duopoly Nash equilibrium to the public leadership equilibrium. When  $(c, \lambda) \in F_2$  the public firm is better off in the private leadership equilibrium rather than in the mixed duopoly Nash equilibrium. So, point iii) of Lemma 3 applies and the Stackelberg equilibrium with the private firm acting as leader is the unique SPE of the endogenous timing game.

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<sup>25</sup>The threshold  $\underline{c} < \lambda / (3\lambda + 1) \quad \forall \lambda < 5.37228$ . Since  $\lambda$  is a measure of the distortion by taxation, we are comfortable assuming that  $\bar{\lambda}$  is lower than 5.37228. If  $\lambda \geq 5.37228$  we would have that  $W^{PL} < W^{MN}$  always.

iii) When  $(c, \lambda) \in F_1$  the private firm prefers to play as Stackelberg follower rather than to play simultaneously. When  $(c, \lambda) \in F_2$  the public firm prefers to play as Stackelberg follower rather than playing simultaneously. Since  $\forall \lambda > 0$ ,  $\underline{c}(\lambda) < \bar{c}(\lambda)$ , it follows that  $F_1 \subset F_2$ ; then  $F_1 \cap F_2$  coincides with  $F_1$ . So, point ii) of Lemma 3 applies and both Stackelberg equilibria belong to the set of the (pure strategy) SPE of the endogenous timing game.

■

**Proof of Theorem 8.** In order to prove the result we need to consider three cases depending on the fact that boundary solutions may occur in the two sequential equilibria. By comparing the thresholds defined in Section 3.2, we have the following equilibria:

i) when  $(c, \lambda) \in F_1$  and  $c < \frac{\lambda}{1+3\lambda}$ , both Stackelberg equilibria are interior. Then, the values of  $W^{GL}$ ,  $\Pi_p^{GL}$ ,  $W^{PL}$  and  $\Pi_p^{PL}$  of interest are those in the first row of Tables 3 and 4.

ii) when  $(c, \lambda) \in F_1$  and  $\frac{\lambda}{1+3\lambda} < c < \frac{1+2\lambda}{4(1+\lambda)}$ , the public firm does not produce in the private leadership equilibrium while it produces positive quantity in the public leadership equilibrium. Then, the values of  $W^{GL}$  and  $\Pi_p^{GL}$  of interest are those in the first row of Table 3, while for  $W^{PL}$  and  $\Pi_p^{PL}$  we have to consider the values in the second row of Table 4.

iii) when  $(c, \lambda) \in F_1$  and  $c > \frac{1+2\lambda}{4(1+\lambda)}$ , the public firm does not produce in both Stackelberg equilibria. Then, the values of  $W^{GL}$ ,  $\Pi_p^{GL}$ ,  $W^{PL}$  and  $\Pi_p^{PL}$  of interest are those in the second row of Tables 3 and 4.

Applying the criterion (8), straightforward but tedious computations show the result.

■

**Proof of Theorem 10.** In order to prove the result, we need to consider three cases: (i) Nash is the relevant equilibrium of the mixed duopoly; (ii) private leadership is the relevant equilibrium of the mixed duopoly with an interior solution; and (iii) private leadership with the public firm not producing is the relevant equilibrium of the mixed duopoly.

(i) By point i) in Theorem 6 Nash is the relevant equilibrium of the mixed duopoly game when  $(c, \lambda) \in \hat{F}_2$ . So, we have to compare  $W^{MN}$  from Table 2 with  $W^{CN}$  (equation 10). It is easy to show that

$$W^{MN} > W^{CN} \quad \forall (c, \lambda) \in \hat{F}_2.$$

(ii) By points ii) and iii) in Theorem 6, and Theorem 8 the Stackelberg outcome with the private firm as leader is the relevant SPE of the mixed duopoly game when  $(c, \lambda) \in F_2$ . Moreover, when  $c < \frac{\lambda}{3\lambda + 1}$  the public firm produces positive quantity in

the equilibrium. Then, we have to compare the value of  $W^{PL}$  in the first row of Table 4 with  $W^{CN}$  (equation 10). Straightforward computations show that

$$W^{PL} > W^{CN} \quad \forall (c, \lambda) \in F_2, \quad c < \frac{\lambda}{3\lambda + 1}.$$

(iii) When  $c \geq \frac{\lambda}{3\lambda + 1}$ , the public firm does not produce in the private leadership

equilibrium. Thus, we have to compare the value of  $W^{PL}$  in the second row of Table 4 with  $W^{CN}$  (equation 10). It shows that

$$W^{PL} > W^{CN} \quad \forall (c, \lambda) \in F_2, \quad c > \frac{\lambda}{3\lambda + 1}.$$

■

**Proof of Theorem 11.** When the privatized firm achieves full efficiency gains, welfare after privatization is:

$$W^{CN}|_{c=0} = \frac{4 + \lambda}{9} - (1 + \lambda)K_g - K_p. \quad (13)$$

In order to prove the result we have to distinguish between three cases as in Theorem 10.

(i) By point i) in Theorem 6 Nash is the relevant equilibrium of the mixed duopoly game when  $(c, \lambda) \in \hat{F}_2$ . So, we have to compare  $W^{MN}$  from Table 2 with  $W^{CN}$  (equation 13). Straightforward computations show that

$$W^{MN} \geq W^{CN}|_{c=0} \quad \forall (c, \lambda) \in \hat{F}_2, \quad c < \frac{3(1 + 2\lambda)^2 - (1 + 3\lambda)\sqrt{2(3 + 8\lambda(1 + \lambda))}}{3(1 + \lambda)(3 + 8\lambda)}.$$

Thus, we can define the subset

$$J_1 = \left\{ (c, \lambda) \in \hat{F}_2 \mid c < \frac{3(1 + 2\lambda)^2 - (1 + 3\lambda)\sqrt{2(3 + 8\lambda(1 + \lambda))}}{3(1 + \lambda)(3 + 8\lambda)} \right\}.$$

Referring to the definition of the subset  $\hat{F}_2$  in (12), it is easy to check that  $J^1$  is a non-empty set.

(ii) By points ii) and iii) in Theorem 6, and Theorem 8 the private leadership equilibrium is the relevant SPE of the mixed duopoly game when  $(c, \lambda) \in F_2$ .

Moreover, when  $c < \frac{\lambda}{3\lambda+1}$  the public firm produces positive quantity in the equilibrium. Then, we have to compare the value of  $W^{PL}$  in the first row of Table 4 with  $W^{CN}$  (equation 13). First of all, define

$$F_{2a} = \left\{ (c, \lambda) \in F_2 \mid c < \frac{\lambda}{3\lambda+1} \right\}.$$

It is easy to show that:

$$W^{PL} \geq W^{CN} \Big|_{c=0}$$

$$\forall (c, \lambda) \in F_{2a}, \quad 9c^2(1+\lambda)^2(4+9\lambda) - 18c\lambda(1+\lambda)(2+3\lambda) + 4\lambda - 7\lambda^3 > 0.$$

Thus we can define the subset

$$J_{2a} = \left\{ (c, \lambda) \in F_{2a} \mid 9c^2(1+\lambda)^2(4+9\lambda) - 18c\lambda(1+\lambda)(2+3\lambda) + 4\lambda - 7\lambda^3 > 0 \right\}$$

that is non-empty.

(iii) Defining

$$F_{2b} = \left\{ (c, \lambda) \in F_2 \mid c \geq \frac{\lambda}{3\lambda+1} \right\}$$

the public firm does not produce in the private leadership equilibrium. Then, we have to compare the value of  $W^{PL}$  in the second row of Table 4 with  $W^{CN}$  (equation 13). Straightforward computations show that the subset  $J_{2b} \subset F_{2b}$  such that privatization reduces welfare is not empty:

$$J_{2b} = \left\{ (c, \lambda) \in F_{2b} \mid c < \frac{1}{3}\sqrt{1-2\lambda} \Leftrightarrow W^{PrL} - W^{FE} \geq 0 \right\}.$$

Then, the subset of the space of parameters such that a full efficient privatization with full bargaining power to the government reduces welfare is the following:

$$J = J_1 \cup J_{2a} \cup J_{2b}.$$

■

Finally, we show the Tables 2, 3, and 4 that summarize the equilibrium outcome in the cases of: (i) simultaneous move game, (ii) public leadership, and (iii) private leadership.

$q_p^{MN}$	$q_g^{MN}$	$\Pi_p^{MN}$
$c + \lambda \frac{(1-2c)}{(3\lambda+1)}$	$(1+\lambda) \frac{(1-2c)}{(3\lambda+1)}$	$\left( \frac{c + \lambda(1+c)}{3\lambda+1} \right)^2 - K_p$
$W^{MN}$		
$\frac{1-2c(1+\lambda)(1+2\lambda)^2 + c^2(1+\lambda)^2(3+8\lambda) + 2\lambda(3+\lambda(5+\lambda))}{2(3\lambda+1)^2} - (1+\lambda)K_g - K_p$		

Table 2: The mixed duopoly Nash equilibrium (MN) quantities, profits and welfare.

	$q_p^{GL}$	$q_g^{GL}$	$\Pi_p^{GL}$
$c < \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{2c(1+\lambda)+\lambda}{(1+4\lambda)}$	$\frac{(1+2\lambda)-4c(1+\lambda)}{(1+4\lambda)}$	$\frac{(2c(1+\lambda)+\lambda)^2}{(1+4\lambda)^2} - K_p$
$c \geq \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{1}{2}$	0	$\frac{1}{4} - K_p$
$W^{GL}$			
$c < \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{1}{2} \frac{(4\lambda - 2c - 6c\lambda + 4c^2 + \lambda^2 - 4c\lambda^2 + 8c^2\lambda + 4c^2\lambda^2 + 1)}{(1+4\lambda)} - (1+\lambda)K_g - K_p$		
$c \geq \frac{1+2\lambda}{4(1+\lambda)}$	$\frac{3}{8} - (1+\lambda)K_g - K_p$		

Table 3: The public leadership (GL) equilibrium quantities, profits and welfare.

	$q_p^{PL}$	$q_g^{PL}$	$\Pi_p^{PL}$
$c < \frac{\lambda}{3\lambda+1}$	$\frac{1}{2} \frac{(c+\lambda+c\lambda)}{\lambda}$	$\frac{(\lambda-c-3c\lambda)(1+\lambda)}{2\lambda(1+2\lambda)}$	$\frac{1}{4} \frac{(c+\lambda+c\lambda)^2}{\lambda(2\lambda+1)} - K_p$
$c \geq \frac{\lambda}{3\lambda+1}$	$1-c$	0	$c(1-c) - K_p$
$W^{PL}$			
$c < \frac{\lambda}{3\lambda+1}$	$\frac{(4\lambda - 4c\lambda + 4c^2 + 8\lambda^2 + \lambda^3 - 10c\lambda^2 + 17c^2\lambda - 6c\lambda^3 + 22c^2\lambda^2 + 9c^2\lambda^3)}{8(2\lambda+1)\lambda} - (1+\lambda)K_g - K_p$		
$c \geq \frac{\lambda}{3\lambda+1}$	$\frac{1}{2} - \frac{1}{2}c^2 - (1+\lambda)K_g - K_p$		

Table 4: The private leadership (PL) equilibrium values of quantities, profits and welfare.

## References

- Amir, Rabah and Anna Stepanova, "Second-mover advantage and price leadership in Bertrand duopoly," *Games and Economic Behavior*, April 2006, 55 (1), 1–20.
- Amir, Rabah and Isabel Grilo, "Stackelberg versus Cournot Equilibrium," *Games and Economic Behavior*, January 1999, 26 (1), 1–21.

- Barros, Fatima, "Incentive schemes as strategic variables: An application to a mixed duopoly," *International Journal of Industrial Organization*, September 1995, 13 (3), 373–386.
- Beato, Paulina and Andreu Mas-Colell, "The Marginal Cost Pricing Rule as a Regulation Mechanism in Mixed Markets," in Maurice Marchand, Pierre Pestieu, and Henry Tulkens, eds., *The Performance of Public Enterprises*, North-Holland, Amsterdam, 1984.
- Bös, Dieter, *Public Enterprise Economics: Theory and Applications*, Vol. 23 of *Advanced Textbooks in Economics*, Amsterdam & New York: North Holland, 1986.
- Cooper, Russell W., Douglas V. DeJong, Robert Forsthe, and Thomas W. Ross, "Selection criteria in coordination games: Some experimental results," *American Economic Review*, March 1990, 80 (1), 218–233.
- Cornes, Richard C. and Mehrdad Sepahvand, "Cournot Vs Stackelberg Equilibria with a Public Enterprise and International Competition," Working Paper Series, SSRN 2003.
- Cremer, Helmuth, Maurice Marchand, and Jacques-Francois Thisse, "The Public Firm as an Instrument for Regulating an Oligopolistic Market," *Oxford Economic Papers*, April 1989, 41 (2), 283–301.
- Cuervo, Alvaro and Belen Villalonga, "Explaining the Variance in the Performance Effects of Privatization," *Academy of Management Review*, July 2000, 25 (3), 581–590.
- de Fraja, Giovanni and Flavio Delbono, "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, April 1989, 41 (2), 302–311.
- Ellison, Glenn, "Learning, Local Interaction, and Coordination," *Econometrica*, September 1993, 61 (5), 1047–1071.
- Fjell, Kenneth and Debashis Pal, "A mixed oligopoly in the presence of foreign private firms," *Canadian Journal of Economics*, August 1996, 29 (3), 737–743.
- Hamilton, Jonathan H. and Steven M. Slutsky, "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria," *Games and Economic Behavior*, March 1990, 2 (1), 29–46.
- Harsanyi, John C. and Reinhard Selten, *A General Theory of Equilibrium Selection in Games*, Cambridge, MA: The MIT Press, June 1988.
- Hindriks, Jean and Denis Claude, "Strategic Privatization and Regulation Policy in Mixed Markets," *The Icfai Journal of Managerial Economics*, February 2006, 2 (4), 7–26.

- Kandori, Michihiro, George J. Mailath, and Rafael Rob, "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, January 1993, 61 (1), 29–56.
- Laffont, Jean-Jacques and Jean Tirole, "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, June 1986, 94 (3), 614–641.
- Laffont, Jean-Jacques and Jean Tirole, *A Theory of Incentives in Procurement Regulation*, Cambridge, MA: MIT Press, 1993.
- Lipsey, Richard G. and Kelvin Lancaster, "The General Theory of Second Best," *The Review of Economic Studies*, 1956, 24 (1), 11–32.
- Magnus, Eivind G.B. and Atle Midttun, "The Norwegian Model: Competitive Public Economy," in Eivind G.B. Magnus and Atle Midttun, eds., *Electricity Market Reform in Norway*, London, UK: Palgrave Macmillan, May 2000.
- Marchand, Maurice, Pierre Pestieu, and Henry Tulkens, "The Performance of Public Enterprises: Normative, Positive and Empirical Issues," in Maurice Marchand, Pierre Pestieu, and Henry Tulkens, eds., *The Performance of Public Enterprises*, Vol. 33 of *Studies in Mathematical and Managerial Economics*, Amsterdam, NL: Elsevier Science B.V., January 1984, pp. 3–42.
- Martin, Steven, "Globalization and the Natural Limits of Competition," in Manfred Neumann and Jürgen Weigand, eds., *The International Handbook of Competition*, Cheltenham, UK: Edward Elgar, 2004.
- Matsumura, Toshihiro, "Partial privatization in mixed duopoly," *Journal of Public Economics*, December 1998, 70 (3), 473–483.
- Matsumura, Toshihiro, "Stackelberg Mixed Duopoly with a Foreign Competitor," *Bulletin of Economic Research*, 2003, 55 (3), 275–287.
- Meade, J. E., "Price and Output Policy of State Enterprise," *The Economic Journal*, December 1944, 54 (215/216), 321–339.
- Meggison, William L. and Jeffrey M. Netter, "From State to Market: A Survey of Empirical Studies on Privatization," *Journal of Economic Literature*, June 2001, 39 (2), 321–389.
- Newbery, David M., *Privatization, Restructuring, and Regulation of Network Utilities* Walras-Pareto Lectures, Cambridge, MA: The MIT Press, May 2000.
- Pal, Debashis, "Endogenous timing in a mixed oligopoly," *Economics Letters*, November 1998, 61 (2), 181–185.

- Pal, Debashis, and Mark D. White, "Mixed Oligopoly, Privatization, and Strategic Trade Policy," *Southern Economic Journal*, October 1998, 65 (2), 264–281.
- Rees, Ray, *Public Enterprise Economics*, 2nd ed., London: Weidenfeld and Nicolson, 1984.
- Sepahvand, Mehrdad, "Public Enterprise Strategies in a Market Open to Domestic and International Competition.," *Annales d'Economie et de Statistique*, 2004, 75–76, 135–153.
- Stehmann, Oliver and Rob Borthwick, "Infrastructure competition and the European Union's telecommunications policy," *Telecommunications Policy*, November 1994, 18 (8), 601–615.
- Taschdjian, Martin, "Alternative Models of Telecommunications Policy: Services Competition versus Infrastructure Competition," in "Twenty-Fifth Annual Telecommunications Policy Research Conference" Alexandria 1997.
- van Damme, Eric and Sjaak Hurkens, "Endogenous price leadership," *Games and Economic Behavior*, May 2004, 47 (2), 404–420.
- Van Huyck, John B., Raymond C. Battalio, and Richard O. Beil, "Tacit coordination games, strategic uncertainty, and coordination failure," *American Economic Review*, March 1990, 80 (1), 234–248.
- Willner, Johan, "Ownership, efficiency, and political interference," *European Journal of Political Economy*, November 2001, 17 (4), 723–748.