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Data-driven Adaptive Model-Based Predictive Control with Application to Wastewater Systems

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Data-driven Adaptive Model-Based Predictive Control with Application to Wastewater Systems

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Abstract: This paper is concerned with the development of a new data-driven adaptive model-based predictive controller (MBPC) with input constraints. The proposed methods employ subspace identification technique and a Singular Value Decomposition (SVD) based optimisation strategy to formulate the control algorithm and incorporate the input constraints. Both Direct Adaptive Model-Based Predictive Controller (DAMBPC) and Indirect Adaptive Model-Based Predictive Controller (IAMBPC) are considered. In DAMBPC, the direct identification of controller parameters is desired to reduce the design effort and computational load whilst the IAMBPC involves a two-stage process of model identification and controller design. The former method only requires a single QR decomposition for obtaining the controller parameters and uses a receding horizon approach to process input/output data for the identification. A suboptimal SVD-based optimisation technique is proposed to incorporate the input constraints. The proposed techniques are implemented and tested on a 4\textsuperscript{th} order nonlinear model of a wastewater system. Simulation results are presented to compare the direct and indirect adaptive methods and to demonstrate the performance of the proposed algorithms.

Keywords: Activated sludge process, Adaptive control, Model-based predictive control, Subspace identification.
1. Introduction

The classical control design problem is to start with building a model of the systems using physical laws to derive a presentation of the system in forms such as transfer function, matrix fraction, state space, impulse response, etc. While this approach works well for many systems, it has several disadvantages. The process of model building is expensive and cumbersome. Moreover, the models are valid only for a limited operating range and hence cannot capture the time varying or nonlinear behavior of many dynamic systems. As a consequence, many solution methods and algorithms were developed to design more effective controllers. The gain scheduling controllers were formally introduced in the sixties [1], followed by adaptive control approach in the seventies [2], neuro-fuzzy controllers in the eighties [2] and H-infinity robust control design in the nineties [3]. Most of these methods were however model-based and hence requires expensive effort to develop accurate models. Data driven methods use the process input/output data to design a stabilizing controller with satisfactory performance. This can include many of the adaptive control algorithms as well as neuro-fuzzy control design techniques. The data driven approach allows the controller to be designed using data from the actual system to be controlled under realistic operating conditions. Hence, the controller will stabilize the actual system instead of a model of that system. This procedure avoids the needs for modeling the plant under all hypothetical disturbances, and operating conditions, but considers only those that actually occur. A good background and application of data driven control are presented in [4] and [5], respectively.

This paper demonstrates the use of subspace-based techniques for the implementation of indirect and direct adaptive model-based predictive controller with input constraints. Subspace identification techniques have emerged as one of the more popular identification methods for the estimation of state space models. Using these techniques, subspace matrices, which obtained directly from input/output data, are used to obtain prediction of the process outputs. These predictions can subsequently serve as a basis for MBPC design. By continuously updating these predictions models, an adaptive predictive control method can be obtained. In IAMBPC, the controller is designed in two separate steps of model identification and control design. A more attractive alternative to the two-step method is to estimate the control parameters directly from the measurements (i.e. DAMBPC). The direct adaptive control method was developed in the early 70s [6] and widely deployed because of low computation requirement as it combines system identification and control design.
Some previous work has been reported on the design of MBPC using data driven control design method such as model-free Linear Quadratic Gaussian (LQG) and subspace predictive controller [7; 8; 9; 10], or controller designs using the state space model identified through subspace approach [11; 12]. It is shown in [7; 8] that the system identification and the calculation of controller parameters may be replaced by a single QR decomposition and hence a data driven controller can be formulated. Although the idea of combined subspace methods and MBPC as a data driven control design method has been around for few years, designing an adaptive subspace-based MBPC is still an open area of research. Existing methods in subspace-based adaptive MBPC does not include constraints [10], and hence one of the main attraction of predictive control design technique is missing from these methods. Therefore, the objective of this paper is to develop subspace based adaptive MBPC, which includes input constraints, as well as soft nonlinear dynamics. Wang et al. [14] have also employed similar approach but their work differs in term of the identification approach for the design of subspace controller, (e.g. the prediction of the future outputs presented by the past available measurements and past input signals.). Moreover, this paper highlights the advantage of using a suboptimal SVD-based optimisation technique to incorporate the input constraints as compared to QP method. Other approaches for dealing with these types of processes include nonlinear MBPC [13] and neural network based MBPC approaches [14]. The latter method has made few inroads in practice due to its complexity and computational load typically associated with these methods. For industrial applications, however, multiple model linear MBPC approaches tend to be more favoured [15; 16].

The proposed direct adaptive MBPC method can offer an attractive alternative to existing methods for slowly time-varying and nonlinear systems. The method combines the simplicity of linear MBPC with the power of a self-tuning controller to incorporate the hard input constraints. The main advantages are that the usually tedious and time-consuming modelling task can be relaxed and the controller can adapt to changing process conditions while the physical constraints are satisfied. The use of an SVD based method for optimisation reduces the computation burden and ensures that a solution can be found in a desired sampling time. The performance of DAMBPC is compared with IAMBPC using a 4th order nonlinear model of a wastewater system. Simulation results are presented to investigate the effect of prediction horizon on the tracking and disturbance rejection properties of the proposed algorithms.

The paper is organised as follows: Section 2 briefly recapitulates the main concepts of subspace identification and QR decomposition. The proposed constrained adaptive model-based predictive control approach is developed in Section 3, both for DAMBPC and IAMBPC methods. Section 4 describes the application of the proposed method to an activated sludge wastewater treatment plant. Section 5 presents the
simulation results where the performances of the proposed control strategies are compared. The paper ends with some conclusion.

2. The subspace identification method

A linear discrete time-invariant state space system can be represented as:

\[ x(k+1) = Ax(k) + Bu(k) + Kv(k) \]  (1)
\[ y(k) = Cx(k) + Du(k) + v(k) \]  (2)

where \( u(k), y(k) \) and \( x(k) \) are the inputs, outputs and states respectively and \( v(k) \) is a white noise sequence with zero-mean and variance \( E[v(k)v(\ell)^T] = \delta_\mu \). \( A,B,C,D \) and \( K \) are system matrices with appropriate dimensions. We assume that \((A,B)\) is controllable and \((C,A)\) is observable. The following matrix input-output equations [17] play an important role in the problem treated in linear subspace identification and it can be obtained by recursive substitution of Eq. (1) into Eq. (2):

\[ \Gamma_i X_f + H U_f = Y_f \quad \text{and/or} \quad \Gamma_i X_p + H U_p \]  (3)

where the data block Hankel matrices for \( u(k) \) represented as \( U_p \) and \( U_f \) defined as:

\[
U_p = \begin{bmatrix}
  u_0 & u_1 & \cdots & u_{i-1} \\
  u_1 & u_2 & \cdots & u_i \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{i-1} & u_i & \cdots & u_{i+j-2}
\end{bmatrix}
\]  (4)

\[
U_f = \begin{bmatrix}
  u_i & u_{i+1} & \cdots & u_{i+j} \\
  u_{i+1} & u_{i+2} & \cdots & u_{i+j+1} \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{i+j-1} & u_{i+j} & \cdots & u_{2i+j-2}
\end{bmatrix}
\]  (5)

The subscripts \( p \) and \( f \) represent ‘past’ and ‘future’ time. The outputs block Hankel matrices \( Y_p \) and \( Y_f \) can also be defined in the same way. \( i \) is the prediction horizon (or so called \( H_p \)). Then, the data set is broken into \( j \) prediction problem. The following shorthand notation is used for the past input-output data:

\[
W_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}
\]  (6)

The future state sequence is defined as:

\[
X_f = \begin{bmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \end{bmatrix}^T
\]  (7)

The extended observability matrix, \( \Gamma_i \) and the lower block triangular Toeplitz matrix, \( H_i \) are defined as:
\[
\Gamma = \begin{pmatrix}
C & CA \\
CA & CA^{-1}
\end{pmatrix}, \quad H_i = \begin{pmatrix}
CB & 0 & \cdots & 0 \\
CAB & CB & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
CA^{-3}B & CA^{-2}B & \cdots & CB
\end{pmatrix}
\] (8)

The basic idea of subspace identification method is that, from the previously defined matrix input-output Eq. (3), it can be observed that the block data Hankel matrix containing the future outputs, \(Y_f\) is linearly related to the future state sequences, \(X_f\) and the future inputs, \(U_f\). Therefore, the main framework of subspace model identification is to recover the term \(\Gamma, X_f\), whereby from the knowledge of either \(\Gamma\) or \(X_f\), the state space system matrices can be retrieved in a least square sense [18]. Once the system parameters are obtained, they can be used to design the controller.

This identification method is implemented for IAMBPC where the state space system matrices are obtained using online subspace identification algorithm, i.e. Numerical algorithms for Subspace State Space System IDentification (N4SID). For the DAMBPC controller parameters (subspace matrices) are directly derived from the input-output measurement. The difference between two approaches is schematically shown in Fig. 1. The following derivation is focused only for identifying the subspace matrices for the DAMBPC method. The detail derivation on the subspace identification and the use of projection in subspace identification are not presented here. They can be found in [18].

In the case when no noise is present, the actual future output \(Y_f\) lies in the combined row space and the linear predictor equation can be written as:

\[
\hat{Y}_f = L_w W_p + L_u U_f
\] (9)

where \(W_p\) and \(U_f\) are the past inputs and outputs and future inputs, respectively. \(L_w\) and \(L_u\) are subspace matrices corresponding to the states and inputs, respectively. By solving the following least square problem, the output prediction, \(\hat{Y}_f\) can be extracted:

\[
\min_{L_w, L_u} \left\| Y_f - \left( L_w, L_u \right) \left( W_p, U_f \right) \right\|_F^2
\] (10)

The orthogonal projection of the row space of \(Y_f\) into the row space spanned by \(W_p\) and \(U_f\) applied in Eq. (10) gives:
\[ \hat{Y}_{rf} = Y_f \left( \begin{array}{c} W_p \\ U_f \end{array} \right) \]  
(11)

\[ \hat{Y}_{rf} = Y_f l_{u_f} W_p + Y_f l_{w_f} U_f \]  
(12)

from which we have two projections involved in the right hand side in the above equations. The first projection of \( Y_f l_{u_f} W_p \) relates to Kalman filter state of the system and the second projection of \( Y_f l_{w_f} U_f \) relates to Toeplitz matrix. Now, Eq. (12) can be solved efficiently via QR decomposition:

\[
\begin{pmatrix} W_p \\ U_f \\ Y_f \end{pmatrix} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{pmatrix}
\]  
(13)

By posing:

\[ L = (R_{31} \ R_{32}) \begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix}^T \]  
(14)

where \( L = (L_u \ L_u) \) and \( \dagger \) denotes the Penrose-Moore pseudo inverse, Eq. (12) can be written as:

\[ \hat{Y}_{rf} = Y_f \left( \begin{array}{c} W_p \\ U_f \end{array} \right) = L \left( \begin{array}{c} W_p \\ U_f \end{array} \right) \]  
(15)

where \( L_u \) and \( L_u \) in Eq. (12) can be obtained by partitioning \( L \) in Eq. (15):

\[ [L_u \ L_u] = Y_f \left[ \begin{array}{c} W_p \\ U_f \end{array} \right] \]  
(16)

\[ [L_u \ L_u] = Y_f \left[ \begin{array}{c} W_p \\ U_f \end{array} \right] \left( \begin{array}{c} Y_f & [U_f^T \\ W_f^T] \end{array} \right)^{-1} \]  
(17)

where \( N_u \) and \( N_y \) denote the number of input and output, respectively. It can be observed that, the subspace matrices \( L_u \) and \( L_u \) can be retrieved directly from matrix \( R \). The system matrices \( A, B, C, D \) do not have to be calculated explicitly. To enable an adaptive sliding window, QR-updating is performed. A combination of updating and down dating the QR decomposition is performed making use of the rank-one modification \([19]\). The subspace matrices are updated online throughout the updated \( R \) factor from the implemented QR updating.

As mentioned previously, the most interesting part in subspace model identification is that we can obtain Kalman filter state directly from input-output data without having knowledge of system
parameters. The link between Kalman filtering and the projection of the future outputs $Y_f$ into the combined row space of the past inputs and outputs $W_p$ and the future inputs $U_f$ is demonstrated in Eq. (12). We can now exploit the duality between Kalman filter and MBPC controller. For $i, j \rightarrow \infty$, we obtain:

$$Y_f / U_f = L_u W_p = \Gamma \hat{X}_f$$

(18)

$$L_u = H_i$$

(19)

In general, $L_u$ correspond to the determination of Kalman filter state and $L_u$ represents the controller parameters. It can be seen that there is a link between subspace projection and the Kalman filter estimates, $\hat{X}_f$ of the state sequence $X_f$ given by Eq. (18). When $L_u$ is approximated to a lower order matrix using singular value decomposition, it would be a rank deficient matrix of order $n$ if there were no noise. This description gives $L_u \approx U_i \Sigma_i V_i^T$. Therefore, for $\Gamma_i \approx U_i \Sigma_i^{1/2}$ and $\hat{X}_f \approx \Sigma_i^{1/2} V_i^T W_p$:

$$\Gamma_i \hat{X}_f = U_i \Sigma_i^{1/2} \Sigma_i^{1/2} V_i^T W_p$$

$$= U_i \Sigma_i V_i^T W_p$$

$$= L_u W_p$$

(20)

where $\hat{X}_f = \hat{x}_i$ is the steady state Kalman filter estimate and $\Gamma_i$ is the extended observability matrix.

To obtain offset-free tracking, an integral action is included. Previous works that include integrator in the design of subspace controller are given in [9; 11]. The matrix input-output in Eq. (3) can be changed to include an integrator in the predictor and this can be expressed as follows:

$$\Delta Y_f = \tilde{\Gamma}_i S_f + \tilde{H}_i \Delta U_f$$

(21)

$$\Delta \hat{Y}_f = \tilde{\Gamma}_i S_f + \tilde{H}_i \Delta U_f$$

(22)

and

$$\Delta \hat{Y}_f = \tilde{L}_u \Delta W_p + \tilde{L}_u \Delta U_f$$

(23)

where $\tilde{L}_u$ and $\tilde{L}_u$ are obtained directly from the previous identification of $L_u$ and $L_u$. Thus:

$$\hat{Y}_f = \hat{y}_i + \tilde{L}_u \Delta W_p + \tilde{L}_u \Delta U_f$$

(24)

where the current output $Y_f$ has the same dimension as prediction horizon and is defined as:
\[ Y_i = [y_i, y_{i-1}, \ldots, y_1]^T \]  

(25)

It should be noted that in any closed-loop parameter identification scheme, the input signal should be persistently exciting to perturb the main dynamics of the system. This can be usually achieved by injecting a Pseudo Random Binary Sequence Signal (PRBS) at the process set points. In practice for adaptive control algorithm, a degree of perturbation and excitation are also achieved during the identification and control design as the controller parameters changes as the control design is updated.

3. Adaptive Model-Based Predictive Control

The adaptive control scheme investigated here uses subspace identification technique described above. The measured data is collected over a sliding (receding) window. The procedure of using a sliding window for identification is illustrated in Fig. 2. Note that, data window used to identify subspace predictor parameters should be expressed in term of future inputs \( u_f = \{u_{i-1}, \ldots, u_{2i-2}\}^T \) and measurement (past) inputs \( u_p = \{u_0, \ldots, u_{i-1}\}^T \) and outputs \( y_p = \{y_0, \ldots, y_{i-1}\}^T \) as described in Eq. (9). Here, the two prediction problems should be solved at current time instant \( i \) and \( i+1 \) as shown in Fig. 2. The first prediction problem \((t=i)\) represents the case for obtaining the optimal prediction of \( i \) future outputs \( \hat{y}_{i} = (\hat{y}_i, \ldots, \hat{y}_{2i-1})^T \) using the information given in the previously stated data window \( u_p, y_p \) and \( u_f \). The second prediction problem shows that the time instant slides \((t=i \text{ to } t=i+1)\) and similar meaning can be observed, only difference is the data window \((u_p, y_p \text{ and } u_f)\), which is, now slides from left to right. At every time step, for the new input-output data obtained, the subspace predictor parameters are updated online and the new control action is computed. Note that, the linear predictor in Eq. (24) is directly driven by input-output data and contains an integral action. The main advantage of this approach is that the controller parameters are updated at each sample time, which usually means a quicker response to process changes. The main drawback of this method is that a QR-decomposition needs to be computed at each sample instance, which increases the computation load.

In the constraint case, the computational burden is an important issue to be considered. The constrained control problem is usually solved using standard Quadratic Programming (QP) method. This requires heavy computational effort and hence is not suitable for online adaptive control design scheme. To reduce the computational load, an SVD based strategy is proposed here [20]. The structure of the
Two methods of subspace adaptive control scheme (DAMBPC and IAMBPC) are developed for the constrained case using SVD analysis.

### 3.1. Direct method (DAMBPC)

A possible structure for direct adaptive control using MBPC is depicted in Fig. 3. To implement the DAMBPC, consider the linear predictor equation given in Eq. (24):

\[
\hat{y}_f = \hat{L}_w p_u + \hat{L}_n \Delta u_f
\]

(26)

The following MBPC performance index is minimised to calculate the control input increment, \( \Delta u_f \):

\[
J = \sum_{i=1}^{H_p} \left( r(k+i) - y(k+i) \right)^\text{T} Q \left( r(k+i) - y(k+i) \right) + \sum_{i=0}^{H_c-1} \Delta u(k+i)^\text{T} R \Delta u(k+i)
\]

(27)

\[
= \left( r_f - \hat{y}_f \right)^\text{T} Q \left( r_f - \hat{y}_f \right) + \Delta u_f^T R \Delta u_f
\]

where \( H_p \) and \( H_c \) denote the prediction horizon and control horizon, respectively. The output and input weighting matrices are \( Q = diag(\Omega L Q_{H_p}) > 0 \) and \( R = diag(\Omega L R_{H_c} - 1) \geq 0 \). Substituting Eq. (26) into Eq. (27) gives:

\[
J = \left( r_f - \hat{L}_w p_u - \hat{L}_n \Delta u_f \right)^\text{T} \left( r_f - \hat{L}_w p_u - \hat{L}_n \Delta u_f \right) + \Delta u_f^T R \Delta u_f
\]

(28)

where \( r_f - \hat{L}_w p_u = e \) is the tracking error, thus:

\[
J = e^\text{T} Q e - 2 \Delta u_f^\text{T} \hat{L}_w Q p_u e + \Delta u_f^T \Omega \Delta u_f
\]

(29)

where \( \Omega = \hat{L}_w L Q_{H_c} + R \in R^{H_c \times H_c} \). To find the minimum of \( J \), its derivative is set to zero:

\[
\frac{\partial J}{\partial \Delta u_f} = 0
\]

(30)

The DAMBPC control law is therefore defined as:

\[
\Delta u_f = \Omega^{-1} \hat{L}_w^T Q e
\]

(31)

Eq. (31) gives the unconstrained optimal solution and the controller parameter \( \hat{L}_n \) is directly obtained from experimental data.
The implementation of SVD based optimisation for the constrained case is discussed next. This makes the adaptive control scheme considerably faster and easier to implement as compared to QP method, since the Hessian $\Omega$ can be formed directly from the identification step. At each sampling instant, a feasible control sequence is determined by selecting a variable subset of the SVD basis representation. This sequence defined as $\Delta u$ satisfies the gain and rate input constraints of the optimisation problem (Eq. (29)) defined as $u(k)_{\text{min}} \leq u(k) \leq u(k)_{\text{max}}$ and $\Delta u(k)_{\text{min}} \leq \Delta u(k) \leq \Delta u(k)_{\text{max}}$.

To calculate the control input, let the SVD of $\Omega$ be defined as [18]:

$$\Omega = U \Sigma V^T$$

where $\Sigma \in \mathbb{R}^{N_r \times N_r}$ is given as:

$$\Sigma = \begin{pmatrix}
\sigma_1 & 0 & \ldots & 0 \\
0 & \sigma_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \sigma_{N_r}
\end{pmatrix}$$

and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{N_r} \geq 0$. The $\sigma_i$ are the singular values of $\Sigma$ and the vectors $u_i$ and $v_i$ are the $i^{th}$ left singular vector and the $i^{th}$ right singular vector, respectively. In this case, since $\Omega$ is symmetric, the left and right singular vectors are identical. This in turn, yields an important property that is the singular vector matrix $V$ is orthogonal i.e. $V^T V = V V^T = I_{N_r}$. If $V = \begin{pmatrix} v_1, \ldots, v_{N_r} \end{pmatrix}$ is orthogonal, then the $v_i$ form an orthonormal basis vector for $\mathbb{R}^{N_r}$. Therefore, the following control input increment vectors:

$$\Delta u = \begin{pmatrix} \Delta u(k) & \Delta u(k+1) & \ldots & \Delta u(k+H_r-1) \end{pmatrix}^T$$

(34)

can be expressed as a linear combination of the singular vectors, $v_i$ of $\Omega$ given as:

$$\Delta u = V \Delta \tilde{u} = \sum_{i=1}^{N_r} v_i \Delta \tilde{u}_i$$

(35)

where $v_i = \begin{pmatrix} v_1, \ldots, v_{N_r} \end{pmatrix}$ are the columns of $V$, and $\Delta \tilde{u}_i$ are the entries of the input increment vector $\Delta \tilde{u}$.

The performance index, $J$, can now be written in terms of the new increment input vectors as:

$$J = e^T Q e - 2 \Delta \tilde{u}^T \tilde{L}_e\tilde{L}_e^T Q e + \Delta \tilde{u}^T \Sigma \Delta \tilde{u}$$

(36)

This gives the optimal unconstrained control input increment sequence:

$$\Delta \tilde{u}_m = \Sigma^{-1} \tilde{L}_e^T \tilde{L}_e Q e$$

(37)
The increment input vectors for the constrained case can be constructed by considering the modification of the unconstrained solution, \( \Delta \tilde{u}_w \) in Eq. (37). Let us first define the performance index, \( J \) as:

\[
\min J = J_w + \sum_{i=1}^{N_w} \sigma_i \left( \Delta \tilde{u}_i - \Delta \tilde{u}_{w,i} \right)^2
\]  

(38)

where \( \Delta \tilde{u}_{w,i} \) is the \( i^{th} \) entry of vector \( \Delta \tilde{u}_w \). From Eq. (38), it can be observed that whenever \( \Delta \tilde{u}_i = \Delta \tilde{u}_{w,i} \), we obtain \( J = J_w \), which is the unconstrained value. Note that the entries of \( \Delta \tilde{u} \) in Eq. (38) are arranged in decreasing order of magnitude, since \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{H,N_u} \geq 0 \), which starting from the one that influences the performance index the most and ending with the one that influences the performance index the least.

Therefore, to find a feasible solution to the constrained optimisation problem \( J \), we need to consider the components in the entries of vector \( \Delta \tilde{u}_w \) with highest contribution in reducing the magnitude of \( J \), i.e. use those elements of \( \Delta \tilde{u}_w \) with the biggest singular values. \( \Delta \tilde{u}_w \) for the unconstrained solution is:

\[
\Delta \tilde{u}_w = V \Delta \tilde{u}_w = \sum_{i=1}^{N_w} v_i \Delta \tilde{u}_{w,i}
\]  

(39)

The vector \( \Delta \tilde{u}_w \) in Eq. (37) will be ordered from the highest to the smallest singular values and progressively discarding smaller components, until the constraints are satisfied, i.e.:

\[
\Delta \tilde{u}_w = V \left( \Delta \tilde{u}_{1,w} \ldots \Delta \tilde{u}_{m,w} \ldots 0 \ldots 0 \right)^T
\]  

(40)

where \( m \in I_{H,N_u} \setminus \{1 \ldots H,N_u\} \). This does not necessarily gives the best control performance, hence the following control increment vector is defined:

\[
\Delta \tilde{u}_w = \sum_{i=1}^{m} v_i \Delta \tilde{u}_{i,w} + \left( v_{m+1} \alpha \Delta \tilde{u}_{m+1,w} \right)
\]  

(41)

where \( 0 \leq \alpha \leq 1 \) and \( m \in I_{H,N_u} \setminus \{1 \ldots H,N_u\} \). To obtain the best solution \( \alpha \) should be as large as possible while the constraints are satisfied. For \( m = H,N_u \), \( \alpha = 0 \) and the solution is unconstrained. The proposed SVD-based DAMBPC is summarised here:

**Algorithm 3.1 (Direct method)**

**Step 1:** Construct data block Hankel matrix, i.e. \( U_p, U_p, Y_p, Y_f \) from a given input-output data.

**Step 2:** Compute \( L \) by using Eq. (14), then partitioning \( L \) into \( L_u \) and \( L_v \) using Eqs. (16) and (17).

**Step 3:** Compute unconstrained solution, i.e. \( \Delta \tilde{u}_w = \Sigma^{-1} V^T \tilde{L}_n^T Q e \)

**Step 4:** Find the “largest” \( \theta = m+\alpha \), where \( 0 \leq \alpha \leq 1 \) such that the vector: 

\[ \ldots \]
lies on the boundary of the constraint set in $H_{N}$ and $\Delta u_{\text{pad}} \in \Delta u$. The parameters $m$ and $\alpha$ are tuned for the best performance whilst the constraints are satisfied.

**Step 5**: At time $k$, only $\Delta u_{\text{pad}}(1)$ is implemented and the calculation is repeated at each time instant, i.e. $u(k)$ is implemented as: $u(k) = u(k - 1) + \Delta u(k)$

**Step 6**: Update the Hankel matrix using the newest data and go to step 2 and repeat.

### 3.2. Indirect method (IAMBPC)

For IAMBPC, the classical two-step of system identification and control design is performed. Firstly, a suitable model is estimated using subspace algorithm and then the controller parameters are calculated from a design method as shown in Fig. 4. The Numerical Algorithm for Subspace State Space System IDentification (N4SID) [20] is employed here for process identification.

By iterating the model in Eq. (1) - (2), the prediction output is defined as:

$$\hat{y} = \Gamma x + H \Delta u$$

(42)

The quadratic performance index can be expressed as follows:

$$J = e^{T} Q e - 2 \Delta u^{T} H^{T} Q e + \Delta u^{T} \Sigma^{+} \Delta u$$

(43)

where $\Sigma = H^{T} Q H + R$ and $e = r - \Gamma x$. The IAMBPC control law can be found by making the gradient of $J$ zero, therefore:

$$\Delta u = \Sigma^{-1} H^{T} Q e$$

(44)

By using SVD-based strategy, the performance index, $J$ can now be rewritten as:

$$J = e^{T} Q e - 2 \Delta \tilde{u}^{T} V^{T} H^{T} Q e + \Delta \tilde{u}^{T} \Sigma \Delta \tilde{u}$$

(45)

Thus, the unconstrained optimal $\Delta u$ is:

$$\Delta \tilde{u}_{\text{opt}} = \Sigma^{-1} V^{T} H^{T} Q e$$

(46)

The derivation of the constrained solution using SVD-based optimisation strategy is similar to the one described in Section 3.1 and it is not repeated here. The IAMBPC algorithm is summarised here.

**Algorithm 3.2: Indirect method**

**Step 1**: Construct data block Hankel matrix, i.e. $U_{u}$, $U_{p}$, $Y_{p}$, $Y_{f}$ from a given input-output data.

**Step 2**: Compute the matrices $A$, $B$, $C$, $D$ by solving the least square problem.
Step 3: Compute the optimal unconstrained solution \( \Delta \hat{u}_{\text{wc}} = \Sigma^{-1}v^{T}H^{T}Qe \)

Step 4–6: Similar to Algorithm 3.1

4. Control of Activated Sludge Processes (ACT)

In this study, the proposed methods are applied to control a nonlinear activated sludge wastewater treatment plant. This process comprises an aerated tank and a settler as shown in Fig. 5. The bioreactor includes a secondary clarifier that serves to retain the biomass in the system while producing a high quality effluent. Part of the settled biomass is recycled to allow the right concentration of microorganisms in the aerated tank. A component mass balance that yields the following set of nonlinear differential equations was used [22]:

\[
\dot{X}(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \tag{47}
\]

\[
\dot{S}(t) = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \tag{48}
\]

\[
C(t) = K_{La} \frac{\mu(t)X(t)}{Y} - D(t)(1+r)C(t) + K_{La}(C_{in} - C(t) + D(t)C_{in}) \tag{49}
\]

\[
\dot{X}_r(t) = D(t).X_r(t) - D(t).(\beta + r)X_r(t) \tag{50}
\]

where the state variables, \(X(t), S(t), C(t)\) and \(X_r(t)\) represent the concentrations of biomass, substrate, dissolved oxygen (DO) and recycled biomass respectively. \(D(t)\) is the dilution rate, while \(S_{in}\) and \(C_{in}\) correspond to the substrate and DO concentrations of influent stream. The parameters \(r\) and \(\beta\) represent the ratio of recycled and waste flow to the influent flow rate, respectively. The kinetics of the cell mass production is defined in terms of the specific growth rate \(\mu\) and the yield of cell mass \(Y\). The term \(K_{La}\) is a constant. \(C_{in}\) and \(K_{La}\) denote the maximum DO concentration and the oxygen mass transfer coefficient, respectively. The quantity \((W)\) appears in Eq. (49) through the oxygen transfer rate coefficient \(K_{La}\):

\[
K_{La} = \rho W \tag{51}
\]

where \(\rho > 0\). The Monod equation gives the growth rate related to the maximum growth rate, to the substrate concentration, and to DO concentration:

\[
\mu(t) = \mu_{\text{max}} \frac{S(t)}{K_s + S(t)} \frac{C(t)}{K_C + C(t)} \tag{52}
\]

where \(\mu_{\text{max}}\) is the maximum specific growth rate, \(K_s\) is the affinity constant and \(K_C\) is the saturation constant.

In this simulation, two controlled outputs substrate (S) and DO and two manipulated inputs dilution rate (D) and airflow rate (W) are considered. The sampling rate of the MBPC controller and the activated sludge process is chosen as \(T_s = 1\) hr.
An adaptive control design scheme is required for this process as the dynamic is nonlinear and time varying. This is usually caused by the variation in the concentration and composition of the influent to the plant as well as 24-hrs changes in the influent flow. The process exhibits different dynamic under varying weather of dry, rain or storm conditions as well as variation in the daily temperature.

5. Evaluation and simulation results

Simulations were carried out for the two proposed control strategies that is, DAMBPC and IAMBPC methods. Both control methods use sliding identification window to update the parameters. The objective of the control algorithm is to regulate the substrate and DO concentrations from a steady state operating point at outputs 41.23 mg/l and 6.11 mg/l, respectively. For a fair comparison of both methods, the horizons are set to $H_p = 20$ and $H_c = 5$. The weighting matrices were chosen as $Q = \text{diag}(1, 10)$ and $R = \text{diag}(10^4, 10^{-4})$. The length of window is set to $n=400$. The constraints on the input change were allowed to be $|\Delta U_1| \leq 0.001$ and $|\Delta U_2| \leq 3.0$.

Fig. 6 compares the set point tracking performance of the two control strategies. It can be seen that both methods exhibit similar response to set point changes as expected but the first control strategy has a slight overshoot. To be able to see that the controller achieves good disturbance rejection, the measurements of substrate and DO are corrupted with step input disturbance (Amp=0.01) at t=1285 and the performance of both design methods are compared as shown in Fig. 7.

It can be clearly seen that the DAMBPC design method is able to track back to the set point quickly. The controller also rejects the disturbance in a reasonable time. The IAMBPC design method takes longer to reach the set point. This is also evident from large peak in the first output signals (Substrate) before it is slowly track back to the set point. The control signals are also given as shown in Fig. 8. Both control methods can handle constraint effectively, as shown in Fig. 8.

In summary, the disturbance rejection can be improved using DAMBPC design method. This can be seen in Fig. 7 and Fig. 8, whereby for a given input disturbance at t=1285, the DAMBPC reacts quickly and able to compensate the disturbance much faster compared to IAMBPC design method. Regarding the computational issue, it is shown by simulation that the DAMBPC outperform IAMBPC, both for SVD and QP methods. To be able to see the effect of computational load that results from directly calculating the
controller parameters, the summary of computation times (in sec) per sampling instant for different $n$ and $H_p$ for both methods is presented in Table 1 and Table 2, respectively. It can be clearly seen that the computation time required for SVD-based strategy is much lower (10 to 15 times) than the QP method. The SVD-based method shows an excellent performance, both for varying $n$ and $H_p$. The tables also show the DAMBPC is faster than IAMBPC as expected.

### 5.1 Closed-loop Stability

The closed-loop stability of the proposed methods was extensively tested using simulation studies. The critical parameter to ensure the local stability when a local linear model is identified is the prediction horizon, $H_p$, and hence this must be selected carefully. Longer $H_p$ will lead to better stability but this will also increase the computation load, and hence a trade-off should be made. Using the size of overshoot as an indication of stability, simulation results presented in Figs. 6 to 8 demonstrates that the DAMBPC has better stability robustness is than IAMBPC.

The overall closed-loop loop stability should also be analyzed for the nonlinear plant as stable linear local controllers are designed at each new operating condition. For the nonlinear process studied here, no global instability was observed as the system can in practice be stabilized by a set of PID controllers. Assuming the nonlinear plant is piecewise controllable and observable, the local MBPC controllers can then be designed to stabilize the nonlinear plant locally by choosing appropriate values for $H_p$. This does not, however, guarantee the global stability of the nonlinear plant. Johansson [23] shows that piecewise quadratic Lyapunov can be used for rigorous stability analysis of smooth nonlinear systems as well as quadratic stabilization of piecewise linear systems. The application of these techniques to the current problem is however out of scope of this paper.

### 6. Conclusions

The SVD-based data driven direct and indirect adaptive MBPC with input constraints were developed and tested in this paper. The derivation of DAMBPC control law only requires calculation of the subspace matrices $L_u$ and $L_w$. There is no need for explicit calculation of system matrices $A$, $B$, $C$, $D$. The identification step and control design can be done simultaneously by making use of a single efficient calculation of QR decomposition of data block Hankel matrices. Secondly, the constrained case has been successfully incorporated into the algorithm using SVD-based strategy. The DAMBPC shows better performance with respect to input disturbance compared to IAMBPC. The direct approach does also give
better tracking properties as well as disturbance rejection when applied to a nonlinear process. On the other hand, the use of SVD-based strategy in the optimisation structure significantly reduces the online computational time associated with the solution of standard QP method. In this case, the DAMBPC outperform the IAMBPC and it can provide an attractive alternative. For fast dynamical systems such as aircraft or vehicle dynamics, it may still be necessary to develop a version of these algorithms that uses the recursive subspace model identification such as R4SID.

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References


Table 1: CPU average times per sampling instant for different $n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>DAMBPC (SVD)</th>
<th>DAMBPC (QP)</th>
<th>IAMBPC (SVD)</th>
<th>IAMBPC (QP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.004</td>
<td>0.096</td>
<td>0.006</td>
<td>0.010</td>
</tr>
<tr>
<td>400</td>
<td>0.006</td>
<td>0.104</td>
<td>0.012</td>
<td>0.150</td>
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<tr>
<td>900</td>
<td>0.031</td>
<td>0.12</td>
<td>0.087</td>
<td>0.180</td>
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</tbody>
</table>

Table 2: CPU average times per sampling instant for different $H_p$

<table>
<thead>
<tr>
<th>$H_p$</th>
<th>DAMBPC (SVD)</th>
<th>DAMBPC (QP)</th>
<th>IAMBPC (SVD)</th>
<th>IAMBPC (QP)</th>
</tr>
</thead>
<tbody>
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<td>20</td>
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<td>0.165</td>
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<tr>
<td>15</td>
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<td>0.172</td>
</tr>
<tr>
<td>10</td>
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<td>0.120</td>
<td>0.006</td>
<td>0.193</td>
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