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We present direct upper limits on gravitational wave emission from the Crab pulsar using data from the first 9 months of the fifth science run of the Laser Interferometer Gravitational-wave Observatory (LIGO). These limits are based on two searches. In the first we assume that the gravitational wave emission follows the observed radio timing, giving an upper limit on gravitational wave emission that beats indirect limits inferred from the spin-down and braking index of the pulsar and the energetics of the nebula. In the second we allow for a small mismatch between the gravitational and radio signal frequencies and interpret our results in the context of two possible gravitational wave emission mechanisms.

Subject headings: gravitational waves — pulsars: individual (Crab pulsar)

Online material: color figure

1. INTRODUCTION

The Crab pulsar (PSR B0531+21, PSR J0534+2200) has long been regarded as one of the most promising known local sources of gravitational wave emission and is an iconic target for gravitational wave searches (Press & Thorne 1972; Zimmermann 1978). Its high spin-down rate, \( \dot{\nu} \approx -3.7 \times 10^{-10} \) Hz s\(^{-1}\), corresponds to a kinetic energy loss rate of \( \dot{E} = 4\pi^2 I_\nu \dot{\nu} \approx 4.4 \times 10^{31} \) W (using a spin frequency of \( \nu = 29.78 \) Hz and the canonical value of 10\(^{-24}\) kg m\(^2\) s\(^{-1}\) for the principal moment of inertia \( I_\nu \)). This loss is due to a variety of mechanisms, including magnetic dipole radiation, particle acceleration in the magnetosphere, and gravitational radiation. If one assumes that all the energy is being radiated gravitationally, the gravitational wave tensor amplitude at Earth is \( h_\nu^\text{em} = 8.06 \times 10^{-11} I_\nu r_{\text{pc}}^{-1} (|\dot{\nu}|/\nu)^{1/2} \), where \( r_{\text{pc}} \) is the distance to the pulsar in kpc and \( I_\nu \) is the moment of inertia in units of the canonical value (Abbott et al. 2007c). For the Crab pulsar this “spin-down upper limit” is \( h_\nu^\text{em} = 1.4 \times 10^{-24} \), using the canonical moment of inertia and a distance \( r = 2 \) kpc. It has long been known that the Laser Interferometer Gravitational-wave Observatory (LIGO) can achieve this sensitivity by integrating several months of data with the initial design noise spectrum.

The electromagnetic emission and accelerating expansion of the Crab Nebula are powered almost entirely by the rotation of the pulsar. The question now is whether these two loss mechanisms can account for the vast majority of the observed rotational energy loss, or whether gravitational wave emission has a significant part to play.

The bolometric luminosity of the nebula is \( (1-2) \times 10^{39} \) W, which accounts for less than half the spin-down power (e.g., Davidson & Fesen 1985). There have been many attempts to estimate the power involved in the observed acceleration of optical filaments, for example recently by Bejger & Haensel (2002, 2003). However these depend on poorly known factors such as the mass and expansion history of the nebula, and the uncertainties in the estimated power are comparable to the spin-down power itself. Thus electromagnetic observations of the nebula, within their uncertainties, still allow for a substantial fraction of the spin-down power to be emitted in gravitational waves.

The braking index \( n = n_0 n_/ \) of the pulsar further constrains the gravitational wave emission. The observed value \( n = 2.5 \) still is not well understood on theoretical grounds, but since quadrupolar radiation has \( n = 5 \) it implies that only a small fraction of the spin-down power is emitted in gravitational waves. The best estimate in print is by Palomba (2000) who uses a phenomenological model of the spin-down (present and historical) together with the present braking index and known age of the pulsar to estimate that the highest possible \( h_\nu \) today is about 40% of the spin-down limit. This value is consistent with the observations of the nebula, and is also observable with several months of data from LIGO’s fifth science run (S5).

Early directed searches for gravitational waves from the Crab pulsar were performed by Levine & Stebbins (1972) using a 30 m interferometer to give a strain upper limit of \( 3 \times 10^{-17} \), and Hirakawa et al. (1978) using a bar detector. The most recent bar result (Suzuki 1995) gave an upper limit that was still over an order of magnitude above the spin-down limit. The LIGO detectors have improved on these results, with LIGO’s second science run (S2) producing a 95% upper limit of \( h_\nu^\text{S2} = 4.1 \times 10^{-21} \) (Abbott et al. 2005), and the combined data from the S3 and S4 runs produced an upper limit of \( h_\nu^\text{S3+S4} = 3.1 \times 10^{-23} \) (Abbott et al. 2007c) only 2.2 times greater than the spin-down limit.

In this Letter, we describe searches of data from the fifth LIGO science run, which started on 2005 November 4 and ended on 2007 October 1 (Abbott et al. 2007b). During this period the detectors (the 4 km and 2 km detectors at LIGO Hanford Observatory, H1 and H2, and the 4 km detector at the LIGO Livingston Observatory, L1) were at their design sensitivities and had duty factors of 78% for H1, 79% for H2, and \( \sim 66\% \) for L1. The GEO600 detector (Lück et al. 2006) also participated in the S5 run but was much less sensitive at the frequency of the expected signal.

The Crab pulsar was observed to glitch on 2006 August 23 at approximately 04:00 UTC (Lyne et al. 2007; A. G. Lyne 2006, private communication). Since the glitch mechanism is not certain and may involve unpredictable changes in the gravitational wave timing and amplitude, we use this glitch as natural point at which to pause this coherent search for the Crab pulsar. Our data set consists of H1 and H2 data from 2005 November 4 and L1 data from 2005 November 14 up to 2006 August 23. For the two different searches carried out in this analysis, described below, this gives 201, 222, and 158 days.
of data for H1, H2, and L1, respectively, for the single-template search, and 182, 206, and 141 days of data for the multitemplate frequency-frequency first-derivative search, which required larger contiguous segments than the single-template search.

2. METHODS

We use two different methods (see Abbott et al. 2004) to search for gravitational waves from the Crab pulsar to account for different emission scenarios. One method uses a single time domain template for the gravitational wave signal assuming that the gravitational wave period evolves precisely as the electromagnetic pulse period. The other method works in the frequency domain to cover a relatively small, physically motivated range of frequency and spin-down values. The searches use the known frequency and position of the Crab pulsar, as derived from the Jodrell Bank Crab Pulsar Monthly Ephemeris (Lyne et al. 2007). Using this ephemeris and the assumption that the gravitational wave and electromagnetic phase track each other precisely, we can predict the signal phase evolution with negligible uncertainty. Both searches assume that emission will be at or near twice the pulsar’s spin frequency, \( 2\nu = \nu_{GW} \approx 59.56 \text{ Hz} \), which is the frequency of emission by a steadily rotating quadrupolar deformation, i.e., a triaxial star. The Crab pulsar might be emitting at \( \nu_{GW} = 4\nu/3 \) through an \( r \)-mode (Owen et al. 1998) if the mode saturates at a small amplitude and thus is long-lived (e.g., Brink et al. 2005). However, the uncertainty of this frequency is relatively large, of order one part in \( 10^3 \) (Lindblom et al. 1999). Due to this, and the greater instrument noise at this frequency, we did not search for \( r \)-modes. Although \( 2\nu \) is close to the 60 Hz power line frequency, it is sufficiently far away that the searches are relatively unaffected by nonstationary components of the power line noise. The absolute timing accuracy of the LIGO data is sufficiently good that the likelihoods produced for each detector can be combined to give a joint likelihood.

For a given search frequency and spin-down, the four unknown signal parameters are the gravitational wave amplitude \( h_0 \), the initial phase \( \phi_0 \), the spin-axis inclination angle \( \iota \), and the polarization angle \( \psi \). X-ray observations of the Crab Pulsar Wind Nebula provide values of the orientation angle \( \iota \) and polarization angle \( \psi \) of the pulsar. From Ng & Romani (2004, 2008) we use \( \iota = 62.17^\circ \pm 2.195^\circ \) and \( \psi = 125.155^\circ \pm 1.355^\circ \), where we have taken the mean of the best-fit values for the outer and inner tori of the nebula. We use these ranges to put Gaussian priors on these two parameters for both the search techniques.

The single-template search (Dupuis & Woan 2005) assumes a triaxial star emitting gravitational waves at precisely twice the spin frequency, following the electromagnetic pulse phase evolution and taking into account the small variations in phase caused by timing noise (Pitkin & Woan 2007). It uses a standard Bayesian methodology to produce a joint posterior probability volume over the four unknown parameters using data from all three detectors. We use both uniform priors and restricted priors on \( \psi \) and \( \iota \) when calculating the posterior. We marginalize the angle parameters to produce a posterior probability for \( h_0 \) and from this calculate a 95% degree-of-belief upper limit on the gravitational wave amplitude.

A search was also performed at gravitational wave frequencies \( \nu_{GW} \) in a narrow band about \( 2\nu \), based on simple astrophysical arguments. We begin by writing \( \nu_{GW} = 2(1 + \delta) \nu \), where \( \delta \) is a small number. A relation of this form holds if the gravitational waves are produced by a component spinning separately from the electromagnetically emitting one, with the two components linked by some torque which acts to enforce corotation between them on a timescale \( \tau_{\text{coupling}} \). In such a case \( \nu_{GW} \) above also holds if the gravitational waves are produced by free precession of a nearly biaxial star (Jones & Andersson 2002). In such a case \( \delta \sim \alpha (\nu_c - \nu) / \nu_c \), where \( \alpha \) is a factor of order unity dependent on the geometry of the free precession, e.g., the angle between the symmetry axis and angular momentum axis. No clear sig-
nature of free precession has been seen in the radio pulsations of the Crab pulsar, although precession would have little effect on the radio signal if the amplitude of the precession were small.

Together, these scenarios suggest searching over a frequency interval \( \pm \Delta \nu_{GW} \) centered on \( 2\nu \), where \( \Delta \nu_{GW} \sim |\delta| 2\nu \). We have followed such a strategy, using a maximum value of \(|\delta| = 10^{-4}\). In terms of the two-component model, such a \(|\delta| \) value corresponds to \( \tau_{\text{coupling}} \sim 10^{-2} \tau_{\text{spin-down}} \) several months, comparable to the longest timescales seen in glitch recovery where recoupling between the two components might be expected to occur. In terms of free precession, \(|\delta| = 10^{-4}\) is on the high end of the range of deformations that compact objects are thought to be capable of sustaining (Owen 2005; Lin 2007; Haskell et al. 2007).

Using the above estimates as a guide, a band of frequencies \( \pm 6 \times 10^{-7} \) Hz centered on twice the Crab pulsar’s observed frequency was searched over. Corresponding bands in frequency derivatives were motivated via differentiation of the equation for \( \nu_{GW} \), which together with the assumption that \( \delta \) itself evolves no more rapidly than on the spin-down timescale, leads to a band in frequency first derivative of \( \pm 1.5 \times 10^{-13} \) Hz s\(^{-1}\), with searches over higher derivatives being unnecessary.

The multitemplate search method is a maximum likelihood technique, the coherent multidetector \( F \)-statistic derived in Cutler & Schutz (2005). An explicit search is required over a single sky position and second derivative of the frequency, and over the selected ranges of the frequency and of the first frequency derivative. The spacing of the templates is chosen in such a way as to ensure at most a 5% loss in the detection statistic, resulting in a total of \( 3 \times 10^7 \) templates. The detection statistic \( 2F \) is computed for each template. The expected 3 \( \sigma \) range of the largest \( 2F \) value for Gaussian noise (no signal present) and \( 3 \times 10^7 \) templates is 35–49. The largest \( 2F \) value found in the actual search is 37, well within the expected range for noise.

Based on the largest \( 2F \) value, 95% confidence upper limits are produced using a frequentist Monte Carlo injection method, as described in Abbott et al. (2007a). For the unknown parameters uniform distributions and physically informed distributions were used for the injected population of signals, consistent with the choices made for the single-template time domain search.

### 3. Results

In the single-template search the joint (i.e., multidetector) posterior probability distribution for the gravitational wave amplitude peaks at zero, indicating that no signal is visible at our current sensitivity. The joint 95% upper limit on the gravitational wave amplitude, using uniform priors on all the parameters, is \( h_{95\%} = 3.4 \times 10^{-25} \). In terms of the pulsar’s ellipticity, given by \( \epsilon = 0.237 h_{-24} f_{10} p^{-3} I_{38} \) (Abbott et al. 2007c), where \( h_{-24} = h_0 \) in units of \( 1 \times 10^{-24} \), this gives \( \epsilon = 1.8 \times 10^{-4} \) using the canonical moment of inertia and \( r = 2 \) kpc. This is 4.1 times lower than the spin-down upper limit and also 1.6 times lower than the limit estimated by Palomba (2000) (see § 1). Squaring the ratio of the spin-down and direct upper limit shows that less than \( \approx 6\% \) of the total power available from spin-down is being emitted as gravitational waves, assuming the canonical moment of inertia. Using the restricted priors on \( \psi \) and \( \epsilon \) we get an upper limit on \( h_0 \) of \( 2.7 \times 10^{-25} \), which is 1.3 times smaller than that with uniform priors, and corresponds to less than 4% of the spin-down energy available.

With the coherent multitemplate frequency-frequency first-derivative search we set 95% confidence upper limits on \( h_0 \) and ellipticity of \( 1.7 \times 10^{-24} \) and \( 9.0 \times 10^{-4} \), respectively, over the entire parameter space searched. These upper limits are larger than the single-template search limits by roughly a factor of 5. This is to be expected because the larger number of templates raises the number of trials and thus the statistical confidence threshold. Assuming restricted priors on \( \psi \) and \( \epsilon \) yields an improved upper limit of \( 1.2 \times 10^{-24} \), a factor of 1.2 below the spin-down limit, across the entire parameter space searched. This limits the energy budget of gravitational waves to be less than 73% of the available energy. These quoted upper limits are subject to uncertainty in the calibration of the detectors. Amplitude calibration uncertainties for H1, H2, and L1, respectively, are 8.1%, 7.2%, and 6.0% (single-template analysis), and 9.5%, 7.8%, and 8.7% (multitemplate analysis).

### 4. Discussion

Under the assumption that the gravitational wave and the electromagnetic signals are phase-locked, our single-template search results constrain the gravitational wave luminosity to be less than 6% of the observed spin-down luminosity. This beats the indirect limits inferred from all electromagnetic observations of the Crab pulsar and nebula.

Our upper limits are interesting because they have entered the outskirt of the range of theoretical predictions. Normal neutron stars are believed to be mostly fluid with maximum elastic deformations orders of magnitude smaller than the few \( \times 10^{-4} \) of our upper limits, but some theories of quark matter predict solid or mostly solid stars which could sustain such ellipticities (Owen 2005; Lin 2007; Haskell et al. 2007). However, our upper limits do not constrain the composition of the star and cannot constrain any fundamental properties of quark matter. The ellipticity is proportional to the quadrupole strain, which may simply be very low for a given star no matter its composition. The Crab is likely to have an ellipticity at least about \( 10^{-11} \) due to the stresses of its internal magnetic field (Cutler 2002) if the internal field is comparable to the external dipole of \( 4 \times 10^{15} \) G. Our upper limits can be interpreted as direct upper limits of about \( 10^{15} \) G on the internal magnetic field, depending on the ratio of toroidal to poloidal components (Colaiuda et al. 2008).

As discussed in Abbott et al. (2007c) there is considerable uncertainty in the true value of the Crab pulsar’s moment of inertia. The best guesses at its value come from neutron star equation of state models rather than direct measurements. Previous pulsar ellipticity upper limits and spin-down limits have made use of the canonical value of \( I_\nu \). We can however cast our upper limit in a way that makes no assumptions about the moment of inertia, by placing the limit on the neutron star quadrupole moment \( m_{zz}^e \). This then allows us to plot the single-template search results as exclusion regions in the \( I-\epsilon \) plane. The results, with uniform and restricted prior ranges, are plotted in this way in Figure 1. Our upper limits are smaller than the spin-down limit by a factor that varies as \( I_\nu^{-1/2} \). If we take the theoretical upper bound on the moment of inertia to be \( 3 \times 10^{48} \) kg m\(^2\) as in Abbott et al. (2007c) then the result with uniform priors beats the spin-down limit by a factor of 7.2.

Finally, the physical interpretation of our multitemplate search depends on the assumed cause of the splitting \( \nu_{GW} = 2\nu(1 + \delta) \) between gravitational and electromagnetic signals. In the context of the two-component spin-down model, our results show that a gravitational wave emitting component of the star
coupled to the electromagnetic (radio) emitting component on a timescale of a few months or less has a quadrupole asymmetry $I_{y} - I_{z}$ of no more than $9.0 \times 10^{34}$ kg m$^2$. This is about 5 times larger than the bound on $I_{y} - I_{z}$ obtained in the single-template search. If free precession is responsible for the frequency splitting our results instead give an upper limit on the product $D I \sin^2 \theta$, where $D$ is the $I_{z} - I_{x}$ part of the quadrupole moment tensor that participates in the precession and $\theta$ the wobble angle (Jones & Andersson 2002).

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A processing error in the signal template used in this search led to upper limits about 30% lower than we now know is warranted by the early S5 data. We have re-analyzed that data and find new upper limits on the strain parameter $h_0$ of $4.9 \times 10^{-25} / 3.9 \times 10^{-25}$ for uniform/restricted prior assumptions concerning the Crab inclination and polarization angles. These results have now been superseded by upper limits of $2.6 \times 10^{-25} / 2.0 \times 10^{-25}$ based on the full S5 data and presented in Abbott et al. (2009). The multitemplate search was not affected by the error.

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