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Solitary and blow-up electrostatic excitations in rotating magnetized electron–positron–ion plasmas

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Abstract. The nonlinear dynamics of a rotating magnetoplasma consisting of electrons, positrons and stationary positive ions is considered. The basic set of hydrodynamic and Poisson equations are reduced to a Zakharov–Kuznetsov (ZK) equation for the electric potential. The ZK equation is solved by applying an improved modified extended tanh-function method (2008 *Phys. Lett.* A **372** 5691) and its characteristics are investigated. A set of new solutions are derived, including localized solitary waves, periodic nonlinear waveforms and divergent (explosive) pulses. The characteristics of these nonlinear excitations are investigated in detail.

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Contents

1. Introduction

Pair plasmas, i.e. plasmas consisting of equal-mass components, are characterized by physical properties quite different from those of conventional electron–ion plasmas. In the analysis of ordinary e–i plasmas, the ratio of the electron mass to the ion mass is exploited to great effect leading one to distinguish, for example, between high (electron-dominated) and low (ion-dominated) frequency motion. Conversely, with both constituent species possessing the same absolute charge to mass ratio, important symmetries manifest themselves, leading to considerable simplifications in the mathematical description of equal-mass plasmas [1].

Electron–positron (e–p) plasmas are pair plasmas, which exist in many astrophysical environments, such as in the early universe [2], in active galactic nuclei [3], in pulsar magnetospheres [4], in the solar atmosphere [5], in neutron stars [6], at the center of the Milky Way galaxy [7], etc. Since roughly two decades ago, the progress in the production of positron plasmas makes it possible to consider performing laboratory experiments on e–p plasmas [8]. Interestingly, Helander and Ward [9] have discussed the possibility of e–p production in large tokamaks due to collisions between multi-mega-electron-volt runaway electrons and thermal particles.

The presence of a magnetic field in astrophysical plasmas plays an important role in linear and nonlinear plasma dynamics, and affects the stability profile of plasma waves. However, the Lorentz force (due to magnetic fields) is not the only force creating plasma rotation in astrophysical environments. Chandrasekhar [10] suggested that the Coriolis force might also play a role in cosmic phenomena. Latter, it was suggested that a slow rotation, however small, might be related to interesting phenomena in astrophysical environments [11]. Based on such considerations, several authors have attempted to examine the nature of wave propagation in rotating plasmas including a Coriolis force [12, 13].

An e–p plasma usually behaves as a fully ionized gas consisting of electrons and positrons. Since many of the astrophysical plasmas contain ions besides the electrons and positrons, it is important to investigate the nonlinear behavior of electron–positron–ion (e–p–i) plasmas. During the last three decades, e–p and e–p–i plasmas have attracted significant attention among researchers [12]–[18]. Linear and nonlinear excitations in e–p and e–p–i plasmas have been studied by using different models. For example, Popel *et al* [14] investigated ion-acoustic solitons in three-component plasmas, whose constituents are electrons, positrons and singly charged ions. It is found that the presence of the positron component can result in a decrease of the amplitude of ion-acoustic solitary waves. Nejoh [15] studied the effect of ion temperature on large amplitude ion-acoustic waves in an e–p–i plasma. He found that the ion temperature increases the maximum Mach number, decreasing the amplitude of ion-acoustic solitary waves and also reducing the region of existence of ion-acoustic excitations. Gogoberidze *et al* [17] investigated a model for the main observational characteristics of the radio emission of pulsars with well organized drifting subpulses. It was found that the electric field of the drift waves, which is directed along the magnetic field lines, modulates the distribution of the particles and hence the radio emission mechanism. Also, the ratio of the frequency of the eigenmode to the rotation frequency of the star is insensitive to the magnetic field strength and the period of rotation, and is of the order of unity. Later, Gogoberidze *et al* [18] showed that a nonlinear decay of obliquely propagating Langmuir waves into Langmuir and Alfvén waves is possible in a one-dimensional, highly relativistic, streaming pair plasma. It was shown that the characteristic frequency of generated Alfvén waves is much less than the frequency of Langmuir waves and may be consistent with the observational data on the radio emission of pulsars.

Actually, e–p plasmas in nature are in reality relativistic and, in fact, dominated by recombination (annihilation) processes. We have here chosen to neglect both relativistic effects and mutual annihilation processes. The reason for this is twofold. Firstly, we aim at addressing generic nonlinear effects, so simplifying at first approach was part of the scope, in order to pinpoint the building blocks of the theory via a simple model. Therefore, the study at hand can be considered as preliminary research, in view of a more complete and physically relevant case of relativistic pair plasmas, to be carried out in the future. Secondly, and more importantly, part of our inspiration comes from experiments on pair-(fullerene-)ion plasmas [19] which have been carried out recently, allowing us to unveil all the fundamental physics of pair plasmas while at the same time remaining in the classical (non-relativistic) regime, in fact free from pair-species annihilation (reported to be negligible in those experiments). Working with the classical (nonrelativistic) toolbox is therefore meaningful physically.

In the present paper, we shall investigate the existence and properties of nonlinear electrostatic potential structures in nonrelativistic, rotating e–p–i magnetoplasmas. Nonlinear electrostatic structures are shown to exist, in the form of solitary waves, nonlinear periodic wave forms and blow-up pulse excitations. This paper is structured as follows: in section 2, we present the governing equations for the electrostatic waves in our e–p–i plasma. In section 3, the reductive perturbation method [20] is employed to derive a Zakharov–Kuznetsov (ZK) equation [21]. In section 4, we solve the ZK equation via a traveling wave ansatz, to obtain obliquely propagating localized potential pulses. In section 5, we apply an improved modified extended tanh-function (iMETF) method [22] to solve the ZK equation. A set of analytical solutions is obtained, and then used to investigate numerically the effect of the plasma parameters on the nonlinear excitations. The results are summarized in the concluding section 6.

2. Basic equations

We consider a two-dimensional, magnetized, rotating, collisionless three-component plasma consisting of electrons, positrons, and stationary ions. The external magnetic field is directed along the x-axis, i.e. $\boldsymbol{B} = B_0\hat{x}$, ($\hat{x}$ is the unit vector along the x-axis). The propagation of electrostatic waves is governed by the electron and positron (denoted by the indices e and p) continuity equation(s)

$$\frac{\partial n_{\mathrm{e,p}}}{\partial t} + \nabla.(n_{\mathrm{e,p}}\boldsymbol{u}_{\mathrm{e,p}}) = 0, \tag{1}$$

and the e (p) momentum equations,

$$m\left(\frac{\partial}{\partial t} + \boldsymbol{u}_{\mathrm{e}}.\nabla\right)\boldsymbol{u}_{\mathrm{e}} = e\nabla\phi - \frac{1}{n_{\mathrm{e}}}\nabla P_{\mathrm{e}} - \frac{e}{c}(\boldsymbol{u}_{\mathrm{e}} \times B_0\hat{x}) + 2m(\boldsymbol{u}_{\mathrm{e}} \times \Omega_0\hat{x}), \tag{2}$$

$$m\left(\frac{\partial}{\partial t} + \boldsymbol{u}_{\mathrm{p}}.\nabla\right)\boldsymbol{u}_{\mathrm{p}} = -e\nabla\phi - \frac{1}{n_{\mathrm{p}}}\nabla P_{\mathrm{p}} + \frac{e}{c}(\boldsymbol{u}_{\mathrm{p}} \times B_0\hat{x}) + 2m(\boldsymbol{u}_{\mathrm{p}} \times \Omega_0\hat{x}). \tag{3}$$

The Poisson equation for this system is

$$\nabla^2\phi = 4\pi e(n_{\mathrm{e}} - n_{\mathrm{p}} - n_{\mathrm{i}}). \tag{4}$$

In equations (1)–(4), $n_{\mathrm{e,p}}$ is the the electron (positron) number density, $\boldsymbol{u}_{\mathrm{e,p}}$ the electron (positron) fluid velocity, ϕ the electrostatic wave potential, $P_{\mathrm{e,p}}$ the electron (positron) pressure, $\Omega = \Omega_0\hat{x}$ the rotation frequency (angular velocity) vector (Ω_0 is the magnitude of rotation frequency), e the magnitude of electron charge, m the mass of an electron, B_0 the magnitude of the ambient magnetic field and c the speed of light. We shall close the system by using the equation(s) of state $P_{\mathrm{e,p}} \sim n_{\mathrm{e,p}}^{\gamma}$, where $\gamma = (2+f)/f$ (for f degrees of freedom; here $f = 2$). The Boltzmann constant K_{B} may be omitted where obvious.

We stress the fact that, although we shall consider all vectorial physical quantities to evolve in three-dimensions (x, y and z), the spatial dependence of all variable quantities is taken to be two-dimensional only. We consider all dynamical quantities to evolve in the x–y plane (due to the physical symmetry of the problem). The nondimensionalized form of equations (1)–(4) can be rewritten for electrons

$$\frac{\partial \overline{n}_{\mathrm{e}}}{\partial \overline{t}} + \frac{\partial \overline{n}_{\mathrm{e}}\overline{u}_{\mathrm{e}x}}{\partial \overline{x}} + \frac{\partial \overline{n}_{\mathrm{e}}\overline{u}_{\mathrm{e}y}}{\partial \overline{y}} = 0, \tag{5}$$

$$\frac{\partial \overline{u}_{\mathrm{e}x}}{\partial \overline{t}} + \left(\overline{u}_{\mathrm{e}x}\frac{\partial}{\partial \overline{x}} + \overline{u}_{\mathrm{e}y}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{\mathrm{e}x} - \frac{\partial \overline{\phi}}{\partial \overline{x}} + 2\frac{\partial \overline{n}_{\mathrm{e}}}{\partial \overline{x}} = 0, \tag{6}$$

$$\frac{\partial \overline{u}_{\mathrm{e}y}}{\partial \overline{t}} + \left(\overline{u}_{\mathrm{e}x}\frac{\partial}{\partial \overline{x}} + \overline{u}_{\mathrm{e}y}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{\mathrm{e}y} - \frac{\partial \overline{\phi}}{\partial \overline{y}} + 2\frac{\partial \overline{n}_{\mathrm{e}}}{\partial \overline{y}} - \overline{\Omega}_{\mathrm{e}}\overline{u}_{\mathrm{e}z} = 0, \tag{7}$$

$$\frac{\partial \overline{u}_{\mathrm{e}z}}{\partial \overline{t}} + \left(\overline{u}_{\mathrm{e}x}\frac{\partial}{\partial \overline{x}} + \overline{u}_{\mathrm{e}y}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{\mathrm{e}z} + \overline{\Omega}_{\mathrm{e}}\overline{u}_{\mathrm{e}y} = 0, \tag{8}$$

for positrons,

$$\frac{\partial \overline{n}_{\mathrm{p}}}{\partial \overline{t}} + \frac{\partial \overline{n}_{\mathrm{p}}\overline{u}_{\mathrm{p}x}}{\partial \overline{x}} + \frac{\partial \overline{n}_{\mathrm{p}}\overline{u}_{\mathrm{p}y}}{\partial \overline{y}} = 0, \tag{9}$$

$$\frac{\partial \overline{u}_{\mathrm{p}x}}{\partial \overline{t}} + \left(\overline{u}_{\mathrm{p}x}\frac{\partial}{\partial \overline{x}} + \overline{u}_{\mathrm{p}y}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{\mathrm{p}x} + \frac{\partial \overline{\phi}}{\partial \overline{x}} + 2\sigma\frac{\partial \overline{n}_{\mathrm{p}}}{\partial \overline{x}} = 0, \tag{10}$$

$$\frac{\partial \overline{u}_{\mathrm{p}y}}{\partial \overline{t}} + \left(\overline{u}_{\mathrm{p}x}\frac{\partial}{\partial \overline{x}} + \overline{u}_{\mathrm{p}y}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{\mathrm{p}y} + \frac{\partial \overline{\phi}}{\partial \overline{y}} + 2\sigma\frac{\partial \overline{n}_{\mathrm{p}}}{\partial \overline{y}} - \overline{\Omega}_{\mathrm{p}}\overline{u}_{\mathrm{p}z} = 0, \tag{11}$$

$$\frac{\partial \overline{u}_{pz}}{\partial \overline{t}} + \left(\overline{u}_{px}\frac{\partial}{\partial \overline{x}} + \overline{u}_{py}\frac{\partial}{\partial \overline{y}}\right)\overline{u}_{pz} + \overline{\Omega}_p \overline{u}_{py} = 0, \quad (12)$$

and the system is closed by the Poisson equation

$$\overline{\nabla}^2 \overline{\phi} = \overline{n}_e - \overline{n}_p - \overline{n}_i. \quad (13)$$

The physical quantities appearing in equations (5)–(13) have been appropriately normalized. Specifically, n_j ($j = \mathrm{e, p, i}$) is normalized by the unperturbed electron density n_0, $\boldsymbol{u}_{e,p}$ by the electron thermal speed $C_{se} = (K_B T_e/m_e)^{1/2}$, ϕ by $(K_B T_e/e)$, the rotation frequency Ω_0 and the electron (positron) gyrofrequency $\omega_c = eB_0/mc$ by the electron plasma period $\omega_{pe} = (4\pi e^2 n_0/m)^{1/2}$, the space and time variables are in units of the electron Debye radius $\lambda_{De} = (K_B T_e/4\pi e^2 n_0)^{1/2}$ and of the electron plasma period ω_{pe}^{-1}, respectively. Here, $\sigma = T_p/T_e$, $\overline{\Omega}_e = 2\overline{\Omega}_0 - \overline{\omega}_c$, $\overline{\Omega}_p = 2\overline{\Omega}_0 + \overline{\omega}_c$, where T_e and T_p are the electron and positron temperatures, respectively. The quasineutrality condition at equilibrium reads

$$1 = \delta + \beta, \quad (14)$$

where $\delta = n_p^{(0)}/n_0$ and $\beta = n_i^{(0)}/n_0$ denote the unperturbed density ratios of positrons and ions to electrons, respectively. The (two-component) electron–positron plasma is recovered in the limit $\beta = 0$. The overbar in (5)–(13) will be omitted in the following, hence dimensionless quantities are to be understood everywhere.

3. Derivation of a ZK equation

The independent variables can be stretched as

$$X = \varepsilon^{1/2}(x - \lambda t),\ Y = \varepsilon^{1/2} y \text{ and } T = \varepsilon^{3/2} t, \quad (15)$$

where ε is a small parameter and λ the wave propagation speed to be determined by compatibility requirements. The dependent variables are expanded as

$$\Psi = \Psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^n \Psi^{(n)}, \quad (16)$$

where

$$\Psi = (n_e, n_p, u_{ex}, u_{ez}, u_{px}, u_{pz}, \phi)^T, \quad (17)$$

$$\Psi^{(0)} = (1, \delta, 0, 0, 0, 0, 0)^T. \quad (18)$$

The transverse velocities in the y- and z-axes are given by

$$u_\alpha = \varepsilon^{3/2} u_\alpha^{(1)} + \varepsilon^2 u_\alpha^{(2)} + \varepsilon^{5/2} u_\alpha^{(3)} + \cdots, \quad (19)$$

where the subscript α indicates either e or p for the y- or z-vector coordinate (all four combinations).

Substituting the stretching (15) and the expansions (16)–(19) into equations (5)–(13), the lowest order in ε yields

$$n_e^{(1)} = \frac{-1}{\lambda^2 - 2}\phi^{(1)}, \qquad u_{ex}^{(1)} = \frac{-\lambda}{\lambda^2 - 2}\phi^{(1)}, \qquad u_{ez}^{(1)} = \frac{-\lambda^2}{\Omega_e(\lambda^2 - 2)}\frac{\partial \phi^{(1)}}{\partial Y}, \quad (20)$$

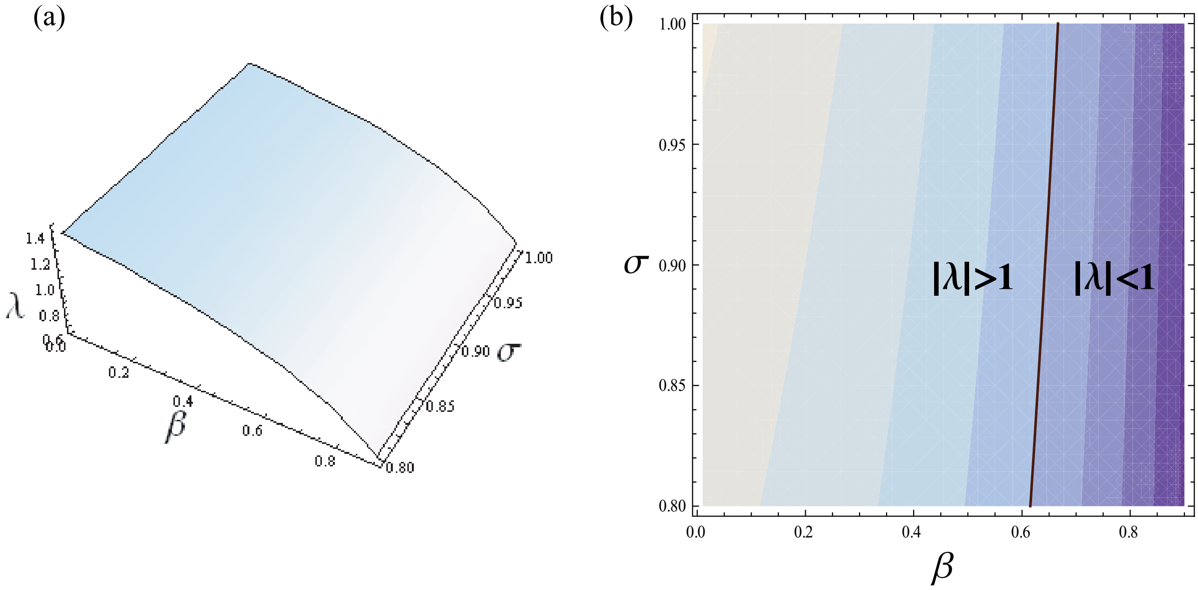

Figure 1. The variation of phase velocity λ [given by equation (22*b*)] with β and σ is depicted. Note that lighter regions in (b) show higher values of the pulse speed λ. The separatrix (thick line) in (b) depicts the sonic limit, i.e. $|\lambda| = 1$.

$$n_{\mathrm{p}}^{(1)} = \frac{\delta}{\lambda^2 - 2\delta\sigma}\phi^{(1)}, \qquad u_{\mathrm{p}x}^{(1)} = \frac{\lambda}{\lambda^2 - 2\delta\sigma}\phi^{(1)}, \qquad u_{\mathrm{p}z}^{(1)} = \frac{\lambda^2}{\Omega_{\mathrm{p}}(\lambda^2 - 2\delta\sigma)}\frac{\partial\phi^{(1)}}{\partial Y}. \tag{21}$$

The Poisson equation gives the compatibility condition

$$\frac{1}{\lambda^2 - 2} + \frac{\delta}{\lambda^2 - 2\delta\sigma} = 0, \tag{22a}$$

which determines the propagation speed λ as

$$\lambda^2 = \frac{2(1-\beta)(\sigma+1)}{2-\beta}. \tag{22b}$$

A solution for λ exists, provided that $\lambda \neq \pm\sqrt{2}$, $\pm\sqrt{2\delta\sigma}$ (which excludes propagation in temperature-symmetric electron–positron plasmas, viz $\delta = \sigma = 1$). The dependence of the phase velocity λ [represented by equations (22*b*)] on the ratios of the ion-to-electron density β and positron-to-electron temperature σ is depicted in figure 1. It is seen that the phase velocity λ is affected by the ion-to-electron density ratio β, and in fact decreases with increasing β. A supersonic solitary pulse propagates for small values of β ($\beta < 0.6$–0.7), however, if the positive ion concentration increases ($\beta > 0.6$) only a subsonic solitary pulse may propagate in the system. Also, the positron-to-electron temperature ratio σ affects the behavior of the phase velocity λ in a certain range of the ion-to-electron density ratio β ($\beta \lesssim 0.6$); see figure 1(b). For higher values of β ($\beta \gtrsim 0.6$), σ plays no important role in the properties of the phase velocity λ. In general, it is seen that only supersonic pulses are affected by the positron-to-electron temperature ratio σ, whereas the latter plays no important role in the behavior of subsonic pulses.

The next order in ε yields

$$\frac{\partial n_{\mathrm{e}}^{(2)}}{\partial X}=-\frac{1}{\lambda^2-2}\frac{\partial\phi^{(2)}}{\partial X}-\frac{2\lambda}{(\lambda^2-2)^2}\frac{\partial\phi^{(1)}}{\partial T}+\frac{3\lambda^2}{2(\lambda^2-2)^3}\frac{\partial\phi^{(1)2}}{\partial X}+\frac{\lambda}{\lambda^2-2}\frac{\partial u_{\mathrm{ey}}^{(2)}}{\partial Y},\tag{23}$$

$$u_{\mathrm{ey}}^{(2)}=\frac{-\lambda^3}{\Omega_{\mathrm{e}}^2(\lambda^2-2)}\frac{\partial^2\phi^{(1)}}{\partial X\partial Y},\tag{24}$$

$$\frac{\partial n_{\mathrm{p}}^{(2)}}{\partial X}=\frac{\delta}{\lambda^2-2\delta\sigma}\frac{\partial\phi^{(2)}}{\partial X}-\frac{2\lambda}{(\lambda^2-2\delta\sigma)^2}\frac{\partial\phi^{(1)}}{\partial T}+\frac{3\lambda^2}{2(\lambda^2-2\delta\sigma)^3}\frac{\partial\phi^{(1)2}}{\partial X}+\frac{\lambda}{\lambda^2-2\delta\sigma}\frac{\partial u_{\mathrm{py}}^{(2)}}{\partial Y},\tag{25}$$

$$u_{\mathrm{py}}^{(2)}=\frac{\lambda^3}{\Omega_p^2(\lambda^2-2\delta\sigma)}\frac{\partial^2\phi^{(1)}}{\partial X\partial Y},\tag{26}$$

$$\frac{\partial^2\phi^{(1)}}{\partial X^2}+\frac{\partial^2\phi^{(1)}}{\partial Y^2}=n_{\mathrm{e}}^{(2)}-n_{\mathrm{p}}^{(2)}.\tag{27}$$

Eliminating the second-order perturbed quantities and making use of the first-order results, we obtain a nonlinear partial-derivative equation in the form of the Zakharov–Kuznetsov (ZK) equation,

$$\frac{\partial\phi^{(1)}}{\partial T}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial X}+\frac{\partial}{\partial X}\left(B\frac{\partial^2}{\partial X^2}+C\frac{\partial^2}{\partial Y^2}\right)\phi^{(1)}=0.\tag{28}$$

The nonlinearity and diffusion coefficients, A, B and C, respectively, read

$$A=-\frac{3\lambda^2B}{(1-\beta)^2}\left[\frac{1+(1-\beta)^2}{(\lambda^2-2)^3}\right],$$

$$B=\frac{1-\beta}{2\lambda}\left[\frac{(\lambda^2-2)^2}{2-\beta}\right],$$

$$C=B\left\{1+\frac{\lambda^4}{(\lambda^2-2)^2}\left[\frac{1}{\Omega_{\mathrm{e}}^2}+\frac{1}{\Omega_{\mathrm{p}}^2(1-\beta)}\right]\right\}.$$

Note that the dispersion coefficients B and C are always positive (recall that $\beta<1$ by definition), whereas the sign of A is positive for λ below $\sqrt{2}$ (λ is assumed to be positive throughout), and negative above. As one may already anticipate, this leads to positive (negative) pulses prescribed below (above) a critical Mach number (scaled pulse speed) of $\sqrt{2}$.

The ZK equation (28) constitutes the key outcome of this model. The anticipated balance among dispersion and nonlinearity within the ZK equation gives rise to localized solitary wave solutions. Some of these solutions will be reviewed in the following.

4. Traveling wave analysis—pulse-shaped localized solutions

We shall use the traveling wave transformation $\zeta=L_xX+L_yY-\vartheta T$, where ϑ is a real variable (representing a constant speed, scaled by the electron thermal speed), L_x and L_y are the directional cosines of the wave vector k along the X- and Y-axes, so that $L_x^2+L_y^2=1$, $A_0=AL_x$ and $B_0=L_xR$, where $R=BL_x^2+CL_y^2$. Note that this ansatz represents propagation in a direction oblique with respect to the magnetic field, at an angle higher than zero

($|L_x| > 0$ is assumed everywhere) and up to a 90° (viz $|L_x| \leqslant 1$). Equation (28) is thus reduced to the ordinary partial differential equation

$$-\vartheta u' + A_0 u u' + B_0 u''' = 0, \tag{29}$$

where we have replaced $\phi^{(1)}$ by $u = u(\zeta)$ for simplicity, and the prime in equation (29) denotes the derivative with respect to ζ.

Integrating equation (29) once, by assuming the boundary conditions u, u' and $u'' \to 0$ for $\zeta \to \pm\infty$, we obtain the pseudo-energy-balance equation

$$\tfrac{1}{2} u'^2 + S(u) = 0. \tag{30}$$

This relation suggests that the evolution of a solitary excitation is analogous to the problem of motion of a unit mass in a (Sagdeev-like) pseudo-potential, given by

$$S(u) = \frac{1}{B_0}\left(\frac{-\vartheta}{2} u^2 + \frac{A_0}{6} u^3\right). \tag{31}$$

The solitary wave solution of equation (29) exists if $\mathrm{d}^2 S/\mathrm{d}u^2 < 0$ at $u = 0$. A value of $\mathrm{d}^2 S/\mathrm{d}u^2$ smaller than zero predicts the formation of solitary structure in an e–p–i plasma. Therefore, we have

$$\mathrm{d}^2 S/\mathrm{d}u^2 = -\vartheta/B_0 < 0. \tag{32}$$

It is noted that equation (32) is always satisfied, except for $\lambda = \lambda_c = \sqrt{2}$ (assuming that $\lambda > 0$, with no loss of generality); the case $\beta = 1$ (implying no positrons, hence no pair plasma, physically) is excluded. One can thus conclude that stationary solitary waves can always propagate in this plasma system except for $\lambda = \sqrt{2}$; where solitons do not exist.

Assuming the boundary conditions $\phi^{(1)} \to 0$ and $\mathrm{d}\phi^{(1)}/\mathrm{d}\zeta \to 0$ at $|\zeta| \to \infty$, we obtain a solitary wave solution of equation (28) as

$$\phi^{(1)} = \phi_0 \,\mathrm{sech}^2(\zeta/W), \tag{33}$$

where $\phi_0 = 3\vartheta/AL_x$ is the maximum amplitude of the potential perturbation and $W = \sqrt{4L_x(BL_x^2 + CL_y^2)/\vartheta}$ measures its spatial extension (width). The localized pulses predicted via this form may be either positive or negative, depending entirely on the sign of the nonlinearity coefficient A. The characteristics of these pulses will be discussed below.

5. Solution of the ZK equation via an improved modified extended tanh-function (iMETF) method

We shall now go beyond the 'traditional' solution of the ZK equation (28) by quadrature (see the previous section) by adopting an alternative method, such as the iMETF method introduced in [22].

According to the iMETF method, we anticipate that equation (29) has the following solution:

$$u(\zeta) = a_0 + a_1\omega + a_2\omega^2 + \frac{b_1}{\alpha_1 + \omega} + \frac{b_2}{(\alpha_2 + \omega)^2}, \tag{34a}$$

with

$$\frac{\mathrm{d}\omega}{\mathrm{d}\zeta} = k + \omega^2, \tag{34b}$$

where a_0, a_1, a_2, b_1, b_2 and k are arbitrary constants to be determined later, and ω, α_1 and α_2 are functions of ζ.

The Riccati equation (34*b*), has the general solutions: for $k < 0$

$$\omega = -\sqrt{-k}\,\tanh(\sqrt{-k}\zeta) \quad \text{or} \quad \omega = -\sqrt{-k}\,\coth(\sqrt{-k}\zeta), \tag{35}$$

for $k = 0$,

$$\omega = -\frac{1}{\zeta}, \tag{36}$$

and for $k > 0$,

$$\omega = \sqrt{k}\,\tan(\sqrt{k}\zeta) \quad \text{or} \quad \omega = -\sqrt{k}\,\cot(\sqrt{k}\zeta). \tag{37}$$

Substituting equation (34*a*) in equation (29) and making use of equation (34*b*), we obtain a polynomial equation in $\omega(\zeta)$. Equating the coefficients of ω to zero will result in an over determined system of algebraic differential equations with respect to a_0, a_1, a_2, b_1, b_2, α_1, α_2, k, L_x, L_y and ϑ. Combining with equations (35)–(37), we obtain a complete new set of solutions, which will be presented in the following.

For $k < 0$, two different solutions are obtained

$$u(\zeta) = \frac{\vartheta - 8kB_0}{A_0} + \frac{12kB_0\tanh^2(\sqrt{-k}\zeta)}{A_0}, \tag{38}$$

or

$$u(\zeta) = \frac{\vartheta - 8kB_0}{A_0} + \frac{12kB_0}{A_0}\left[\tanh^2(\sqrt{-k}\zeta) + \coth^2(\sqrt{-k}\zeta)\right]. \tag{39}$$

For $k = 0$,

$$u(\zeta) = \frac{\vartheta}{A_0}\left(1 - \frac{12B_0}{\zeta^2\vartheta}\right). \tag{40}$$

For $k > 0$,

$$u(\zeta) = \frac{\vartheta - 8kB_0}{A_0} - \frac{12kB_0\,\tan^2(\sqrt{k}\zeta)}{A_0}, \tag{41}$$

or

$$u(\zeta) = \frac{\vartheta - 8kB_0}{A_0} - \frac{12kB_0}{A_0}\left[\tan^2(\sqrt{k}\zeta) + \cot^2(\sqrt{k}\zeta)\right]. \tag{42}$$

Finally, in the particular case where $k = -1$ and $\vartheta = 4B_0$, we obtain

$$u(\zeta) = \frac{12B_0}{A_0}(1 - c^2)\left[1 - \frac{2c}{c - \tanh\,\zeta} - \frac{1 - c^2}{(c - \tanh\,\zeta)^2}\right], \tag{43}$$

where $\alpha_1 = \alpha_2 = c$ [cf (34*a*)] is here an arbitrary function of ζ. Note that $A_0 \neq 0$ and $B_0 \neq 0$ is understood everywhere in the above solutions. It may be pointed out for rigor that, although solutions (38)–(42) can also be obtained by the modified extended tanh-function method [25], the latter cannot recover solution (43).

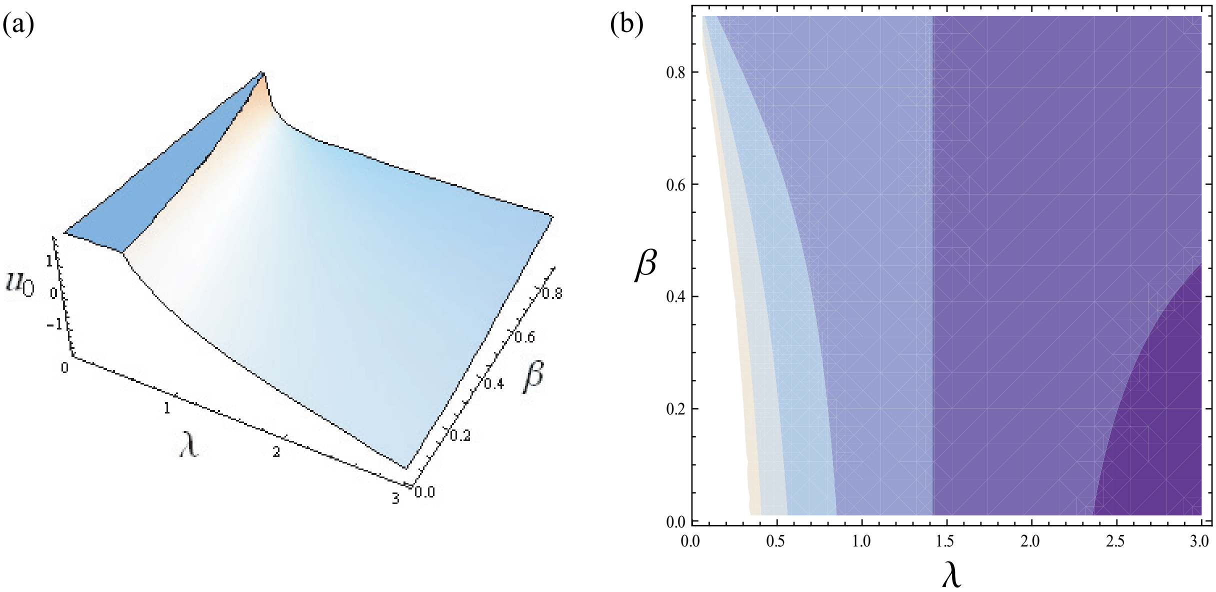

Figure 2. The variation of the soliton amplitude u_0 [defined in equation (44)] with β and λ for $L_x = 0.3$. Light-colored regions in (b) correspond to higher values of the soliton amplitude u_0.

5.1. Pulse-shaped localized solutions via the iMETF method

Anticipating localized solutions, which vanish far from the origin, we may impose u, u' and $u'' \to 0$ for $\zeta \to \pm\infty$, to find that the only parameter choice allowing for a localized solution is $k = -\vartheta/4B_0$. In this case, solution (38) reduces to

$$u(\zeta) = u_0 \,\mathrm{sech}^2(\zeta/W). \tag{44}$$

It is straightforward to see that equation (44) is the only localized solution from (38). Here, $u_0 = 3\vartheta/A_0$ is the maximum (potential perturbation) amplitude and $W = \sqrt{4B_0/\vartheta}$ is the spatial extension (width) of the localized pulse. We note that this coincides with expression (33) above, here recovered via the iMETF method. However, the method employed in this section also provides alternative types of solutions, in addition to the latter one, as we shall see below.

To investigate the nature of the solitary structure (represented by equation (44)), we have numerically analyzed the potential amplitude u_0 and investigated how the phase velocity λ and the ion-to-electron density ratio β change the profile of the maximum potential perturbation.

From figure 2, it is seen that increasing the propagation speed λ leads to a decrease in the soliton amplitude u_0 for a given (any, fixed) value of the ion-to-electron density ratio β. At a critical velocity $\lambda = \sqrt{2}$, the sign of A—and hence of the pulse amplitude u_0—shifts to negative, so the effect is inversed: higher speed λ then corresponds to a higher negative pulse amplitude (i.e. absolute value of u_0). The ratio β also has a two-fold effect on the potential amplitude u_0, i.e. for low propagation speed $\lambda < 1.44$ the amplitude increases as the ion-to-electron density ratio β increases. However, for high propagation speed $\lambda > 1.44$, increasing β leads to an increase of the soliton amplitude u_0.

The dependence of the spatial extension (width) W on the phase velocity λ, the ion-to-electron density ratio β, the rotation frequency Ω_0 and the electron (positron) gyrofrequency ω_c are displayed in figures 3 and 4. In figure 3, it is noticed that increasing the propagation speed λ

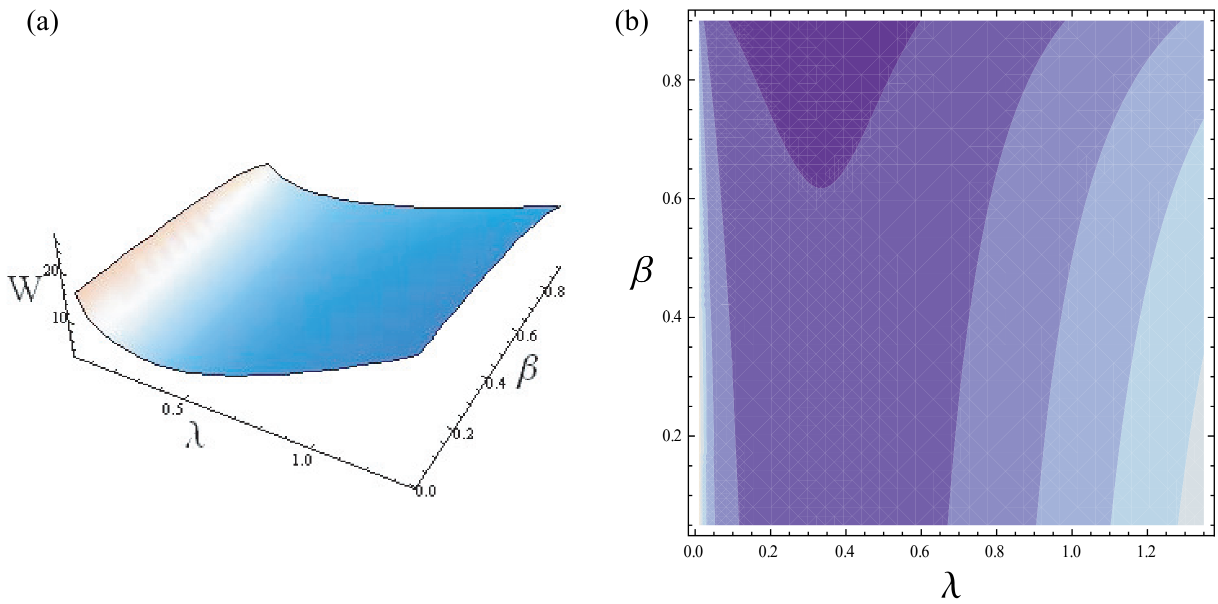

Figure 3. The variation of the soliton width W [defined in equation (44)] with β and λ is depicted, for $L_x = 0.3$, $\Omega_0 = 0.1$, and $\omega_c = 0.3$. Light-colored regions in (b) correspond to higher values of the soliton width W.

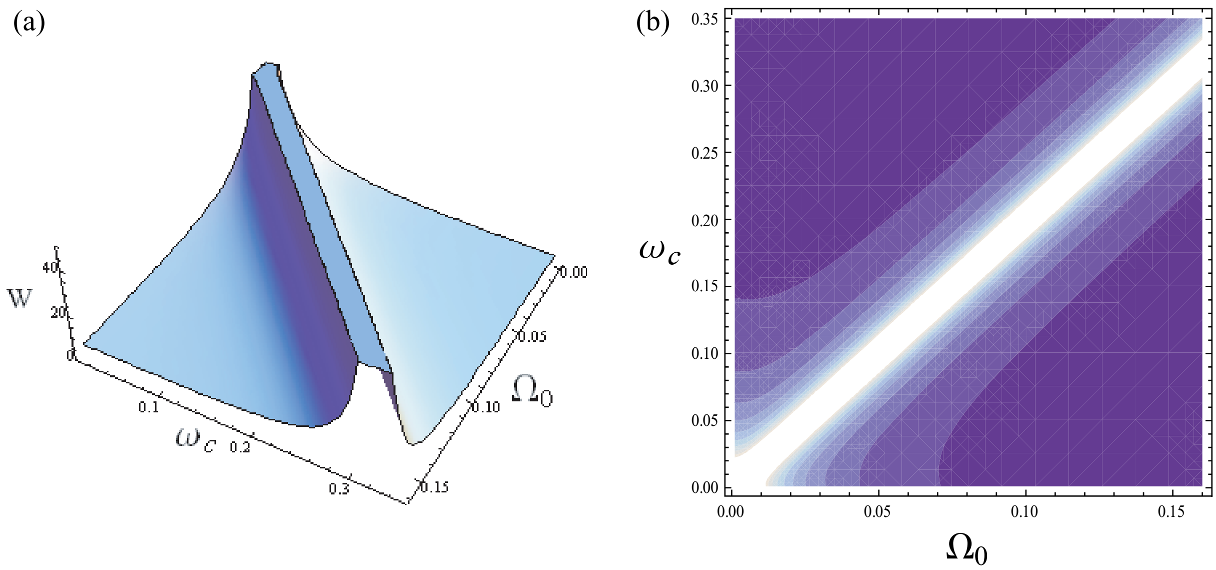

Figure 4. The variation of the soliton width W [defined by equation (44)] with Ω_0 and ω_c is depicted, for $\beta = 0.4$, $\lambda = 0.6$, and $L_x = 0.7$. Light-colored regions in (b) correspond to higher values of the soliton width W.

($\lambda \lesssim 0.6$) leads to a decrease of the width W. For $\lambda \gtrsim 0.6$, by increasing the phase speed λ the width increases. However, increasing the ion-to-electron density ratio β leads to a decrease of the soliton width. For certain values of λ (low phase speed) and β (high ion-to-electron density ratio), one can notice a depletion of the width; see figure 3(b). In figure 4, we have investigated the effect of the rotation frequency Ω_0 and the electron (positron) gyrofrequency ω_c on the

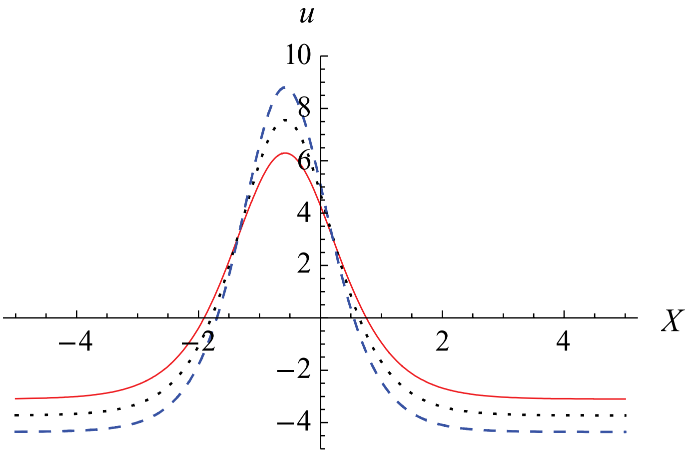

Figure 5. The profile of the solution (38) is depicted, for $k=-1$ (solid line), $k=-1.2$ (dotted line) and $k=-1.4$ (dashed line). Here, $L_x=0.866$, $L_y=0.5$, $\lambda=1.1$, $\beta=0.55$, $\omega_c=0.15$, and $\Omega_0=0.149$.

properties of the soliton width W. It is shown that the width W is influenced by the rotation frequency Ω_0 and the electron (positron) cyclotron frequency ω_c. In particular, an interesting effect is witnessed when $\omega_c-2\Omega_0\to 0$, where the width W diverges, leading to the dispersion-nonlinearity-managed excitations are expected to be not satisfied at these values of ω_c and Ω_0, notice the white region in figure 4(b). After the nonexistence region, the width decreases with increasing ω_c and Ω_0.

Now, we shall investigate the behavior of equation (38) if $k\neq-\vartheta/4B_0$. Recall that k is an arbitrary negative value. We shall set $k=-1$, -1.2 and -1.4 and investigate the form of the solution (38); cf figure 5. It is obvious that the pulse does not vanish at infinity although it has a localized form. Therefore, the arbitrary value k plays a role in obtaining a pulse-shaped localized solution.

5.2. Alternative solutions

First of all, in view of the analysis and interpretation of our results, we should point out that the solutions (39)–(42), at any instant t, allow for the existence of some singular point where the solutions are infinite. It should be stressed that the ZK equation is derived using perturbation theory, and thus assumes weak-amplitude excitations and in fact fails in the vicinity of singular points. Our predictions on singular solutions can therefore inevitably not claim to apply quantitatively to all physically relevant situations, despite their qualitative interest and novelty, as pointed out in this paper.

Solution (40) is a rational solution, which may be helpful to explain some physical phenomena; see the discussion in [23]. However, it is obviously (upon inspection) a divergent localized form, i.e. it accounts for an infinite potential perturbation at the center ($\zeta=0$), and is thus physically rather questionable with respect to electrostatic plasma excitations.

Solutions (39), (41) and (42) are singular periodic solutions, which develop a singularity at a finite point, i.e. for any fixed $t=t_0$ there exists a value of ζ_0 at which these solutions blow up; see figure 6. Note that these excitations never reach zero, except in a very specific combination

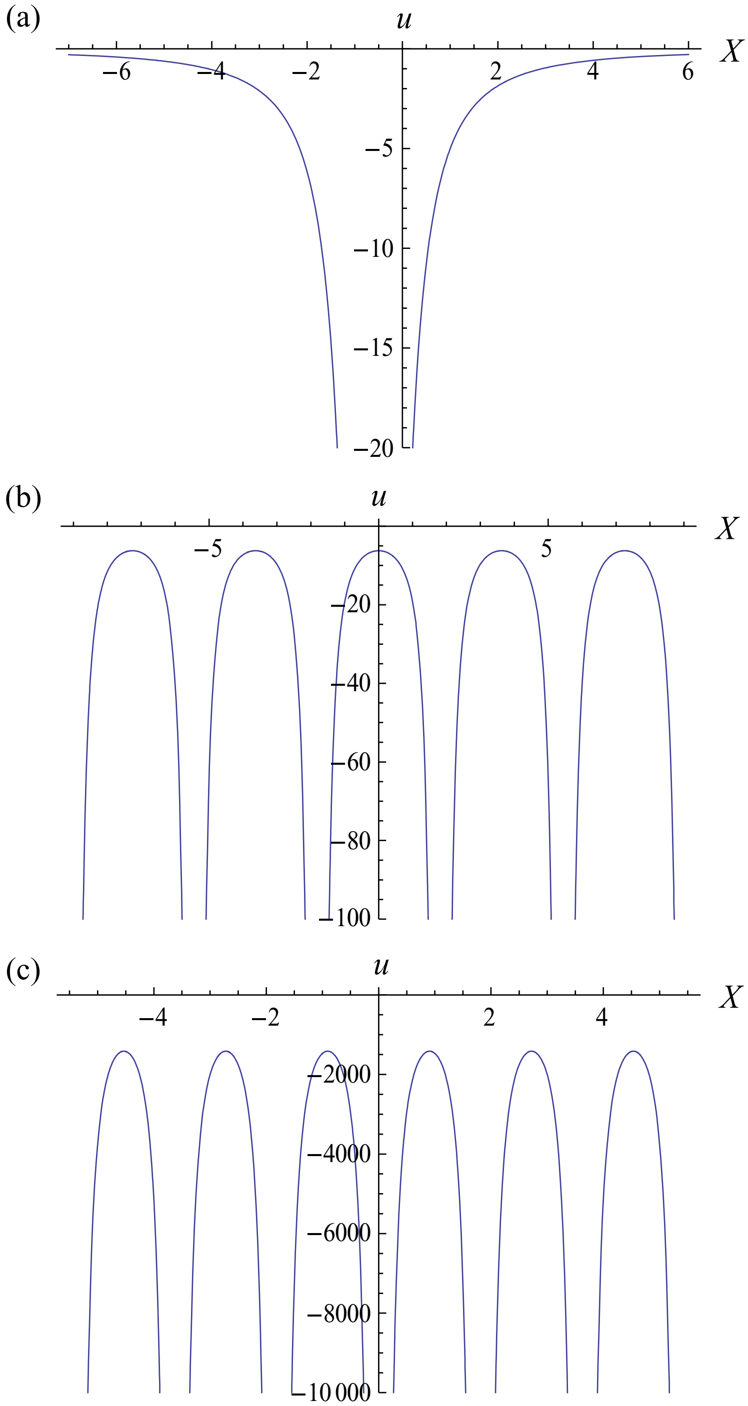

Figure 6. The profile of explosive (divergent) pulses predicted is depicted; (a) localized explosive pulse, as defined by equation (39), (b) periodic solutions, as defined by equation (41) and (c) as defined by equation (42). Here, $L_x = 0.866$, $L_y = 0.5$, $\lambda = 1.1$, $\beta = 0.55$, $\omega_c = 0.15$, and $\Omega_0 = 0.149$.

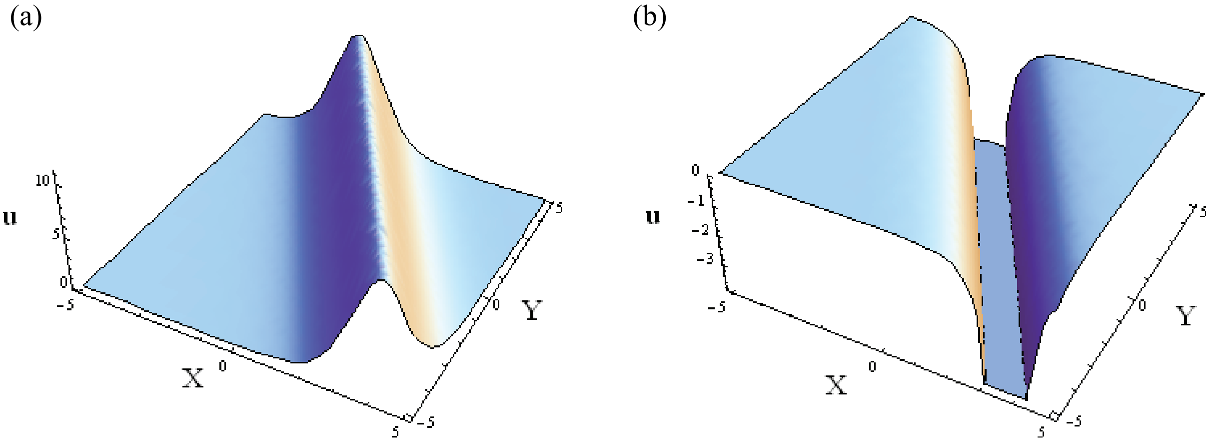

Figure 7. (a) The profile of the solitary pulse for $c = \lambda = 1.2$ and $\beta = 0.414$ and (b) an explosive/blow-up pulse for $c = \lambda = 0.4$ and $\beta = 0.89$ [defined by equation (43)] are depicted in the X–Y plane. Here, $\omega_c = 0.15$, and $\Omega_0 = 0.149$.

of parameter values. Our prediction for a potential excitation blowup indicates that an instability in the system may occur due to the effect of nonlinearity (which in our case depends on the ion concentration β and the propagation speed λ). In simple terms, the balance between dispersion and nonlinearity [24] may be disturbed by variations of plasma quantities (e.g. temperature, pressure, density, etc). This might locally destroy localized excitation stability leading to an amplitude increase to very high values; since this represents an increase in the electric potential, it might lead to an acceleration of the moving particles.

We shall now study the behavior of the solution (43). The latter can support a bell-shaped profile for $c > 1$, while for $c < 1$ an explosive pulse is expected to propagate. For $c = 1$, equation (43) is a simply constant solution. Recalling that c is an arbitrary constant, which can be assumed any physical parameters such as propagation angle L_y, propagation speed λ, ion-to-electron concentration ratio β, rotation frequency Ω_0, electron and positron gyrofrequency ω_c. We have numerically analyzed the solution (43) in figure 7 and investigated how the constant c changes the profile of the pulse (from solitary pulse to explosive pulse). It turns out that the bell-shaped solitary pulse can exist for $c = 1.2$, however, for $c = 0.4$ an explosive pulse exists. An explosive mode cannot be obtained from the solution (38), while the new solution (43) indicates that either solitary or explosive pulses can propagate in the system depending on the physical parameters. This result could not have been obtained from the original modified extended tanh-function method [25].

6. Summary

We have studied the nonlinear propagation of electrostatic excitations in rotating magnetized electron–positron–ion plasmas. We have found that both subsonic and supersonic waves may propagate in the system. A supersonic solitary pulse propagates for small values of the ion-to-electron density ratio; however, if the positive ion concentration increases beyond a certain threshold, only a subsonic solitary pulse may propagate in the system. Furthermore, only supersonic pulses are affected by the positron-to-electron temperature ratio, whereas the latter plays no important role on the behavior of subsonic pulses.

The evolution of the system is governed by a ZK equation. The latter has been solved analytically and investigated numerically using an improved modified extended tanh-function method. New solutions of the evolution equation were obtained, which allow the propagation of either solitary or blow-up pulses.

The present results may be useful in understanding the nonlinear properties of e–p–i plasmas in astrophysical environments, such as in active galactic nuclei, in pulsar magnetospheres, in the solar atmosphere, in neutron stars, etc.

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