# On Preconditioning Strategies for Geotechnics

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ABSTRACT: Iterative solvers are of increasing interest in geomechanics with the move towards 3D finite element modelling. Potentially, these methods can lead to reduced computational complexity as, unlike direct methods, they do not require the full system matrix to be assembled. In general, however, iterative solvers have not been widely adopted in geomechanics due to problems with convergence. This paper reviews the background to iterative methods for elastic and elasto-plastic material models. In some cases, existing numerical methods can be taken from research in the mathematics community. For other systems, further work is needed. The paper provides demonstrations of the capabilities of some strategies.

## Introduction

In FE calculations the majority of computing resources are spent solving the linear system $\mathbf{Ku} = \mathbf{f}$ where $\mathbf{u}$ is the unknown vector of displacements, $\mathbf{f}$ is the force vector and $\mathbf{K}$ the coefficient matrix. Iterative solution of the FE equations

using the conjugate gradient (PCG) method requires preconditioning, i.e. we solve instead …

$$\mathbf{P}^{-\frac{1}{2}}\mathbf{K}\,\mathbf{P}^{-\frac{1}{2}}\left(\mathbf{P}^{\frac{1}{2}}\mathbf{u}\right) = \mathbf{P}^{-\frac{1}{2}}\mathbf{f}$$

where $\mathbf{P}$ is the preconditioner. Choosing a preconditioner requires knowledge of the coefficient matrix, $\mathbf{K}$. In geomechanics we are often dealing with a "difficult" $\mathbf{K}$ and the choice of preconditioner is not clear.

In linear elasticity problems the effect of Poisson's ratio on speed of convergence using PCG is well-known. Fig. 1 shows iteration count against nos. of degrees of freedom (n) for a simple LE problem solved using PCG and diagonal scaling, for the two simplest triangular FEs.

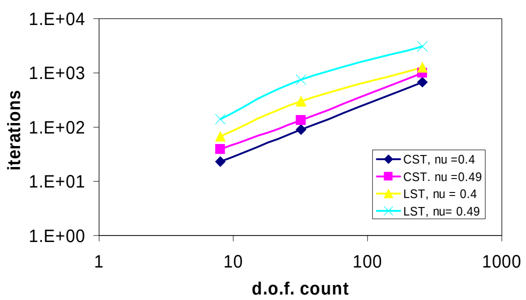


*Figure 1: Iteration counts against problem size for LE problem*

Our initial studies, discussed in the paper, indicate that the effect of Poisson's ratio may dominate system condition, for elasto-plasticity problems as well. Only when there is an appreciable proportion of the domain with stress points on the yield surface does this situation change.

## Element-based preconditioning

Since geotechnical FE modelling is often characterized by a mixture of material models and property values, stiff inclusions such as foundations and tunnels, and analyses where stiffness changes (due to yield) it seems improved preconditioning methods might be developed working **element-by-element (EBE)** instead, thus including element level information that should improve the preconditioner. The most prominent EBE method is due to Hughes et al. (1983) where revised element stiffness matrices $\mathbf{K}_e$ are formed which are ensured to be positive definite by a process called regularisation,the preconditioner is then built from a factorization of $\overline{\mathbf{K}}_e$ .

## Performance of EBE methods

Figure 2 shows the performance of two EBE preconditioning methods, compared to plain diagonal scaling, for a plane strain footing problem with linear elastic material properties

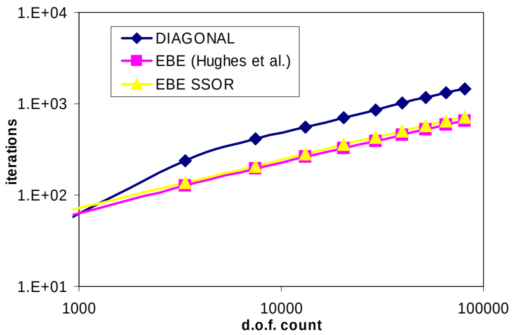


*Figure 2: Iteration counts against problem size for LE footing problem*

The noticeable features are (1) the improvements offered by EBE approaches and (2) the similar (unattractive) convergence rates of all methods. Moving to elasto-plastic problems should alter the system condition and hence change the iteration count. Table 1 shows iteration counts at particular steps in incremental analyses of a 2D plane strain footing and a 2D thick cylinder under internal pressure. In both cases, for simplicity, the material model is elastic-perfectly plastic, with a von Mises yield criterion (which gives a symmetric coefficient matrix). Iteration counts are provided when the domain is completely elastic, at first yield and at the full applied load or prescribed displacement.

| | Footing | | | Cylinder | | |
|---|---|---|---|---|---|---|
| | DIAG | EBE-HLW | EBE-SSOR | DIAG | EBE-HLW | EBE-SSOR |
| First step | 1095 | 453 | 503 | 254 | 98 | 103 |
| First yield | 1096 | 414 | 505 | 258 | 99 | 104 |
| Final step | 1555 | 589 | 705 | 355 | 130 | 147 |

*Table 1: iteration counts for elasto-plastic problems*

Onset of plasticity has no great effect on iteration counts. Ill-conditioning due to plasticity is significant only when there are large zones of yielding (as is the case with each problem at the final step).

## Conclusion

These simple tests appear to show that change in Poisson's ratio in the elastic part of a constitutive model is much more likely to affect iteration count for a PCG solution of linear systems arising from explicit FE calculations than is the presence of plasticity. Further work is underway to confirm this for a range of soil models