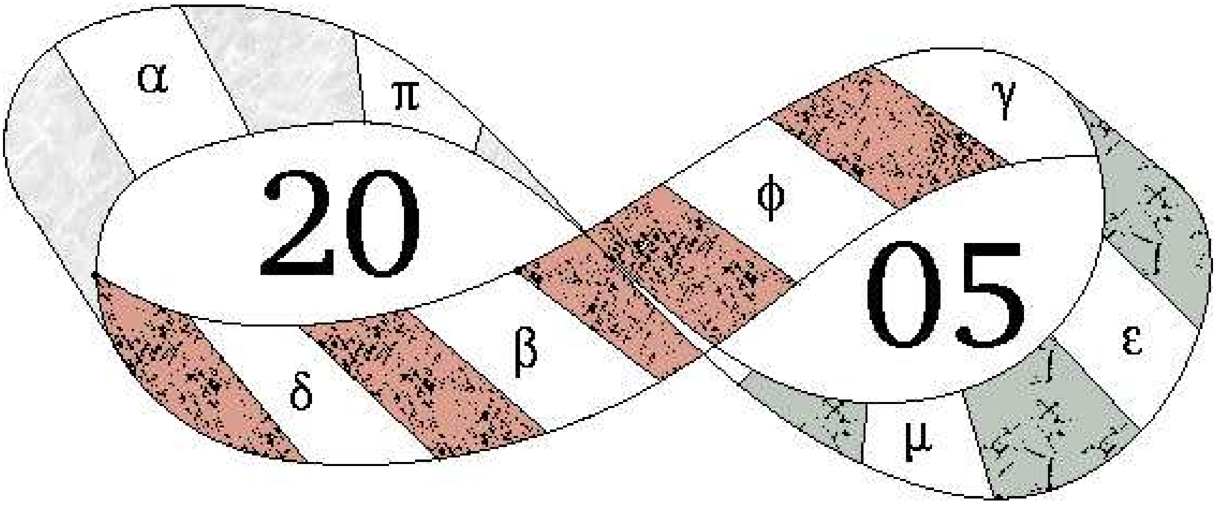
EDINBURGH MATHEMATICAL SOCIETY
POSTGRADUATE STUDENTS' MEETING



The Burn, Edzell

10th – 12th May 2005

Organising Committee

Martin Bees (Glasgow)
Christina Cobbold (Glasgow)
Sophie Huczynska (St. Andrews)
Adam McBride (Strathclyde)

	Tuesday 10 May
1600	TEA
1645	**Björn Assmann** Representations of polycyclic groups
1715	**Sarah Hulton** Renormalization of the correlations in the fluctuations for the generalized Harper equation
1745	**Steven Johnstone** Fourier multipliers
1815	**Matthew Mustard** Cyclic Field Vole populations: investigating the role of season length *Session Chairs: Kirsty Gordon & Miles Gould*
1900	DINNER
2015	**Adam McBride** "Icebreaker" (bring pen, paper & brain. . .)

	Wednesday 11 May
0900	BREAKFAST
1000	**Robert Brignall** Finite basis results of wreath products
1030	**Gavin Dunn** The non-isothermal spreading of a thin drop on a heated or cooled horizontal substrate
	Session Chairs: Richard Vale & Erik Jan de Vries
1100	COFFEE
1130	**Henry Cove** Two-loop string theory on branched covers
1200	**Martin Hamilton** Minimal generating sets of groups, rings and fields
1230	**Chris Smith** Mathematical modelling of blood flow in the human placenta
	Session Chairs: Sam Halliday & Peter Gallagher
1300	LUNCH
1900	DINNER
2015	**Agata Wronka** Mathieu equation
2045	**Katie Russell** Exciting topological solitons
	Session Chairs: Michael Graham & James Ferguson

Thursday 12 May

0900	BREAKFAST
1000	**Martin Hille** Gluing at infinity
1030	**Alan Walker** Dynamical fluctuations in SmA and SmC liquid crystals *Session Chairs: Chris McCaig & Enrique Covarrubias*
1100	COFFEE
1130	**David Salisbury** An introduction to the (polynomial) invariants of finite groups
1200	**Helen Waugh** Mathematical modelling in diabetic wound healing
1230	**Graeme McGuiness** A method for extending a class of fragmentation equations *Session Chairs: Ignacio Ramis Conde & Peter Davidson*
1300	LUNCH

Reserve session chairs: Revaz Kurdinani, Mateja Presern, Jan Hilmar.

Abstracts

Representations of polycyclic groups
Björn Assmann, St. Andrews

A group is called polycyclic if it has a subnormal series with cyclic factors. In the late 1950s it was conjectured by Philip Hall that every polycyclic group can be represented as a matrix group over $\mathbb{Z}$. This was proved to be true by Louis Auslander about ten years later.
In this talk I want to give an overview of Auslanders' proof. Further I want to report on a recent algorithm by Ostheimer and Lo for computing a representation for a given polycyclic group.

Finite basis results of wreath products.
Robert Brignall, St. Andrews

A pattern class of permutations which is closed under pattern involvement may be described in terms of its basis. The wreath product construction $X \wr Y$ of two closed pattern classes X and Y is also closed, and we aim to investigate classes with the wreath finite basis property, i.e. those classes Y such that $X \wr Y$ is finitely based for all finitely based classes X.

The profile of a permutation has previously been used to to reduce permutations by contracting consecutive increasing symbols into a single symbol. Here we extend this notion to the Y-profile, where we reduce permutations by contracting patterns from a closed class Y. The relation between the Y-profile and the wreath product is demonstrated, which reduces the wreath finite basis property to a simpler bounding problem, relying on considering pairs of symbols from basis elements of the class X involved within a given Y-profile.

Pairs of symbols, within a permutation, partition the permutation into nine regions. They also describe their own unique minimal block (the smallest interval containing both symbols), and we see these concepts are fundamental to the success or failure of the bounding problem. This motivates the concept of extensions and extended blocks, which can then be used to produce a generalised Wreath Finite Basis Property result.

Two-loop string theory on branched covers.

Henry Cove, Heriot-Watt

A description of the two loop calculation of the discrete light cone quantised bosonic string free energy at finite temperature.

The non-isothermal spreading of a thin drop on a heated or cooled horizontal substrate.

Gavin Dunn, Strathclyde

We revisit the spreading of a thin drop of incompressible Newtonian fluid on a uniformly heated or cooled smooth planar surface. The dynamics of the moving contact line are modelled by a Tanner Law relating the contact angle to the speed of the contact line. The present work builds on an earlier theoretical investigation by Ehrhard and Davis (*JFM*, **229**, 365–388 (1991)) who derived the non-linear partial differential equation governing the evolution of the drop. The (implicit) exact solution to the two-dimensional version of this equation in the limit of quasi-steady motion is obtained. Numerically calculated and asymptotic solutions are presented and compared. In particular, multiple solutions are found for a drop hanging beneath a sufficiently cooled substrate. If time permits, some basic models for evaporative spreading will be considered.

Minimal generating sets of groups, rings and fields.

Martin Hamilton, Glasgow

I will introduce the notion of a minimal generating set, and give some specific examples. Then I will show that not every group, ring or field will have one.

Gluing at infinity.

Martial Hille, St. Andrews

Let $M_\epsilon := H^2/G_1 * \gamma_\epsilon G_2 \gamma_\epsilon^{-1}$ be the surface obtained by gluing two hyperbolic surfaces $M_1 = H^2/G_1$ and $M_2 = H^2/G_1$ at some point ξ on the circle at infinity. In here γ_ϵ refers to the reflection at the geodesic of Euclidean distance ϵ to ξ. One finds that the spectrum of the Laplacian $\sigma(\Lambda(M_\epsilon))$ for the glued surface converges in a certain sense to the spectrum of the disjoint union of M_1 and M_2. The talk will clarify this type of convergence and will give a sketch the proof.

Renormalization of the correlations in the fluctuations for the generalized Harper equation.

Sarah Hulton, Stirling

A renormalization analysis is presented for a generalized Harper equation

$$(1 + \alpha \cos(2\pi(\omega(i + 1/2) + \phi)))\psi_{i+1} \\ + (1 + \alpha \cos(2\pi(\omega(i - 1/2) + \phi)))\psi_{i-1} + 2\lambda \cos(2\pi(i\omega + \phi))\psi_i = E\psi_i \,.$$

For ω having periodic continued-fraction expansion, we construct the periodic orbits of the renormalization strange set in function space that governs the correlations of the fluctuations of the solutions of the generalized Harper equation for the strong-coupling limit $\lambda \to \infty$.

Fourier multipliers.

Steven Johnstone, Strathclyde

The Fourier transform is a useful tool for studying constant coefficient differential operators and, more generally, translation invariant operators on $\mathbb{R}^d$. In particular, any constant coefficient differential operator can be realised on the "Fourier side" as a polynomial multiplier operator. In essence, calculus problems can be turned into easy to handle algebraic problems.

For a large class of Lie groups there is an analogue to the Fourier transform termed the Plancherel transform. The main aims of this talk are to give a very brief introduction to the Plancherel transform and to give examples of how calculus problems on Lie groups can be transformed into (non-commutative) algebraic problems.

A method for extending a class of fragmentation equations.

Graeme McGuiness, Strathclyde

Fragmentation equations arise in a variety of guises in the natural sciences. One example is the break-up of rocks when a suitable force is applied. By adopting methods from functional analysis, I shall show that known results on weighted Lebesgue spaces can be extended to related spaces of generalised functions. Hence, we can then consider analogous initial-value problems where the initial condition is of the form of a translated Dirac delta.

Cyclic Field Vole populations: investigating the role of season length.

Matthew Mustard, Heriot–Watt

Many wild rodent populations exhibit multi-annual population cycles of 2-5 years. Field studies are yet to reveal what makes the different populations and species cycle, and whether or not they cycle for the same reasons. For the cyclic populations of Field Voles in Kielder forest (North England) studies have revealed that the length of the breeding season correlates strongly with the vole density 9 months previously. Can this delayed density dependent effect alone produce multi-annual population fluctuations of the correct period?

Exciting topological solitons.

Katie Russell, Heriot–Watt

Many nonlinear classical field theories admit exact solutions, called topological solitons, which are stable and topologically interesting. Topological solitons have a localised energy, this is one of the properties which makes them like particles. Examples which will be briefly discussed are magnetic monopoles and Skyrmions. It has been shown that a particular type of magnetic monopole retains energy from an excitation for a (relatively) long time after being perturbed. It is in this sense that the adjective 'exciting' comes into the title, though the other sense holds for many mathematical physicists...

An introduction to the (polynomial) invariants of finite groups.

David Salisbury, Aberdeen

I will give an introduction to the polynomial invariants of finite groups including the transfer method and the Dickson algebra. If there is time I will give some results about the invariants under finite groups of the divided powers algebra.

Mathematical modelling of blood flow in the human placenta

Christopher Smith, Strathclyde

We are investigating maternal blood flow to try to understand the key factors influencing the transport of oxygen and nutrients to the baby, and waste and carbon dioxide in the other direction. As the group led by Prof. Oliver Jensen at the University of Nottingham with whom we are collaborating have found, it is not easy to view the flow of blood in the placenta. Hopefully our mathematical model will give an indication of what is actually happening within the placenta.

We present a simple mathematical model that describes the maternal blood flow inside the placenta. Specifically, we consider steady two-dimensional flow of maternal blood through a single placental compartment filled with a uniform, porous medium (a greatly simplified representation of the dense villous trees filling the compartment).

We use the well-established Darcy Law to describe the maternal blood flow. We use conformal mappings to provide analytical solutions for the pressure and velocity in the special cases of infinitely deep and infinitely wide compartments, and numerical methods to compute the corresponding results for a finite-sized compartment. Darcy's Law (which is valid only for flows with negligible inertia) is then generalised to Forchheimer's equation, and the numerically computed results indicate that inertia plays a significant role in the penetration of the oxygen-rich maternal blood into the compartment. Finally, we use an appropriate advection-diffusion equation to compute the transport of oxygen from the mother to the baby.

This work is supported by the Carnegie Trust for the Universities of Scotland via a Carnegie Scholarship.

Dynamical fluctuations in SmA and SmC liquid crystals

Alan J. Walker, Strathclyde

We apply incompressible dynamic theory for SmA and SmC [1,2] to wave-like fluctuations of the smectic layers. A comparison with compressible dynamic theory will be made, especially with earlier theories presented in papers by Martin, Parodi & Pershan [3], Weinan E [4], and Sukumaran & Ranganath [5]. An identification of dominant viscosity coefficients will be made. Polar plots, which are of relevance to physical data, will be presented. These depict the propagation modes as a function of the direction of propagation. A discussion of the related power spectrum and integrated intensity will also be made.

References

1. Stewart, I.W., The Static and Dynamic Continuum Theory of Liquid Crystals, 2004 (London: Taylor and Francis).

2. De Gennes, P.G. and Prost, J., The Physics of Liquid Crystals, 2nd Edn., 1993, (Oxford: Clarendon).

3. Martin, P.C., Parodi, O., Pershan, P.S., 1972, Phys. Rev. A, 6, 2401

4. E, W., 1997, Arch. Rat. Mech. Anal., 137, 159

5. Sukumaran, S., Ranganath, G.S., 1998, Phys. Rev. E, 57, 5597

Mathematical modelling in diabetic wound healing

Helen Waugh, Heriot-Watt

Diabetes is a disease affecting around 100 million people worldwide, and due to the nature of the disease complications result. One of the most common complications is poor wound healing, and the resulting wounds are often difficult to encourage to heal without extensive treatment. Indeed, in 20% of cases the only way to solve this problem is to amputate the affected limb.

Mathematical modelling can be used to understand aspects of wound healing in diabetes and explore possible reasons as to why these wounds do not follow normal wound healing trajectories.

Mathieu equation

Agata Wronka, Edinburgh

Consider an equation

$$\ddot{x} - (a + b\cos(t))x) = 0 \tag{1}$$

with two real nonnegative parameters a, b. This equation was derived by Mathieu in 1868 in studies on vibrations of an elliptic membrane.

From the Floquet theory it follows that there are regions in (a, b)-plane where a solution of the equation (1) is either bounded or unbounded. Stable and unstable regions are separated by the *transitions curves* which are given by those values of parameters a, b for which a solution is periodic.

The aim of this talk is to present the background for the Floquet theory, and also to consider an approximation of the distance between two transition curves for fixed b. In other words determine an approximation of the length of the interval within which a pair (a, b) gives unstable solution of the equation (1).

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