Enhanced Monopulse Radar Tracking Using Fractional Fourier Filtering in the Presence of Interference

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***Abstract*— Monopulse radars are used to track a target that appears in the look direction beam width. Significant distortion is produced when manmade high power interference (jamming) is introduced to the radar processor through the radar antenna main lobe (main lobe interference) or antenna side lobe (side lobe interference). This leads to errors in the target tracking angles that may cause target mistracking. A new monopulse radar structure is presented in this paper which addresses this problem. This structure is based on the use of optimal Fractional Fourier Transform (FrFT) filtering. The improved performance of the new monopulse radar structure over the traditional monopulse processor is assessed using standard deviation angle estimation error (STDAE) for a range of simulated environments. The proposed system configurations with the optimum FrFT filters is shown to reduce the interfered signal and to minimize the STDAE for monopulse processors.**

Keywords-component; monopulse radar; noise interference; fractional Fourier filtering;

I. INTRODUCTION

Monopulse radars are commonly used in target tracking because of their angular accuracy. However, these radars are affected by different types of interference which affects the target tracking process and may lead to inaccurate tracking [1].

A high power interference (jamming) is introduced to the radar processor through the radar antenna main lobe (main lobe interference) or antenna side lobe (side lobe interference). The resultant distortion due to this interference will affect the induced target error voltage and consequently the radar tracking ability. Seliktar et al. [2] suggests monopulse processors to decrease the effect of the noise interference before the target information is extracted. In our paper we propose the use of an optimal fractional Fourier transform (FrFT) filter to reduce the interference signals introduced from the main lobe or from the side lobe [3, 4]. Following a brief introduction to FrFT the paper will describe the structure of the new monopulse structure using the proposed filtering technique in the optimum FrFT showing the reduction in the interfered signal and the minimization in the STDAE for monopulse processors. The superior performance of the new algorithm will be demonstrated for different interference scenarios.

II. MONOPULSE RADAR PROCESSORS

A. Monopulse Radar Structure

The typical monopulse radar is chirp radar with pulsed chirp signal $c(t)$ is produced from the waveform generator. This is up-converted to the radar carrier frequency, amplified and passed through the duplexer to be transmitted.

$$c(t) = \exp(j\pi(\frac{F_{stop} - F_{start}}{T})(t - \frac{T}{2})^2) \tag{1}$$

where t is the time, T is the chirp time duration (pulse duration), F_{start} is the chirp start frequency, and F_{stop} is the chirp stop frequency. The down-converted received signal passes through a band limited Gaussian filter before passing through the chirp matched filter to maximize the target return signal. The target information parameters (azimuth angle, elevation angle, and target range) are then calculated by the monopulse processor from the filtered signal. The structure of monopulse radar is repeated N times (N equal to the of array antenna elements). Thus each antenna will have its own complete receiving system and all the output data will be processed in only one monopulse processor.

B. Monopulse Radar processors

1) The conventional processor is a non adaptive system comprising two sets of weights set to the sum and difference steering vectors defined as [5]:

$$w_{\Sigma} = a(\nu_l)\,, \qquad w_{\Delta} = \left.\frac{\partial a(\nu)}{\partial \nu}\right|_{\nu_l} \tag{2}$$

where $a(\nu)$ is the centre phase normalized steering vector in the look direction, N is the number of antenna, ν is the spatial steering frequency, and ν_l is the spatial steering frequency snapshot at time instant *l*.

2) The spatial processor is an adaptive system comprising an adaptive sum and difference beams formed by applying sum and difference unity gain constraints in the look direction, the sum and difference weights may be written in the following form [5]:

$$w_\Sigma = \frac{R_x^{-1} v_\Sigma}{v_\Sigma^H R_x^{-1} v_\Sigma} \quad , \qquad w_\Delta = \frac{R_x^{-1} v_\Delta}{v_\Delta^H R_x^{-1} v_\Delta} \tag{3}$$

where R_x is the covariance matrix of the input data, v_Σ and v_Δ are the spatial steering frequency for the sum and difference channel respectively and H indicates the Hermitian.

3) The sum and difference outputs are given in terms of the respective processors,

$$z_\Sigma(l) = w_\Sigma \mathbf{x}(l) \,, \qquad z_\Delta(l) = w_\Delta \mathbf{x}(l) \tag{4}$$

where $\mathbf{x}(l)$ is the $N \times 1$ spatial snapshot at time instant *l*. The real part of the ratio of difference to sum outputs is known as the error voltage defined as [2, 5]

$$\varepsilon_v(l) = \Re\left\{\frac{z_\Delta(l)}{z_\Sigma(l)}\right\} \tag{5}$$

This error voltage conveys purely directional information that must be converted to an angular form via a mapping function [2].

4) The standard deviation of the angle error (STDAE) [2] is determined using a target that is injected randomly across the range and angle within the main beam. The corresponding angle error is then averaged over the range and is defined as :

$$\sigma_{\varepsilon_\phi} = \sqrt{E\{|\varepsilon_\phi|^2\}} \tag{6}$$

where $\varepsilon_\phi = \hat{\phi} - \phi$, ϕ is the target angle, and $\hat{\phi}$ is the measured angle.

III. Fractional Fourier Transform

The FrFT is the generalized formula for the Fourier transform that transforms a function into an intermediate domain between time and frequency. The signals with significant overlap in both the time and frequency domain may have little or no overlap in the fractional Fourier domain. The fractional Fourier transform of order a of an arbitrary function $x(t)$, with an angle α, is defined as [6]:

$$X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(t,u)\, dt \tag{7}$$

where $K_\alpha(t,u)$ is the transformation Kernel and $\alpha = a\pi/2$ with $a \in \Re$. The optimum order value for a for a chirp signal may be written as:

$$a_{opt} = -\frac{2}{\pi} \tan^{-1}\left(\frac{\delta f}{2\gamma \times \delta t}\right) \tag{8}$$

where δf is the frequency resolution, δt is the time resolution, and γ is the chirp rate parameters. The optimum angle α_{opt} is calculated from:

$$\alpha_{opt} = \frac{\pi}{2} a_{opt} \tag{9}$$

The peak position P_p of a chirp signal in the FrFT domain is defined as [7]:

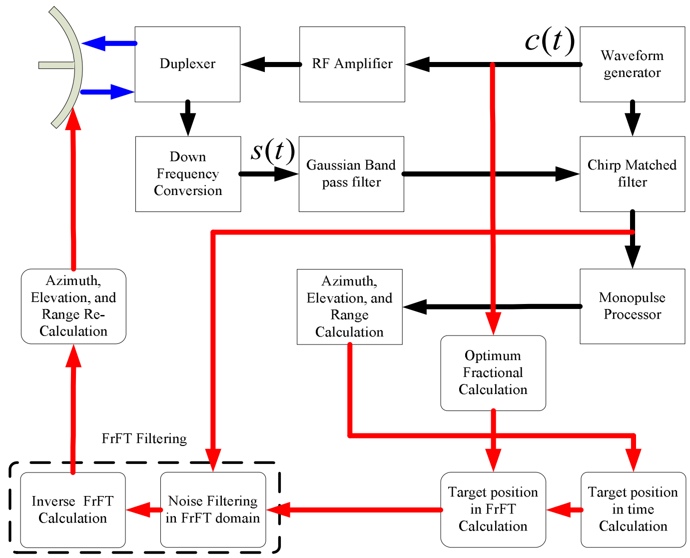

Fig 1. New structure of the proposed monopulse radar

$$P_p = \sin(\alpha)\left[\frac{F_{start}}{\delta f} + \frac{W(L/M_T)}{2 \times \delta f}\right] - \cos(\alpha)\, t_{st} \tag{10}$$

where W is the chirp bandwidth, L is the number of samples in the time window, M_T is the number of samples in the chirp signal duration T, and t_{st} is the chirp start time sampling number. The peak position in the FrFT domain depends on the chirp start position, the chirp signal bandwidth, and the chirp start frequency F_{start}.

IV. A New Structure Of Monopulse Radar

The proposed FrFT based monopulse radar is illustrated in Fig.1. It comprises a conventional monopulse subsystem along with addition FrFT and related processing blocks A pulsed chirp signal $c(t)$ defined in Eq 1 is produced from the waveform generator and is up-converted to the radar carrier frequency, amplified and passed through the duplexer to be transmitted. The down-converted received signal passes through a band limited Gaussian filter. The received signal $s(t)$ may be expressed in the baseband as:

$$s(t) = \begin{cases} \left[\left(Ae^{-j2\pi\phi_o}\, e^{j\pi\left(\frac{F_{stop}-F_{start}}{T}\right)\left(t-T_{start}-\frac{T}{2}\right)^2}\right).F_d\right] \times F_\phi & T_{start} < t < T_{start}+T \\ 0 & elsewhere \end{cases} \tag{11}$$

where A is the received signal amplitude, ϕ_o is a random phase shift, and T_{start} is the start time of the returned pulse, passes through a band pass Gaussian filter. The start time T_{start} depend on the target range R_t.

The optimal FrFT filtering (dashed block in Fig. 1) consists of N receiving channel in which the received signal from each of the N antenna elements will fill L range gates. The total radar data size is therefore equal to $N \times L$ for each pulse return. The optimum FrFT domain is calculated for each receiving channel data with size $1 \times L$ to filter the signal in the fractional domain. The resultant filtered data (useful signal) is converted back from the optimum FrFT domain using inverse FrFT processor to the time domain. The $1\times L$ data output from N FrFT processors are re-processed to re-determine the target information parameters.

The following steps are involved in the proposed algorithm that may be used to cancel the noise interference signal:

1. Determine the optimal fractional domain for the tracked target signal from the information supplied from the wave generator.
2. Identify the samples occupied by the target in the time domain to determine St_n.
3. Calculate the peak position of the target in the FrFT domain.
4. Filtering the received data by keeping the target data (peak position sample and its adjacent samples) and force all the rest of the samples in tracking window to be equal to zero.
5. Use the inverse FrFT with the known optimal order to get the signal back to time domain after filtering.
6. Recalculate the target information.

The mathematical description for the steps is now described. The target received signal is a chirp signal given by Eq 11. A signal model of our radar system [3] is:

$$\boldsymbol{z} = \boldsymbol{x} + \boldsymbol{n}_j \tag{12}$$

where the useful signal $\boldsymbol{x}$ is the tracked target signal and the distortion signal $\boldsymbol{n}_j$ is interference signal.

Barrage noise jamming is the most common form of hostile interference. Such interference emanates from a spatially localized source and is temporally uncorrelated from sample to sample as well as from PRI to PRI. It is modelled as the Kronecker product of a white Gaussian $\boldsymbol{n}_j(t)$ noise vector with a spatial steering vector,

$$\boldsymbol{n}_j = \boldsymbol{n}_j(t) \otimes a(\nu) \tag{13}$$

where the power of each component of $\boldsymbol{n}_j(t)$ is σ_j^2.

The optimum FrFT order a_{opt} for this chirp can be computed by applying Eq 8 to the radar system as:

$$a_{opt} = -\frac{2}{\pi}\tan^{-1}\left(\frac{F_s^{\,2}\times T}{(F_{stop}-F_{start})\times L}\right) \tag{14}$$

where F_s is the sampling frequency.

From the target position on the return radar window, the chirp start time sampling number t_{st} is determined. So the target peak position in the optimum FrFT can be calculated from Eq 10 and the other entire variable in this equation are known and can be re-written as:

$$P_p = \sin(\alpha_{opt})\left[\frac{-(W/2)}{(F_s/L)} + \frac{W\,(L/M_T)}{2\times(F_s/L)}\right] - \cos(\alpha_{opt})\,t_{st} \tag{15}$$

Peak position sample and the adjacent samples (5 samples in both sides) are kept and all the rest of the samples in the tracking window to be equal to zero to get the filtered data in the optimal FrFT domain $\boldsymbol{z}'$.

The filtered signal $\boldsymbol{x}'$ in the time domain is introduced by applying inverse FrFT using the same optimal operator a_{opt} as:

$$\boldsymbol{x}' = F^{-a_{opt}}(\boldsymbol{z}') \tag{16}$$

All the output signals from the N FrFT filters are then re-processed to get the target information parameters after applying the proposed filtering technique using Eqs 4-6 as described in section II.

V. Simulation Results

A monopulse radar with an array of 14 elements spaced 1/3 meters apart is simulated. The radar pulse width is 100 microseconds and the pulse repetition interval of 1.6 milliseconds uses a 435 MHz carrier. A 200 kHz Gaussian band pass filter exists at the front end of each of the N receivers. These are used to filter the incoming data returns prior to sampling. The incoming base band signals are sampled at 1 MHz. Also it is assumed that the radar operating range is 100:200 range bins with a starting window at 865 microseconds and a window duration of 403 microseconds. The desired target is known to exist at range bin=150 at angle 32^o from the look direction with target signal to noise ratio (SNR) set to 70 dB and a Doppler frequency of 150 Hz. A jamming signal with interference noise ratio (INR) set to 82 dB with two angles scenario, first at angle 32^o from the look direction (main beam jamming) and second at angle 62^o from the look direction (side lobe beam jamming) are introduced.

The jamming interference causes deviations in the monopulse error voltages from their original values (no jamming) to distorted curves. This distortion affects the tracking angle of the tracked target resulting in a probable mistracking outcome.

From the monopulse radar parameters, Eq 14 is used to compute the order of the optimal FrFT domain a_{opt} as 1.7074. Following the steps mentioned in section IV, the target in the time domain St_n occurs at bin 150.

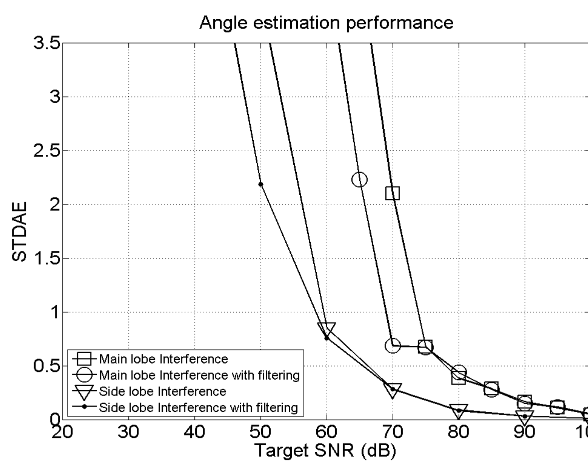

Fig 2. Conventional processor at main lobe and side lobe Interference

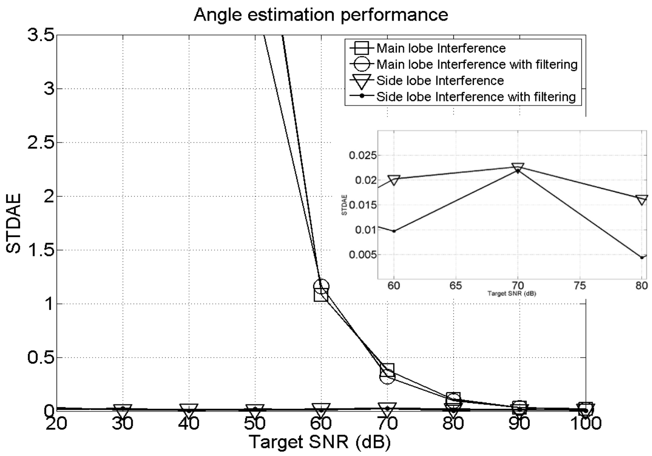

Fig 3. Spatial processor at main lobe and side lobe Interference

The target sample peak position in the optimum fractional domain is computed to exist at 205. All the samples in tracking window in the fractional domain are forced to be zero except the samples from 200 to 210 (peak position sample and its adjacent samples).

The inverse FrFT with a_{opt} equal to -1.7074 transforms the signal back to time domain after filtering. The azimuth, elevation target angles and processor output can then be computed.

Fig 2 shows the STDAE curves for the conventional processor in both cases of the interference scenarios. STDAE is calculated for different target SNR (from 20-100 dB). From Fig. 2 it is seen that the processor starts tracking at an STDAE value of 3.5 for a target SNR of 66.6 dB for a main beam interference. The tracking performance is enhanced when using the FrFT filtering technique where the tracking starts at an STDAE value of 3.5 at an SNR of 61.6 dB. Also from Fig 2 it is clear that in case of side lobe interference the processor starts tracking at a STDAE value of 3.5 for SNR equal 51.5 dB while the tracking performance is enhanced when using the FrFT filtering technique where the tracking starts at an STDAE value of 3.5 at an SNR of 46.5 dB. The results indicate an improvement of 5dBs in both cases when using the FrFT based Monopulse radar.

The STDAE curves for the spatial adaptive processor in both cases of the interference scenarios are shown in Fig 3 for different target SNR (from 20-100 dB). It is shown that the processor starts tracking at a STDAE value of 3.5 for SNR equal 54 dB while the tracking performance is enhanced when using the FrFT filtering technique where the tracking starts at an STDAE value of 3.5 at an SNR of 51 dB. This indicates an improvement of 3dBs when using the FrFT based Monopulse radar. Also it is clear that in case of side lobe interference the processor succeeds in reducing the inference (very low STDAE about 0.02) but still getting lower STDAE values for using the filtering in the optimum FrFT domain shown in the zoomed figure in Fig 3.

All previous STDAE values of the new monopulse based FrFT filtering structure are superior to those obtained from the conventional and the spatial processors at different target SNR using no filtering. For higher SNR the new system does not introduce a large enhancement to the tracking performance due to the fact that the target signal is much higher than the interference signal. The proposed system will work effectively when one target is in the look direction with and without interference without any additional constrains on the radar processor so it can work with any monopulse processor.

VI. Conclusion

The distortion resulting from interference appearing in the monopulse main lobe and side lobe has been investigated. The proposed FrFT based monopulse radar system configuration with the optimum *N* FrFT filters successfully reduces the interference noise signal and minimizes the STDAE for the both considered monopulse processors compared to the monopulse radar without filtering.

An improvement in the radar tracking ability for different SNR (lower STDAE) is gained by using the suggested cancelling technique (starts tracking at a STDAE value of 3.5 for SNR less than the conventional monopulse structure by about 5 dB for both the conventional and the spatial adaptive processors). One of the key advantages of the proposed system is that it will work efficiently even when only one target in the look direction (normal case) as well as when more high power interference exists in the look direction. This will be investigated in future work.

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