Al-Hanafy, Waleed and Hussin, Mohamed Nuri and Weiss, S. (2010) Incremental rate maximisation power loading with BER improvements. In: The 18th European Signal Processing Conference (EUSIPCO-2010), 23-27 August 2010, Aalborg, Denmark.

INCREMENTAL RATE MAXIMISATION POWER LOADING WITH BER IMPROVEMENTS

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ABSTRACT

This paper aims to maximise the rate over a MIMO link using incremental power and bit allocation. Two different schemes, greedy power allocation (GPA) and greedy bit allocation (GBA), are addressed and compared with the standard uniform power allocation (UPA). The design is constrained by the target BER, the total power budget, and fixed discrete modulation orders. We demonstrate through simulations that GPA outperforms GBA in terms of throughput and power conservation, while GBA is advantageous when a lower BER is beneficial. Once the design constraints are satisfied, remaining power is utilised in two possible ways, leading to improved performance of GPA and UPA algorithms. This redistribution is analysed for fairness in BER performance across all active subchannels using a bisection method.

1. INTRODUCTION

Transmission resources (bit and power) allocation according to channel conditions in multichannel systems has been proved to significantly enhance the overall system performance provided that channel state information (CSI) is known to the transmitter [1, 2]. This includes the achievement of either higher data rates or lower power requirements under one or more practical/design constraints known respectively in the literature as *rate maximisation* [3], or *margin maximisation* [4]. Multiple transmission channels arise for example in multicarrier systems such as OFDM and for MIMO systems using spatial multiplexing based on e.g. the singular value decomposition (SVD). In both cases a number of subchannels with different gains is obtained over which a reliable communication is to be established. The parameters to be considered in such loading problems are the bit error ratio (BER), the data rate and the total expended transmit power. The sum-rate of a multichannel system with different subchannel gains is of particular interest from the system design point of view which can be optimised using bit and/or power loading schemes.

Optimal standard water-filling solutions assume infinite modulation orders and real-valued data rates which is realistically infeasible and leads to a final rounding remedy step [3] that degrade the overall performance. Power and bit allocation problems are usually phrased as closed form expressions with respect to either channel capacity [5] or bit error probability [6]. Alternatively, so-called incremental or greedy approaches optimising sum-rate using power [7] and bit [8] loading schemes can achieve higher rates at the expense of computational complexity.

In this paper, the data rate maximisation is considered using both power and bit loading schemes. Two different greedy approaches are examined and compared, both are trying to maximise the overall rate with the same set of constraints. However, one of these algorithms considers greedy power allocation (GPA) that achieves the target BER to its maximum desirable value. The other algorithm [8] uses the greedy approach but with bit loading (power is uniform distributed among all subchannels) and has to carefully consider achieving of the average BER not to exceed the target BER, we call this algorithm: greedy bit allocation (GBA). Both approaches are compared with the standard uniform power allocation (UPA) scheme.

While achieving target BER, both GPA and UPA schemes would save some unused (excess) power, this power can be redistributed for BER improvements. Two power redistribution algorithms are considered, one simply allocate power equally among all active subchannels while the other achieves fair BER across these subchannels. The rest of this paper is organised as follows. In Sec. 2 the rate maximisation problem of our system model is formulated, while the greedy approach solutions are given in Sec. 3. BER improvement algorithms using excess power redistribution are proposed in Sec. 4. Simulation results evaluating system performance are highlighted in Sec. 5 and conclusions are drawn in Sec. 6.

2. PROBLEM FORMALISATION

We consider the problem of maximising the throughput of a narrowband MIMO system with N_T transmit and N_R receive antennas characterised by an $N_R \times N_T$ channel matrix $\mathbf{H}$ under the constraints of: a fixed total transmit power budget P_{budget}, a specified target BER $\mathscr{P}_b^{\text{target}}$, and fixed QAM modulation orders

$$M_k = \begin{cases} 2^{b_k} & 1 \le k \le K, \\ 0 & k = 0, \end{cases} \tag{1}$$

where the maximum constellation size $M_K = 2^{b^{\max}}$, with $b_k \in \{0, 1, 2, \cdots, b^{\max}\}$, is limited.

By means of a SVD, the channel matrix $\mathbf{H}$ can be decoupled into an N independent subchannels with gains of descending order $\sigma_i^2, 1 \le i \le N$, where $N = \text{rank}(\mathbf{H}) \le \min(N_R, N_T)$ and σ_i are the singular values of $\mathbf{H}$. This maximisation can be defined by the optimisation problem

$$\max \sum_{i=1}^{N} b_i, \tag{2}$$

subjected to the constraints

$$\sum_{i=1}^{N} P_i \le P_{\text{budget}} \quad \text{and} \quad \overline{\mathscr{P}}_b = \mathscr{P}_b^{\text{target}} \tag{2a}$$

or

$$\sum_{i=1}^{N} P_i = P_{\text{budget}} \quad \text{and} \quad \overline{\mathscr{P}}_b \leq \mathscr{P}_b^{\text{target}}, \tag{2b}$$

where b_i and P_i are, respectively, the number of bits and amount of power allocated to the ith subchannel. The average BER is defined as

$$\overline{\mathscr{P}}_b = \frac{\sum_{i=1}^{N} b_i \mathscr{P}_{b,i}}{\sum_{i=1}^{N} b_i} \tag{3}$$

with $\mathscr{P}_{b,i}$ being the BER of the ith subchannel. The aim of this paper is to explore the effect of these two different constraints on the overall data rate by using greedy algorithms that perform power or bit allocation, respectively. Moreover, BER improvement is the second stage of interest after achieving the maximum system throughput.

The channel-to-noise ratio of the ith subchannel is given by

$$\text{CNR}_i = \frac{\sigma_i^2}{\mathscr{N}_0}, \tag{4}$$

where $\mathscr{N}_0$ is the total noise power at the receiver, while its signal-to-noise ratio (SNR) is

$$\gamma_i = P_i \times \text{CNR}_i \,. \tag{5}$$

Closed form expressions and solutions of the throughput in (2) are extensively considered in the literature, see for example [9, 10] for a review. Based on the concept of the SNR-gap approximation [11], a closed form for b_i is given by [10]

$$b_i = \log_2\left(1 + \frac{\gamma_i}{\Gamma}\right), \tag{6}$$

where Γ denotes the SNR-gap that signifies the loss in SNR of a particular transmission scheme when compared to the theoretical channel capacity. For QAM modulation schemes, this SNR-gap is given by

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{\mathscr{P}_{s,i}}{4}\right)\right]^2, \tag{7}$$

where Q^{-1} is the inverse of the well-known Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$, and $\mathscr{P}_{s,i}$ is the symbol error ratio (SER) of the ith subchannel. It is clear from (7) that Γ is not fixed for all subchannel but depends on the subchannel SER which in turn depends on b_i and γ_i of (6). This dependence has to be taken into account whenever the rate or the gain in (6) is changed. Nevertheless, this approximation is valid only for very low BER, typically 10^{-6}, and higher QAM orders which is not usually the case for realistic applications [3].

Direct optimisation of (6) with the constraints in (2a) or (2b) under consideration leads to the well-know water-filling solution [5]. However, the resultant bit allocation obtained by the water-filling is real-valued and requires rounding off to the nearest integer value. This quantisation leads to an overall loss in performance. Alternatively and more accurately, greedy approaches [10] have been proved to be optimal in this sense [12, 13].

We assume *M-ary* QAM modulation where the BER is given by [14]

$$\mathscr{P}_{b,i} = \mathscr{F}(\gamma_i, M_k) = \begin{cases} Q\left(\sqrt{2\gamma_i}\right) & \text{for BPSK}, \\ \dfrac{1-\left[1-2\left(1-\frac{1}{\sqrt{M_k}}\right)Q\left(\sqrt{\frac{3\gamma_i}{M_k-1}}\right)\right]^2}{\log_2 M_k} & \text{for } M_k \text{ QAM}. \end{cases} \tag{8}$$

By allocating the power equally among all subchannels, the subchannels SNR γ_i in (8) is given by

$$\gamma_i = P_i \times \text{CNR}_i = \frac{P_{\text{budget}}}{N} \times \text{CNR}_i \,. \tag{9}$$

According to (8) and by assuming the existence of the inverse of $\mathscr{F}$, the minimum SNR that is required to achieve a throughput $b_k = \log_2 M_k$ with BER of $\mathscr{P}_b^{\text{target}}$ is

$$\gamma_k^{\text{QAM}} = \mathscr{F}^{-1}\left(\mathscr{P}_b^{\text{target}}, M_k\right) \tag{10}$$

3. INCREMENTAL BIT AND POWER LOADING

3.1 Incremental Bit Loading

In [8], an incremental bit loading approach is proposed to maximise the throughput and efficiently fulfil the quality-of-service (QoS) in terms of the mean BER, i.e., the constraints in (2b). However in order to achieve this, a power allocation scheme has to be predefined across all subchannels which was chosen to be a simple uniform power allocation (UPA). The algorithm then starts with filling all subchannels with the highest modulation order M_K and then iteratively remove bits from the worst subchannels in order to achieve the mean BER of (3) not to violate the constraint $\overline{\mathscr{P}}_b \leq \mathscr{P}_b^{\text{target}}$. This solution can be described as a greedy bit allocation (GBA) scheme, however, it lacks the beneficial of the efficient power distribution as power is equally distributed among all subchannels. In the following, we will introduce an efficient (greedy) power allocation scheme.

3.2 Greedy Power Allocation (GPA) Scheme

By adjusting the transmit power to exactly fulfil the target BER $\mathscr{P}_b^{\text{target}}$ across all subchannels $\mathscr{P}_{b,i} = \mathscr{P}_b^{\text{target}}$, the GPA algorithm is trying to maximise the throughput with the constraints in (2a). In order to achieve this, an initialisation step of a UPA has to be done first to load all subchannels with QAM orders M_{k_i} according to their γ_i in (9) and by using (10), where the index k_i is obtained such that

$$k_i: \quad \gamma_i \geq \gamma_k^{\text{QAM}} \quad \text{and} \quad \gamma_i < \gamma_{k+1}^{\text{QAM}}, \tag{11}$$

with $\gamma_0^{\text{QAM}} = 0$ and $\gamma_{K+1}^{\text{QAM}} = +\infty$ (cf. Fig. 1). The throughput of this UPA scheme is therefore

$$B_{\text{upa}} = \sum_{i=1}^{N} b_i^{\text{upa}} = \sum_{i=1}^{N} \log_2 M_{k_i} \tag{12}$$

while the difference (saved) power from the total budget is

$$P_d^{\text{upa}} = \sum_{i=1}^{N} \frac{\gamma_i - \gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i} = P_{\text{budget}} - \sum_{i=1}^{N} \frac{\gamma_{k_i}^{\text{QAM}}}{\text{CNR}_i} \tag{13}$$

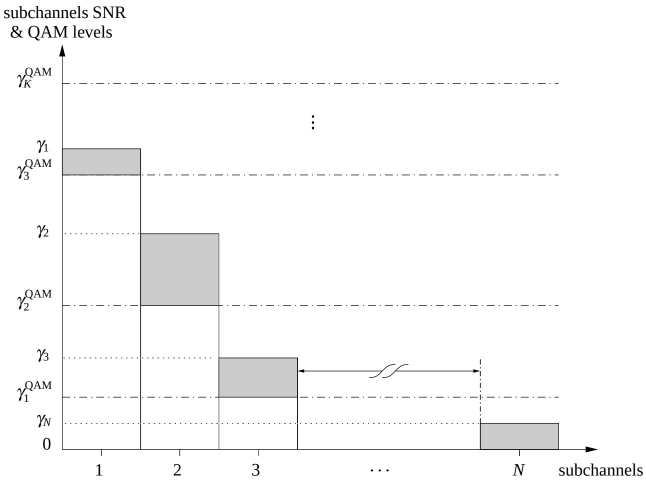

Figure 1: Subchannels residing into QAM levels according to their SNRs and UPA with excess power shown by the shadowed areas.

Table 1: Bit Loading using GPA - Constraint (2a)

Initialisation:
Initiate GPA with $P_d^{gpa} = P_d^{upa}$ in (13)
For each subchannel i do the following:
  Set $b_i^{gpa} = b_i^{upa}$ and k_i using (12) and (11), respectively
  Cal. the min required upgrade power $P_i^{up} = \frac{\gamma_{k_i+1}^{QAM} - \gamma_{k_i}^{QAM}}{\text{CNR}_i}$
Recursion:
while $P_d^{gpa} \geq \min(P_i^{up})$ **and** $\min(k_i) < K$
  $j = \underset{1\leq i \leq N}{\text{argmin}}(P_i^{up})$
  Update $k_j = k_j + 1$, $P_d^{gpa} = P_d^{gpa} - P_j^{up}$
  if $k_j = 1$
    $b_j^{gpa} = \log_2 M_1$, $P_j^{up} = \frac{\gamma_2^{QAM} - \gamma_1^{QAM}}{\text{CNR}_j}$
  elseif $k_j < K$
    $b_j^{gpa} = b_j^{gpa} + \log_2\left(\frac{M_{k_j}}{M_{k_j-1}}\right)$, $P_j^{up} = \frac{\gamma_{k_j+1}^{QAM} - \gamma_{k_j}^{QAM}}{\text{CNR}_j}$
  else
    $b_j^{gpa} = b_j^{gpa} + \log_2\left(\frac{M_{k_j}}{M_{k_j-1}}\right)$, $P_j^{up} = +\infty$
  end
end
Evaluate $B_{gpa} = \sum_{i=1}^N b_i^{gpa}$ and P_d^{gpa}

The procedures of the GPA algorithm based on the UPA initialisation is illustrated by Fig. 1 and given completely in Table 1. Then power difference from P_d^{upa} is collected and iteratively allocated to subchannels that do not yet reach their maximum allowable QAM level K. The throughput of this algorithm B_{gpa} and its final power difference from P_{budget}, P_d^{gpa}, are evaluated. The usage power of both UPA and GPA algorithms are therefore, respectively,

$$P_{used}^{upa} = P_{budget} - P_d^{upa}, \tag{14a}$$
$$\text{and} \quad P_{used}^{gpa} = P_{budget} - P_d^{gpa}, \tag{14b}$$

this is a useful measure of how efficient, in terms of power utilisation, both algorithms are. Note that this quantity is not defined for the GBA scheme as it uses, by definition, the total power budget.

4. BER IMPROVEMENT VIA EXCESS POWER REDISTRIBUTION

Since UPA and GPA algorithms presented in Sec. 3.2 cannot attain the complete usage of the total power budget due to the constraint of the fixed modulation orders, in addition that BER has to be tied to a given (target) value $\mathscr{P}_b^{target}$ for mathematical tractability. Therefore it is proposed in this Section to utilise the remaining excess power for BER performance improvement. This is done by redistribution of the difference power of both UPA and GPA algorithms, which can be achieved in two distinctive possible algorithms presented in the following.

4.1 Uniform Power Redistribution (UPR)

The simplest and most straightforward way to redistribute the excess power that is left unused by the UPA and GPA algorithms is to equally allocate these powers across all active subchannels regardless of how much BER improvement is attained by each subchannel. We call this power redistribution algorithm: uniform power redistribution (UPR). The excess powers P_d^{upa} and P_d^{gpa} are utilised for BER improvement of both UPA and GPA, respectively. The algorithm can be described for the UPA as follows:

1. Determine the active subchannels $i : b_i^{upa} \neq 0$ and their respective allocated modulation orders M_{k_i} that is occupied by the UPA, where k_i, as above, is the index of the QAM order M_k that is assigned to the subchannel i.
2. Calculate the minimum required SNR to achieve $\mathscr{P}_b^{target}$ across these active subchannels using (10) as $\gamma_{k_i}^{QAM} = \mathscr{F}^{-1}\left(\mathscr{P}_b^{target}, M_{k_i}\right)$.
3. Equally allocate the excess power P_d^{upa} among all active subchannels and compute the subchannels' new SNRs as
$$\gamma_i = \gamma_{k_i}^{QAM} + \frac{P_d^{upa}}{N_a} \times \text{CNR}_i\,, \tag{15}$$
where N_a is the number of active subchannels.
4. Calculate the subchannels' new BERs using (8) as $\mathscr{P}_{b,i}^{upa} = \mathscr{F}\left(\gamma_i, M_{k_i}\right)$ and then the mean BER $\overline{\mathscr{P}_b^{upa}}$ using (3).

The same procedures are applied for the GPA algorithm to redistribute P_d^{gpa} and obtain $\overline{\mathscr{P}_b^{gpa}}$.

4.2 Fairness-BER Power Redistribution (FPR)

The UPR presented above equally allocates the excess power among all active subchannels results in an unequal subchannels' BERs which depend on subchannels CNR_i and their occupied modulation orders M_{k_i}. Therefore, the expected mean BER $\overline{\mathscr{P}_b^{upa}}$ or $\overline{\mathscr{P}_b^{gpa}}$ may be dominated by the worst individual subchannel's BER as a result. Moreover, it is desirable to achieve same BER performance across all subchannels for fairness in QoS or link reliability applications. Therefore in this Section we adapt the power redistribution for an algorithm that can achieve this QoS fairness across all active subchannels for both UPA and GPA algorithms, this algorithm is referred here as fairness-BER power redistribution (FPR). Compared to the UPR algorithm, a new factor $\alpha_i \in \mathbb{R}, 1 \leq i \leq N_a, \sum_i \alpha_i = 1$ is introduced to the last term of the r.h.s of (15) to adjust the power redistribution conditions for equal BERs across all active subchannels. This can be mathematically formulated as

$$\begin{aligned} \text{solve for} \quad \alpha &= [\alpha_1, \alpha_2, \cdots, \alpha_{N_a}] \\ \text{that results in} \quad \gamma_i &= \mathscr{F}^{-1}\left(\mathscr{P}_b^F, M_{k_i}\right) \quad \forall i, \end{aligned} \tag{16}$$

where

$$\gamma_i = \gamma_{k_i}^{\mathrm{QAM}} + \alpha_i \cdot P_d \cdot \mathrm{CNR}_i \tag{17}$$

is the new subchannels' SNRs and $\mathscr{P}_b^F$ is the fair (constant) BER across all active subchannels. From (16) and (17), the entries of the unknown vector α are given by

$$\alpha_i = \frac{\mathscr{F}^{-1}\left(\mathscr{P}_b^F, M_{k_i}\right) - \gamma_{k_i}^{\mathrm{QAM}}}{P_d \cdot \mathrm{CNR}_i}, \quad 1 \leq i \leq N_a. \tag{18}$$

Since $\sum_{i=1}^{N_a} \alpha_i = 1$ and by defining the function

$$f\left(\mathscr{P}_b\right) \stackrel{d}{=} \sum_{i=1}^{N_a} \frac{\mathscr{F}^{-1}\left(\mathscr{P}_b, M_{k_i}\right) - \gamma_{k_i}^{\mathrm{QAM}}}{P_d \cdot \mathrm{CNR}_i} - 1, \tag{19}$$

it is possible to find a solution (root) $\mathscr{P}_b^F$ of $f\left(\mathscr{P}_b\right)$ such that $f\left(\mathscr{P}_b\right)|_{\mathscr{P}_b = \mathscr{P}_b^F} \simeq 0$. The bisection method is used to find such solution. The complete FPR algorithm is given as follows:

1. Given the active subchannels $i : 1 \leq i \leq N_a$ and their respective M_{k_i} as well as CNR_i and P_d for either UPA or GPA algorithm, calculate $\gamma_{k_i}^{\mathrm{QAM}} = \mathscr{F}^{-1}\left(\mathscr{P}_b^{\mathrm{target}}, M_{k_i}\right)$.
2. Locate two possible appropriate BER points that return the function $f\left(\mathscr{P}_b\right)$ in (19) with two opposite-sign values that are close to zero. These $\mathscr{P}_b$ points exists in the domain $\left(0, \mathscr{P}_b^{\mathrm{target}} - \varepsilon\right]$, where $\varepsilon \to 0^+$.
3. Use the bisection method to find the root $\mathscr{P}_b^F$ that returns $f\left(\mathscr{P}_b^F\right) \to 0$. This BER solution is denoted by $\mathscr{P}_b^{F,\mathrm{upa}}$ for the UPA algorithm and by $\mathscr{P}_b^{F,\mathrm{gpa}}$ for the GPA algorithm.

Note that, the complexity of this algorithm is dominated by the root finding search method. Faster methods can be located in the literature, however the bisection method is selected for its relative simplicity.

5. SIMULATION RESULTS

A 4x4 MIMO system of frequency-flat channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ with entries $h_{ij} \in \mathscr{CN}(0,1)$ is considered in this simulations. A target BER of $P_b^{\mathrm{target}} = 10^{-3}$ is to be achieved through the bit loading schemes presented in this paper. Fixed QAM modulation orders of $\left\{2^1, 2^2, \cdots, 2^{b^{\max}}\right\}$, where $b^{\max} = 6$ bits, are constrained by the system under consideration. Both GBA algorithm of Wyglinski *et. al* [8] and our proposed GPA algorithm presented in Sec. 3.2 along with the UPA scheme are conducted in this simulation.

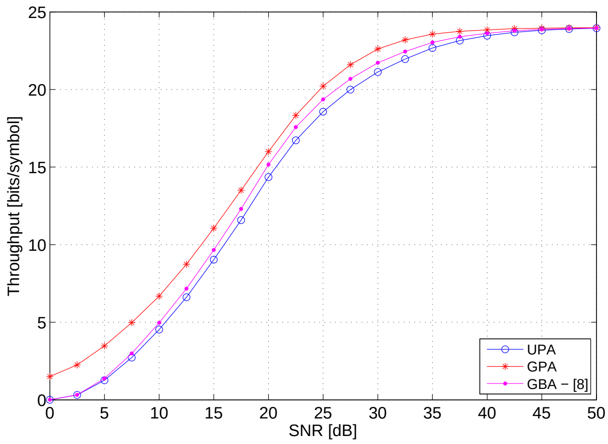

Figure 2: Throughput results for a 4x4 MIMO system with $\mathscr{P}_b^{\mathrm{target}} = 10^{-3}$ and varying SNR.

It is shown from the throughput results in Fig. 2 that GPA algorithm performs better than both GBA and UPA algorithms. An explanation to this is as follows: since the power allocation of the GBA algorithm is done using the UPA which is an inefficient power allocation scheme, therefore wasting power for unnecessary improvement (compared to the requirement of $\mathscr{P}_b^{\mathrm{target}}$) of the mean BER $\overline{\mathscr{P}}_b < \mathscr{P}_b^{\mathrm{target}}$. On the other hand, the GPA algorithm is efficiently utilise the total power budget P_{budget} (power is allocated according to the greedy approach) to maximise the overall throughput while achieving BER to its maximum requirements, $\mathscr{P}_{b,i} = \mathscr{P}_b^{\mathrm{target}}, \forall i$. This means better investment of the total power towards the rate maximisation problem.

In Fig. 3, the power usage of UPA and GPA algorithms are compared, in conjunction with the achieved rate in Fig. 2, which shows better performance of GPA over UPA algorithm. Note that GBA algorithm (shown as the P_{budget} curve) cannot be compared here as it spends all power budget getting improvement in the achieved average BER as will be shown in Fig. 4. Once the throughput reaches its expected maximum of $4(\text{subchannels}) \times 6\,\text{bits} = 24\,\text{bits}$, extra power is no longer required. Therefore, the effective used power for both UPA and GPA algorithms in (14a) and (14b), respectively, starts to saturate to the minimum power that is theoretically required to achieve the maximum bit loading $b^{\max}$ for all subchannels, i.e. $\sum_i \frac{\gamma_K^{\mathrm{QAM}}}{\mathrm{CNR}_i}$, which is found to be $\approx 38.17\,\mathrm{dB}$ and highlighted by the dashed line in Fig. 3.

As proposed in Sec. 4 and demonstrated by Fig. 3, the excess power of UPA and GPA algorithms are redistributed to improve the BER performance. Fig. 4 shows these improvements for both power redistribution algorithms UPR and FPR compared to the actual achieved BER of the GBA algorithm. Mean BER is investigated against varying SNR showing BER improvements compared to the target BER (of 10^{-3}). Interestingly, both UPA and GPA algorithms with excess power redistribution can achieve better performance than the GBA algorithm of [8], again these results are in conjunction with the achieved rates in Fig. 2. It is also noted that FPR performs better than UPR if applied to the UPA, while the situation is inverted for the GPA algorithm. This can be attributed to that since the excess power of the UPA

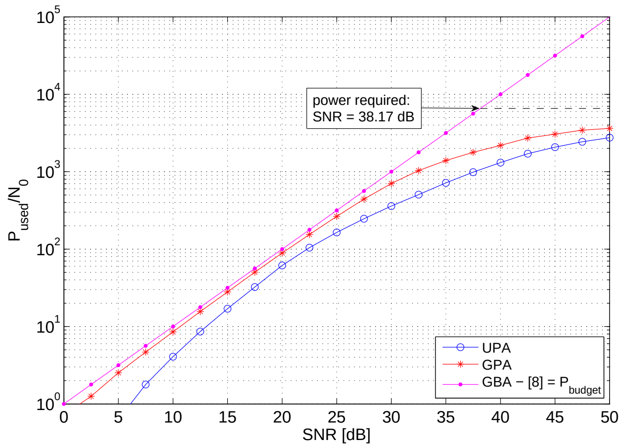

Figure 3: Power usage for a 4x4 MIMO system with $\mathscr{P}_b^{\text{target}} = 10^{-3}$ and varying SNR.

algorithm is greater than that of the GPA, it is most likely that mean BER of UPA-UPR is dominated by subchannels of poor CNR_i while FPR algorithm is advantageous in this case because of its inherent fair BER property. On the other hand, for the GPA algorithm since the excess power is relatively small and another constraint of balancing BERs across all active subchannels, most of the redistributed power will be seized by subchannels in lower QAM levels leading to lower BER performance compared to that obtained by the UPR algorithm.

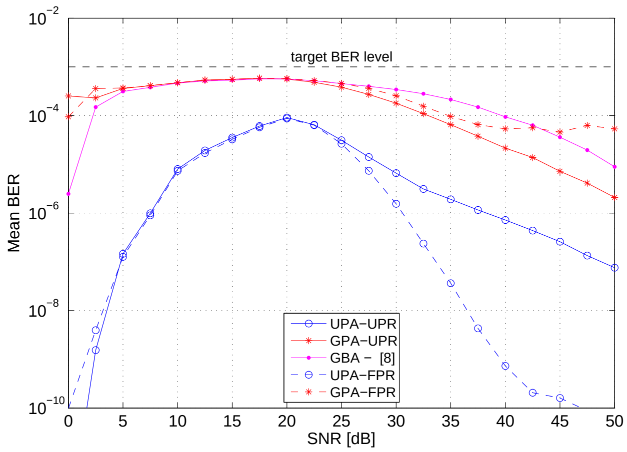

Figure 4: BER improvements of UPA and GPA algorithms.

6. CONCLUSIONS

Inefficient uniform power allocation (UPA) scheme leads to poor throughput performance of multichannel systems with constrained-loading parameters. This can be improved through rate maximisation using greedy power GPA and bit GBA allocation schemes. However, since GBA approach sacrifices power utilisation by adapting UPA for BER improvements, degradation in achieved data rate is expected as a result. By optimising power allocation, GPA demonstrates optimal performance in the rate maximisation sense. Another aspect of UPA and GPA schemes is the saving power in achieving target BER, this power can be redistributed for better BER with different design aspects. Simulation results show that GPA can achieve better BER performance compared to the GBA scheme.

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