Effective Scraping in a Scraped Surface Heat Exchanger: Some Fluid Flow Analysis

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Abstract

An outline of mathematical models that have been used to understand the behaviour of scraped surface heat exchangers is presented. In particular the problem of the wear of the blades is considered. A simple model, exploiting known behaviour of viscous flow in corners and in wedges, and accounting for the forces on the blade is derived and solutions generated. The results shows initial rapid wear but that the wear rate goes to zero.

Introduction

Scraped surface heat exchangers (SSHE) are used in a variety of food processes. Central to the operation is the careful design of the scraping blades as these generate circulating flow field to continuously renew the food material on the outer heat exchange surface. The developments by the current research team are created within a multidisciplinary research framework and a number of mathematical models have been used to create a theoretical framework to aid understanding of the flow regimes within SSHEs. Our general approach is to consider foods that are shear-thinning, heat-thinning fluids and then to study the resulting flow using a blend of numerical techniques and analytical approaches. In addition we have considered a number of elucidating paradigm problems that involve some geometric or rheological simplification in order to gain insight though analytical solutions. Within this introduction we give a brief description of the various aspects of our work studying the SSHE.

We have exerted a significant amount of research effort in computing numerical solutions for steady-state flow of heat-thinning and shear-thinning fluids in a prescribed geometry. A 2-D lid-driven cavity [1,2] considers the limit where the blades close off a rectangular or parallelogram region and one surface moves at prescribed velocity over the stationary blades. We have found how the location of the primary eddy's centre is determined by the shear-thinning index and the Reynolds number. A two dimensional annular cross-section of the SSHE with a periodic array of blades has also been simulated [3] and compares well with available data. The angle and length of the blades have been varied to understand their influence on the flow pattern. In all these calculations a small amount of fluid has been allowed to travel under the blade in order to remove the stress singularity that occurs where the sliding surface is in contact with the blade. One of the natural extensions to [2] is being presented at this meeting [4] where a steady 3D simulation of the SSHE is examined.

In the aforementioned numerical approaches the blades are given a fixed position and shape. Because of the central role of the blades in the scraping process we have also considered the movement of freely pivoted blades. In particular we have looked at a periodic array of blades [5] in a slender channel where the reduced Reynolds number of the flow is sufficiently small that the assumption of lubrication flow is appropriate. We have found that in such cases the blade can have multiple steady-state positions. However, due to the assumption of no moment on the blades the blade tips can never come into contact with the channel walls. When contact does occur the force due to the outer moving surface can be included but a singularity in the stress then arises. We have considered a number of different theories to alleviate this problem including use of a fluid slip condition, a power law fluid and, as in this paper, a small flow through the blade contact region.

Other problems of relevance to SSHE behaviour include the paradigm problem of parallel flow shear flow in a channel for a heat-thinning and shear-thinning fluid [6]. There is a particularly interesting limit where pressure gradient along the channel tends to zero. It is well known that when this pressure gradient is non-zero there can exist multiple steady state solutions and even non-existence of a steady states [7,8], and that for zero-pressure gradient there is a unique solution in cases where the wall velocity is prescribed. We have been studying the structure of these various solutions, particularly in the non-unique case.

Another class of paradigm problem we have considered are high Peclet-number flows in a lid-driven cavity [9]. There are a number of analytical simplification that can be made to help understand the underlying mechanisms that drive heat transfer in an SSHE. These approaches also will help clarify issues related to numerical diffusion that can cause difficulties with our numerical solutions particularly in the narrow thermal boundary layers present in SSHEs.

Finally, the problem that we will discuss in more detail in this paper are how the resultant forces acting upon the blade affect the quality of scraping. Blade wear is very important and poorly designed blades can lead to excess fouling on the heated/cooled surface and unwanted significant blade fragments in the finished food product.

The Blade Wear Problem

The wear of scraper blades is due to rubbing between the blade tip and the moving hard outer surface. To simplify the analysis consider the fluid in the exchanger to be Newtonian and neglect any of the effects of temperature on either the fluid or the wear rate. The basic idea is to consider the fluid dynamics around the scraper and to use this to identify the forces acting on the tip. The flow under the blade tip changes as the blade wears thereby altering the forces on the blade and we show how these effects interact to determine the tip wear rate.

In order to simplify the analysis, but still remain relatively close to the physical problem, consider the scraper to be straight, pivoted at its end point and to be “touching” the outer moving surface. The scraper will be taken to be at a small angle, α, to the moving surface under the scraper. In addition the flow above the scraper will be taken to be very simple (the scraper is assumed to be a long way from the inner surface) so that we can take the flow to be at pressure p_a (this infers that the scraper must be held on the outer surface due to suction of the fluid behind the blade rather than by fluid pressing to get over the blade).

One of the more controversial aspects of the modelling is to decide the mechanisms whereby fluid can get under the tip, between the scraper and the moving surface. Here the ideas usually applied to flow of fluid in cracked rock seem applicable. The tip behaviour will first be modelled by presuming that where the scraper touches the outer surface there is still a small tortuous gap of small height h^* that the fluid can pass through. This height will effectively be determined

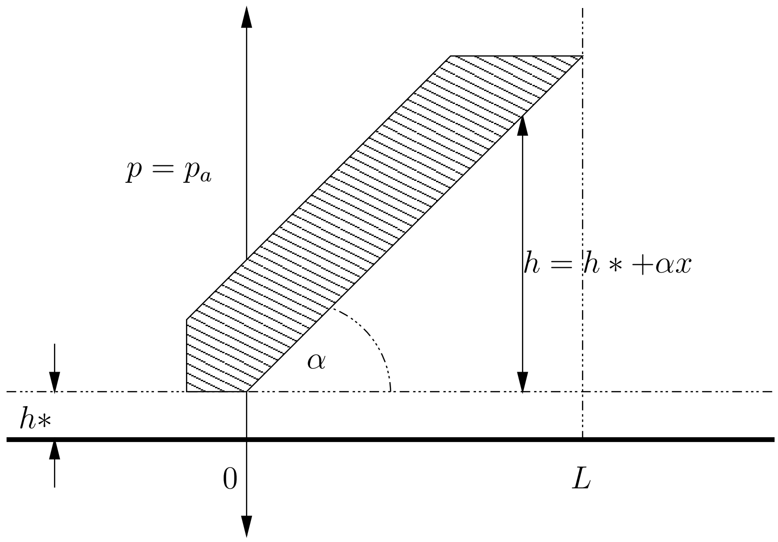

Figure 1: The basic geometry of the scraper blade being considered

by the asperity height on the hard outer surface. A simple and realistic model is to take h^* to be the asperity height and hence, because of the sliding action, this is independent of the force applied to the blade. There is extensive discussion about the appropriate law for flow through such a set of touching surfaces but, in general, they employ a "tortuocity" parameter to account for the tortuous path the fluid takes.

A second modelling difficulty is to identify the mechanisms that cause wear. Again there are many different models but the basic parameters in these are usually the normal stress applied between the two surfaces and the relative tangential speed of the two surfaces (the aim being to estimate the shear stress induced in the asperities). The simplest model is to take the wear rate as proportional to the normal stress times the sliding velocity

The Model

The basic problem to be studied is outlined in Figure 1. Here the tip is "touching" the outer surface in the region $-l(t) < x < 0$ where $l(t)$ is the length of the contact region between the tip and the moving outer surface which will increase as the scraper wears. Using the basic ideas given above we can write the following system of equations down. In the contact region under the blade tip, $-l(t) < x < 0$, the flux of fluid, $Q(t)$, is given by

$$Q(t) = \frac{Uh^*}{2} - \frac{\beta(h^*)^3}{12\mu}\frac{(p(0,t) - p(-l(t),t))}{l(t)} \,, \tag{1}$$

where the constant $\beta < 1$ is the tortuocity parameter, and we have to determine the pressures at the two ends $p(0,t)$ and $p(-l(t),t)$.

To find the pressures we consider the flow in front of the blade and in the wedge under the blade. We consider the addition of two classic fluid flow problems concerning flow in a wedge. These are a "driven corner flow" (one stationary wall and one moving wall) together with a low Reynolds number version of "Jeffrey-Hamel flow" (both walls stationary and a given flow into the corner) [9]. These solutions depend on the radial distance from an assumed sharp corner. For this problem we take the solutions to be valid until a radial distance equal to h^* where we anticipate the solution changes to the solution in the contact region. Hence the flow will only be approximate. In addition the pressure in the corner depends on both the radial distance and the angular variable. We have taken an average of the pressure over this angle as representing the pressure exerted at the end of the contact region. For the flow in the wedge of angle α under the blade such analysis gives

$$p(x,t) = p_a - A_\alpha \frac{\mu U}{x+h^*} + B_\alpha \frac{\mu Q}{(x+h^*)^2} \ . \tag{2}$$

while for the flow in front of the blade, where the wedge angle is $\pi/2$ we get the result

$$p(-l(t),t) = p_a + A_{\frac{\pi}{2}} \frac{\mu U}{h^*} - B_{\frac{\pi}{2}} \frac{\mu Q}{h^{*2}}. \tag{3}$$

Here A_α and B_α are constants defined as :

$$A_\alpha = 2\,\frac{1-\cos\alpha}{\alpha - \sin\alpha}, \quad B_\alpha = \ 2\frac{\sin\alpha}{\sin\alpha - \alpha\cos\alpha}. \tag{4}$$

The force applied in the region of contact is due to the moment around the pivot point. This consists of an integral of the pressure in the wedge under the blade (2) and to the pressure in the contact region under the tip. Hence the stress taken by solid-solid contact in the tip area is given by

$$\sigma = \frac{1}{lL}\int_0^L (L-x)\left(\frac{A_\alpha \mu U}{x+h^*} - \frac{B_\alpha \mu Q}{(x+h^*)^2}\right)\,dx + \frac{(A_\alpha - A_{\frac{\pi}{2}})\mu U}{2h^*} - \frac{(B_\alpha - B_{\frac{\pi}{2}})\mu Q}{2h^{*2}}. \tag{5}$$

Finally we have to model how the contact length, $l(t)$, will change due to the wearing of the blade. If the rate at which the blade material is worn away is given by $\gamma U\sigma$, where γ is a coefficient related to the hardness of the blade material, then the behaviour of length of the contact region is determined by

$$\frac{dl}{dt} = \frac{\gamma U}{\sin\alpha}\sigma \ . \tag{6}$$

From the pressure equations and the definition of $Q(t)$ we get

$$Q(t) = \frac{\dfrac{Uh^*}{2} + \dfrac{\beta U (h^*)^2}{12\,l(t)}(A_\alpha + A_{\frac{\pi}{2}})}{1 + \dfrac{\beta h^*}{12\,l(t)}(B_\alpha + B_{\frac{\pi}{2}})} \ . \tag{7}$$

We wish to solve the equations (5), (6) and (7) for $\sigma(t)$, $l(t)$, and $Q(t)$ given the initial condition that the blade is new and hence $l(0) = 0$.

Non-Dimensional Formulation To get some basic understanding of the behaviour of this system we non-dimensionalise the equations. To do this we take

$$x = L\bar{x}, \qquad l = l_0\,\bar{l}, \qquad \sigma = \frac{\mu\,U}{h^*}\,\bar{\sigma}, \qquad t = \frac{h^*\,l_0\,\sin\alpha}{\gamma\,\mu\,U^2}\,\bar{t}, \qquad Q = U\,h^*\,\bar{Q}\ .$$

Doing some elementary simplifications we therefore get

$$\frac{d\bar{l}}{d\bar{t}} = \frac{\delta}{\bar{l}(\bar{t})}\int_0^1 (1-x)\left(\frac{A_\alpha}{x+\epsilon} - \frac{\epsilon B_\alpha \bar{Q}(\bar{t})}{(x+\epsilon)^2}\right)\,dx + \frac{(A_\alpha - A_{\frac{\pi}{2}})}{2} - \frac{(B_\alpha - B_{\frac{\pi}{2}})\bar{Q}(\bar{t})}{2}, \tag{8}$$

where

$$\bar{Q}(\bar{t}) = \frac{12\bar{l}(\bar{t}) + \delta\beta(A_\alpha + A_{\frac{\pi}{2}})}{12\bar{l}(\bar{t}) + \delta\beta(B_\alpha + B_{\frac{\pi}{2}})}\,. \tag{9}$$

with $\bar{l}(0) = 0$ and where we have introduced the two non-dimensional parameters

$$\epsilon = \frac{h^*}{L}, \qquad \delta = \frac{h^*}{l_0}.$$

The solution therefore is determined by the values of ϵ, δ and α. All these parameters are conventionally very small in practice and numerous approximations can be made to exploit these.

Discussion

The mathematical modelling we have done has revealed the following. We have identified the time scale of the wear and it's dependence on the basic physical parameters. We have also found the basic behaviour of the wear. There is a short initial period where the flux through the contact region is controlled by the huge pressure difference between the two ends and this flux is constant. During this time the contact region increases in length like the square root of time. This behaviour quickly gives way to the flux being dominated by fluid transported by the moving surface. Interestingly the flux in this period is also constant, but different, corresponding to $\bar{Q} \approx 1$. During this later period, because ϵ is small the downward force on the blade is dominated by the large pressures in the wedge very near the contact point and the governing equation is approximately.

$$\frac{d\bar{l}}{d\bar{t}} \approx \frac{\delta\left(-A_\alpha - B_\alpha - A_\alpha - \epsilon B_\alpha\right)\ln\epsilon}{\bar{l}(\bar{t})} + \frac{(A_\alpha - A_{\frac{\pi}{2}} - B_\alpha + B_{\frac{\pi}{2}})}{2} \tag{10}$$

This is easily solved and solutions for a typical practical case are given in Figure 2.

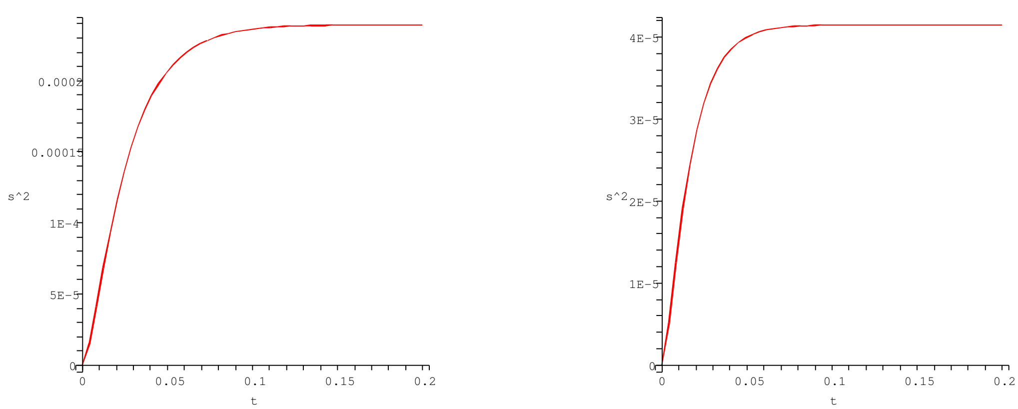

Figure 2: The length of the contact region as a function of time (in nondimensional units) for the cases i) $\epsilon = 10^{-4}$, $\delta = 10^{-2}$, $\alpha = \pi/6$ and ii) $\epsilon = 10^{-3}$, $\delta = 10^{-2}$, $\alpha = \pi/6$

We again find that the contact region length increases proportional to the square root of time. However, as time progresses the force due to the pressure under the contact region increases and the wear rate decreases rapidly with the contact region tending to a fixed length of

$$l_\infty = \frac{2\left(A_\alpha + B_\alpha + \left(A_\alpha + \epsilon B_\alpha\right)\ln\epsilon\right)}{\left(A_\alpha - A_\pi - B_\alpha + B_\pi\right)}$$

These formula give insight into the setting of the blade angle, α, and how this will affect the wear process. We are currently investigating the stability of the solutions for the contact region length in order to understand the possible reduction in scraping efficiency. In particular we anticipate that as the contact region reaches its maximum length the blade will become very sensitive to pressure variations due to the dependence of the motion on the pressure in the contact region. We are also considering the inclusion of elastic deformation effects in the blade and how they interact with the flow problem outlined here.

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