Nonlinear electromagnetic wave equations for superdense magnetized plasmas

Nitin Shukla,[a)] G. Brodin, M. Marklund, P. K. Shukla,[b)] and L. Stenflo[c)]
Department of Physics, Umeå University, Umeå SE-90187, Sweden

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By using the quantum hydrodynamic and Maxwell equations, we derive the generalized nonlinear electron magnetohydrodynamic, the generalized nonlinear Hall-MHD (HMHD), and the generalized nonlinear dust HMHD equations in a self-gravitating dense magnetoplasma. Our nonlinear equations include the self-gravitating, the electromagnetic, the quantum statistical electron pressure, as well as the quantum electron tunneling and electron spin forces. They are useful for investigating a number of wave phenomena including linear and nonlinear electromagnetic waves, as well as three-dimensional electromagnetic wave turbulence spectra and structures arising from mode coupling processes at nanoscales in dense quantum magnetoplasmas. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3184571]

I. INTRODUCTION

Superdense quantum plasmas are ubiquitous in compact astrophysical objects[1–5] (e.g., the interior of massive white dwarf stars, magnetars, Jupiter's interior) and in the next generation intense laser-solid density plasma experiments.[6–10] In dense plasmas the degenerate electrons follow Fermi–Dirac statistics, and there are quantum tunneling[11–16] and spin[17–20] forces due to the spread in the electron probability wave function. The quantum statistical electron pressure and quantum Bohm forces produce wave dispersions at nanoscales. Accordingly, there has been a great deal of interest[12,13,16,19–23] in investigating linear and nonlinear waves/structures at quantum scales in very dense quantum plasmas. The collective x-ray scattering measurements of plasmons in solid-density plasmas[8] reveal the signature of quantum effects (both statistical pressure and quantum Bohm force) on the dispersion of the modified electron plasma wave.

It is well known that electrons and antielectrons (positrons) are degenerate in superdense magnetoplasmas that are found in the vicinity of pulsars and magnetars,[4] as well as in the interior of massive white dwarf stars and Jupiter.[2–4] Accordingly, the dynamics of electrons and antielectrons in Fermi degenerate plasmas is greatly affected by the inclusion of the Lorentz force, the quantum statistical pressure, the quantum Bohm force, and electron and antielectron $-1/2$ spin effects. Thus, electromagnetic wave theories (both linear and nonlinear) in dense magnetized plasmas have to be developed, accounting for all quantum effects mentioned above. A recent paper by Haas[24] develops nonlinear quantum Hall-magnetohydrodynamic (Q-HMHD) equations, accounting for the Lorentz force, the quantum statistical pressure, and quantum Bohm force in an electron-ion dense magnetoplasma. Quantum ideal magnetostatic stationary equilibria without ion flow have also been presented.

In this paper, we derive a set of nonlinear equations for finite amplitude electromagnetic waves in superdense self-gravitating dense magnetoplasmas, taking into account the combined effects of the Lorentz force, the quantum statistical pressure, the quantum Bohm force, and the electron $-1/2$ spin. Specifically, we shall use the generalized Q-MHD equations to obtain compact nonlinear equations for the electron MHD (EMHD), HMHD, and dust HMHD plasmas, and show how the density, fluid velocity, and magnetic fields are coupled in a nontrivial manner. The present equations shall be useful for studying numerically[25] the linear and nonlinear wave phenomena at quantum scales in superdense laboratory and astrophysical plasmas. While in the latter, the quantum statistical pressure and gravitational forces can play a major role, both the quantum Bohm and spin forces acting on the electrons are relevant for the next generation intense laser-solid density plasma experiments and for quantum free electron lasers.[26] Furthermore, it should be emphasized that the electron $-1/2$ spin force should play a crucial role in plasma assisted nanoscale structures in plasmonic devices.[27,28]

II. DERIVATION OF THE NONLINEAR EQUATIONS

The governing nonlinear equations for electromagnetic waves in dense plasmas are the quantum magnetohydrodynamic equations composed of the continuity equation,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = 0, \quad (1)$$

the electron momentum equation,

[a)]Also at GoLP/Instituto de Plasmas Fusão Nuclear, Instituto Superior Técnico, Universidade Técnica do Lisboa, 1049-001 Lisboa, Portugal. Electronic mail: nitshukla@yahoo.com.
[b)]Also at Scottish Universities Physics Alliance (SUPA), Department of Physics, University of Strathclyde, Glasgow G4 ONG, U.K.; GoLP/Instituto de Plasmas Fusão Nuclear, Instituto Superior Técnico, Universidade Técnica do Lisboa, 1049-001 Lisboa, Portugal; School of Physics, University of KwaZulu-Natal, 4000 Durban, South Africa; Max-Planck Institut für Extraterrestrische Physik und Plasmaphysik, D-85740 Garching, Germany. Electronic mail: ps@tp4.rub.de.
[c)]Also at Department of Physics, Linköping University, SE-58183 Linköping, Sweden. Electronic mail: lennart.stenflo@physics.umu.se.

$$n_e m_e\left[\frac{\partial \mathbf{u}_e}{\partial t}+\mathbf{u}_e\times\nabla\mathbf{u}_e\right]=-n_e e\left[\mathbf{E}+\frac{1}{c}\mathbf{u}_e\times\mathbf{B}\right]-\nabla p_e+\mathbf{F}_{Qe}, \quad (2)$$

the Faraday law,

$$c\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \quad (3)$$

the Maxwell equation including the magnetization spin current,

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}(\mathbf{J}_p+\mathbf{J}_m)+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \quad (4)$$

and the momentum equations,

$$n_\sigma m_\sigma\left[\frac{\partial \mathbf{u}_\sigma}{\partial t}+\mathbf{u}_\sigma\times\nabla\mathbf{u}_\sigma\right]=q_\sigma n_\sigma\left[\mathbf{E}+\frac{1}{c}\mathbf{u}_\sigma\times\mathbf{B}\right]-n_\sigma m_\sigma\nabla\phi_g, \quad (5)$$

where we have denoted the pressure for a nonrelativistic degenerate electron gas,

$$p_e=\left(\frac{4\pi^2\hbar^2}{5m_e}\right)(3/8\pi)^{2/3}n_e^{5/3}, \quad (6)$$

and the sum of the quantum tunneling and spin forces,

$$\mathbf{F}_{Qe}=\frac{\hbar^2}{2m_e}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)-n_e\mu_B\tanh(\xi)\nabla B. \quad (7)$$

We should point out that there are generalizations of the tunneling force that modify the expression for the stress tensor (see Ref. 11), such that quantum corrections can be incorporated also in the energy equations, see Ref. 11. Furthermore, there are extensions of the spin force that includes higher order effects in an $\hbar$ expansion (see Ref. 17).

The gravitational force is

$$\nabla^2\phi_g=4\pi G\sum_{\sigma=i,d}m_\sigma n_\sigma, \quad (8)$$

where G is the gravitational constant. Equations (1)–(8) are the generalization of the Q-HMHD equations[24] by including the self-gravitation force, the electron spin force, as well as the magnetization and displacement currents.

In Eqs. (1)–(7), n_j is the number density of the particle species j (j equals e for the electrons, i for the ions, and d for the dust grains), $\mathbf{u}_e$ is the electron fluid velocity, $\mathbf{u}_\sigma$ is the fluid velocity of the species σ ($\sigma=i,d$, the index i and d stand for the ions and charged dust grains), m_j is the mass, $q_\sigma=Z_i e$ for the ions and $\epsilon Z_d e$ for the dust grains, Z_i is the ion charge state, e is the magnitude of the electron charge, Z_d is the number of charge on dust, $\epsilon=-1(+1)$ for negative (positive) dust, c is the speed of light in vacuum, $\mu_B=e\hbar/2m_e$ is the Bohr magneton, $\hbar$ is the Planck constant divided by 2π, and $B=|\mathbf{B}|$. We have denoted plasma current density $\mathbf{J}_p=-n_e e\mathbf{u}_e+n_i e\mathbf{u}_i+\epsilon Z_d e n_d\mathbf{u}_d$ and the electron magnetization spin current density $\mathbf{J}_m=\nabla\times\mathbf{M}$, where the magnetization for dynamics on a time scale much slower than the spin precession frequency reads $\mathbf{M}=n_e\mu_B\tanh(\xi)\hat{\mathbf{B}}$. Here $\tanh(\xi)=B_{1/2}(\xi)$, $B_{1/2}$ is the Brillouin function with argument 1/2 describing electrons of spin 1/2 $\xi=\mu_B B/k_B T_{\mathrm{Fe}}$, $\hat{\mathbf{B}}=\mathbf{B}/B$, k_B is the Boltzmann constant, and T_{Fe} is the Fermi electron temperature. We have assumed that the spin orientation has reached the thermodynamical equilibrium state in response to the magnetic field, which accounts for the $\tanh(\xi)$ factor. On a time scale shorter than the spin relaxation time scale, the individual electron spin is conserved, and thus $\tanh(\xi)$ can be taken as constant for an initially inhomogeneous plasma.

Let us first present the generalized nonlinear EMHD equations for dense plasmas. Here the ions and dust grains form the neutralizing background. The wave phenomena in the EMHD plasma will occur on a time scale much shorter than the ion/dust plasma and gyroperiods. In equilibrium, we have[29]

$$n_{e0}=Z_i n_{i0}+\epsilon Z_d n_{d0}, \quad (9)$$

where the subscript 0 stands for the unperturbed value.

The relevant nonlinear EMHD equations are

$$\frac{\partial n_e}{\partial t}+\nabla\cdot(n_e\mathbf{u}_e)=0, \quad (10)$$

the electron momentum equation (2), Faraday's law (3), and the electron fluid velocity given by

$$\mathbf{u}_e=\frac{\mathbf{J}_m}{en_e}-\frac{c(\nabla\times\mathbf{B})}{4\pi}+\frac{\partial\mathbf{E}}{\partial t}. \quad (11)$$

We observe that the quantum tunneling and spin forces play an important role if there are slight electron density and magnetic field inhomogeneities in dense plasmas. The nonlinear EMHD equations are useful for studying collective electron dynamics in metallic and semiconductor nanostructures.[30]

Second, we derive the modified nonlinear HMHD equations in a self-gravitating dense electron-ion plasma with immobile charged dust grains. The HMHD equations shall deal with the wave phenomena on a time scale larger than the electron gyroperiod. The relevant nonlinear HMHD equations are the electron and ion continuity equations, the inertialess electron momentum equation,

$$\mathbf{E}+\frac{1}{c}\mathbf{u}_e\times\mathbf{B}+\frac{\nabla p_e}{n_e e}-\frac{\mathbf{F}_{Qe}}{n_e e}=0, \quad (12)$$

Faraday's law (3), the ion momentum equation,

$$n_i m_i\left[\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\times\nabla\mathbf{u}_i\right]=Z_i e n_i\left[\mathbf{E}+\frac{1}{c}\mathbf{u}_i\times\mathbf{B}\right]-n_i m_i\nabla\phi_g, \quad (13)$$

with

$$\nabla^2\phi_g=4\pi G m_i n_i, \quad (14)$$

and the electron fluid velocity given by

$$\mathbf{u}_e=\frac{Z_i n_i\mathbf{u}_i}{n_e}+\frac{\mathbf{J}_m}{en_e}-\frac{c(\nabla\times\mathbf{B})}{4\pi}, \quad (15)$$

where we have neglected the displacement current since the HMHD plasma deals with electromagnetic waves whose phase velocity is much smaller than the speed of light in vacuum.

We now eliminate the electric field from Eq. (13) by using Eq. (12), obtaining

$$n_i m_i\left[\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\right]=Z_i e n_i\left[\frac{1}{c}(\mathbf{u}_i-\mathbf{u}_e)\times\mathbf{B}-\frac{\nabla p_e}{n_e e}+\frac{\mathbf{F}_{Qe}}{n_e e}\right]-n_i m_i\nabla\phi_g. \tag{16}$$

Furthermore, eliminating $\mathbf{u}_e$ from Eq. (16) by using Eq. (15), we have

$$n_i m_i\left[\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\right]=Z_i e n_i\left\{\frac{1}{c}\left[\frac{\epsilon Z_d n_d}{n_e}\mathbf{u}_i-\frac{\mathbf{J}_m}{e n_e}+\frac{c(\nabla\times\mathbf{B})}{4\pi}\right]\times\mathbf{B}-\frac{\nabla p_e}{n_e e}+\frac{\mathbf{F}_{Qe}}{n_e e}\right\}-n_i m_i\nabla\phi_g, \tag{17}$$

where $n_e=Z_i n_i+\epsilon Z_d n_d$ and $Z_d n_d$ is constant.

Finally, by using Eq. (12) we can eliminate $\mathbf{E}$ from Eq. (2) obtaining

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\left[\frac{Z_i n_i\mathbf{u}_i\times\mathbf{B}}{n_e}+\frac{\mathbf{J}_m\times\mathbf{B}}{e n_e}-\frac{c}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}\right]. \tag{18}$$

The ion continuity equation, Eqs. (17) and (18), together with Eq. (14) and the quasineutrality condition $n_{i1}=n_{e1}$, where $n_{e1,i1}\ll n_{e0,i0}$ are the desired generalized nonlinear equations for the low-frequency (in comparison with the electron gyrofrequency), low-phase velocity (in comparison with the speed of light in vacuum) in a self-gravitating HMHD plasma with immobile charged dust grains. These equations, which are more general than those in Ref. 24, show nontrivial linear and nonlinear couplings between the density, the ion fluid velocity, the magnetic field fluctuations, and the self-gravitating potential in a dense magnetoplasma. They describe the dynamics of a broad range of electromagnetic waves that are linearly and nonlinearly coupled.

Third, we consider a self-gravitating dense magnetoplasma composed of inertialess electrons and ions, as well inertial dust grains. The dust mass density $n_d m_d$ is supposed to be much larger than the ion mass density $n_i m_i$. The relevant nonlinear dust HMHD equations, valid for the low-frequency (in comparison with the ion gyrofrequency) electromagnetic waves, are then composed of the dust continuity equation,

$$\frac{\partial n_d}{\partial t}+\nabla\times(n_d\mathbf{u}_d)=0, \tag{19}$$

the inertialess electron, and ion momentum equations, which are combined with the dust momentum equation (in which the electric field is eliminated by using the inertialess electron and ion momentum equation),

$$n_d m_d\frac{d\mathbf{u}_d}{dt}=\left(\frac{c}{4\pi}\nabla\times\mathbf{B}-\mathbf{J}_m\right)\times\mathbf{B}-\frac{\epsilon Z_d n_d\nabla p_e}{n_e}+\frac{\epsilon Z_d n_d\mathbf{F}_{Qe}}{n_e}-n_d m_d\nabla\phi_g, \tag{20}$$

with

$$\nabla^2\phi_g\approx 4\pi G n_d m_d, \tag{21}$$

together with Faraday's law (3). Here $d/dt=(\partial/\partial t)+\mathbf{u}_d\cdot\nabla$. In deriving Eq. (20) we have used the modified Ampère's law [e.g., Eq. (4) without the displacement current] in view of the low-phase velocity (in comparison with c) electromagnetic waves.

From Faraday's law (3) and the dust momentum equation, we obtain

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{u}_d\times\mathbf{B})-\frac{c m_d}{q_d}\nabla\times\frac{d\mathbf{v}_d}{dt}. \tag{22}$$

Equations (19), (20), and (22) with $n_e\approx Z_d n_d\gg Z_i n_{i0}$ are the desired dust HMHD equations in self-gravitating dense magnetoplasmas. These equations govern the dynamics of the modified coupled dust cyclotron, dust acoustic, and dispersive dust Alfvén waves. The dispersion arises due to the quantum electron tunneling and the finite frequency (in comparison with the dust gyrofrequency)/or the dust skin effect.

III. SUMMARY AND CONCLUSION

In this paper, we have derived the nonlinear equations for electromagnetic waves in dense self-gravitating magnetoplasmas. For this purpose, we have used the generalized quantum MHD and Maxwell's equations, and obtained three sets of nonlinear equations, which exhibit nontrivial linear and nonlinear couplings between the plasma number density, fluid velocity, magnetic field fluctuation, and self-gravitating potential. The present set of nonlinear equations should be used to investigate numerically[31] the dynamics of obliquely (against the external magnetic field direction) propagating modified electron whistlers within the generalized EMHD plasma model, as well as the modified fast and slow modes/dispersive electromagnetic ion-cyclotron-kinetic Alfvén waves within the generalized HMHD plasma model, and the modified coupled dust-cyclotron-dust acoustic and dispersive dust Alfvén waves within the dust HMHD plasma model. The obtained results would then provide valuable information on multidimensional localized electromagnetic waves (e.g., x-rays and gamma ray bursts) at nanoscales, which may emanate in the interior of massive white dwarf stars and in magnetars,[1–3] as well as in the next generation intense laser-solid density plasma experiments,[5,8,9] in x-ray free-electron lasers,[26] and in plasmonic devices.[27,28,32] In passing, we mention that in a dense magnetized plasma there also exist spin waves, which can be excited by intense neutrino fluxes.[33]

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