Economics of Energy Efficiency

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1. Overview

Improvements in energy efficiency are seen as a key mechanism for reducing energy dependence and meeting sustainability and security of supply goals (Sorrell, 2007; Stern, 2007). However, there is dispute about the way in which the economy responds to such efficiency improvements. An increase in energy efficiency reduces the price of energy, measured in efficiency units, and this has output, income and substitution effects that tend to mitigate, and possibly to offset totally, any energy saving. Mitigation is labelled as “rebound” and an increase in energy use as “backfire”.

Rebound and backfire involve system-wide effects that are difficult to quantify and track. In this chapter we adopt a purely analytical approach that investigates the impact of an improvement in energy efficiency in a stylised open economy. The aim is pedagogic: that is to identify and clarify the nature of the various system-wide factors that can affect the change in energy use that accompanies improvements in energy efficiency.

Section 2 explains the small open economy model used and the resource, technology and sustainability problems that it faces. Section 3 introduces improvements in energy efficiency into the model and discusses measures of energy productivity. Section 4 analyses the way in which energy use will be affected by improvements in energy efficiency. Section 5 discusses how tax policy can adjust the profit maximising energy use after improvements in energy efficiency. Section 6 extends the simple model in three directions so as to analyse variations in the price elasticity of demand for the product, the elasticity of substitution in the production function and the elasticity of supply of non-energy inputs. Section 7 discusses improved energy efficiency in consumption. Section 8 concludes.

2. Small stylised open economy: resource, technology and sustainability constraints

We wish to illustrate the issues raised by concern over energy efficiency in a very simple stylised model of a small open economy. This approach is adopted in order to illustrate the underlying issues that might be obscured in more practical and detailed studies.

In this model it is assumed that an economy produces an output Q of a single commodity by means of a fixed amount of local resources, N, and homogeneous energy used in production, E_P. This output is either consumed locally or exported and energy is wholly imported at a fixed international price. The price of output is taken as the numeraire so that the price of energy is given as p_E. The difference between the output of the economy and the energy imports is a surplus generated in production, S, available for consumption, C. Initially we focus only on production and assume that consumption is of the local output or non-energy imports. This assumption is relaxed in Section 7.

The relationship between local and energy inputs and output is determined, initially at least, by a well-behaved production function. This implies the following. First, with no energy input, there is no output. Second, with fixed amounts of other inputs, an increase in energy use will generate additional output but at a diminishing rate. Third, there are constant returns to scale. The model can therefore be specified as:

(1) $$S = C = Q - p_E E_P$$

(2) $$Q = Q(N, E_P)$$

where:

$$N = \bar{N}$$

and

$$Q(L,0) = 0, \quad \frac{\partial Q}{\partial E_P} > 0, \quad \frac{\partial^2 Q}{\partial E_P^2} < 0$$

This model is illustrated in Figure 1. The upper half, Figure 1a, represents total output and total energy cost as a function of the level of energy inputs. The production frontier Q(E_P) shows the maximum output available for a each energy input, on the assumption that there is a fixed input of local resources and a given well behaved technology. Points on the production frontier are technically efficient: technical inefficiency is represented by points below and to the right of the production frontier. For any such points, there are possible movements to the frontier that will both generate higher output and use lower energy inputs.

FIGURE 1 AROUND HERE

The corresponding maximum consumption levels that are associated with given energy inputs (and resource and technology constraints) are presented in the lower half of Figure 1, Figure 1b. In such a simple model, the government's aim would normally be to maximise consumption.[1] This is achieved for an energy input where the marginal product of energy just equals the price, so that $\frac{\partial Q}{\partial E_P} = p_E$. This corresponds to point A in Figures 1a and 1b, with an output of Q^* and consumption C^*. This rule would apply in a centralised command economy whose aim is to maximise consumption, but would also be the outcome from a decentralised perfectly competitive economy with no market failures, since the equality of the marginal product of energy and the energy price would maximise profits.

However, sustainability issues typically drive current concerns over energy use. That is to say, the present level of energy consumption is thought to be unsustainable. The view that society might value an outcome that differs from the competitive one is represented by the notion of a social welfare function (SWF) (Bergson, 1938; Samuelson, 2004). In this case, the SWF would incorporate consumption as a positive component, but energy use as a negative component.[2]

Imagine that sustainability involves a minimum consumption level, $\underline{C}$, and a maximum energy use in production $\bar{E}_P$. Within the energy input, commodity output space defined by:

$$C > \underline{C}, \quad 0 < E_P < \bar{E}_P$$

there are a family of convex iso-SWF curves where each curve represents combinations of consumption and energy use that produce the same combined level of social welfare. Social welfare will be maximised, here implicitly incorporating sustainability considerations, where the consumption curve in Figure 1b is just tangent to the highest iso-SWF curve. This is shown as point B in Figures 1a and 1b and implies an output and consumption of Q^{**} and C^{**}, where $Q^{**} < Q^{*}$ and $C^{**} < C^{*}$.

Figures 1a and 1b suggest that with fixed resources and technology, achieving technical and allocative efficiency implies sacrificing some consumption.[3] In a decentralised market system this can be achieved through setting a tax on energy use, so as to make the price of energy equal to the slope of the production frontier at B.[4] Of course, incorporating sustainability involves giving positive weight to the utility of future generations. The idea that this requires less consumption for present generations meets some political resistance. The question that is addressed in this chapter is whether changes in energy efficiency can aid the attainment of sustainable goals.

3. **Energy efficiency and energy productivity**

The concept of energy efficiency used here is the notion of energy augmenting technical change. In this framework, an improvement in energy efficiency means an increase in the effective productive services generated by a given amount of energy inputs. This can be conveniently thought of as inputs of energy measured in efficiency units, F, where:

$$F = nE \tag{3}$$

An improvement in energy efficiency, or alternatively energy augmenting technical change, is represented by an increase in n. The idea of measuring energy inputs in efficiency units is similar to the engineering notion of useful work (Patterson, 1996; Sorrell and Dimitropoulos, 2008), where an improvement in energy efficiency is measured as an increase in useful work performed by a given energy input. In this chapter, where we discuss changes in energy use, the implicit assumption is that this is measured in natural units. Where energy measured is measured in efficiency units, this will be referred to explicitly.

There is a convenient way of analysing the impact of energy augmenting technical progress. In the conventional production function, the energy input measured in natural units, E_P, can be simply replaced with the same input measured in efficiency units. That is to say, equation (2) in the model presented in Section 2 can be replaced by equation (4):

(4) $$Q = Q(N, F_P) = Q(N, nE_P)$$

Where energy inputs are still measured in natural units, this has the effect of moving the production frontier upwards and to the left, still anchored at the origin, as shown later in Figure 3a. A central characteristic of an improvement in energy efficiency is that a given output can now be produced with the same level of other inputs but less energy. Also, with a conventional well-behaved production function, a higher output can be generated with the same energy and non-energy inputs.

It is important to draw a distinction between this measure of energy efficiency and the more straightforward measure of energy productivity, Π. This is the average output per unit of energy input, so that:

(5) $$\Pi = \frac{Q}{E_P}$$

The key point is that energy productivity is determined by a combination of energy efficiency and the ratio of energy to local resources used in production.

FIGURE 2 AROUND HERE

Figure 2 shows the production frontier from Figure 1a and identifies the consumption maximising point A, the welfare maximising point B and a further point C. At each point the slope of the line from the origin measures energy productivity: the steeper that slope, the higher the energy productivity. Clearly, moving from A to B shows a measured increase in the energy productivity, but this is unrelated to any change in energy efficiency. The production frontier has not shifted. The increase in energy productivity comes about as a result of the change in the ratio of energy inputs in production. Further, the less energy inputs are employed, with an unchanged technology, the higher the energy productivity will be: point C has a higher measured energy productivity than A or B. With a constant supply of local resources and no change in energy efficiency, an increase in energy productivity necessarily implies a reduction in total output.

4. Increased energy efficiency and energy use.

Figures 3a and 3b show the effect on production and consumption of an increase in energy efficiency in the simple model outlined in Section 2. The proportionate increase in energy efficiency is $\dot{n}$. The figures are constructed for a particular Cobb-Douglas form of the production function.[5] This implies that equation (4) can be written as:

$$(6) \qquad Q = AN^{1-\alpha}F_P^{\alpha} = BF_P^{\alpha} = BE_P^{\alpha}n^{\alpha}$$

where $A, B > 0,\ \ 1 > \alpha > 0$.[6] A is a general productivity parameter, and α a distributional parameter. With marginal productivity factor pricing, α is the share of energy costs in total output. Many of the characteristics of the Cobb-Douglas production function are replicated for other well-behaved functions. However, other characteristics are specific and these will be clearly distinguished in the discussion.

FIGURE 3 AROUND HERE

Figure 3a shows how the production frontier shifts outwards allowing the same output to be produced with less energy. The figure has been constructed so that the sustainable level of energy use can now be achieved with no change in output. The impact on the trade off between consumption and energy use is even more favourable: if output remains constant with lower energy inputs and unchanged prices, consumption will rise as energy costs in efficiency units have fallen. Sustainability can be achieved with a fall in energy use and a simultaneous increase in consumption.[7] However, a key issue in the literature is: will an increase in energy efficiency in itself lead to a reduction in energy use (Brookes, 1978; Jevons, 1865; Khazzoom, 1980; Saunders, 1990)?

As we observed in Section 2, in this simple model the output that would be derived from the free market mechanism will be the one that maximises consumption. The improvement in energy efficiency will allow an increase in consumption. However, with prices constant there is no guarantee that such an increase in consumption will be accompanied by a reduction in energy use. For a well-behaved production function the increase in energy efficiency reduces the price of energy in efficiency units and increases the price of local resources. In general this increases the profit maximising input of energy in efficiency units, so that there must be a degree of "rebound" in this model. That is to say, in this basic variant of the model the use of energy, measured in natural units, cannot fall by the full amount of the increase in energy efficiency.

For backfire, in the present model, with a well behaved, not necessarily Cobb-Douglas, production function, the general issue is straightforward. If at the initial consumption optimising energy use, $E^*_{P,1}$, the efficiency improvement increases the marginal productivity of energy, then the market equilibrium energy use (in natural units) will rise. There seems no strong *a priori* reason for ruling out this case. Moreover, for the Cobb-Douglas production function this condition will always hold.

Under the Cobb-Douglas production function, using the marginal productivity condition and equation (6), the profit maximising energy use, E_P^*, is given as:

(7) $$\left[\frac{\alpha B}{p_E}\right]^{\frac{1}{1-\alpha}} n^{\frac{\alpha}{1-\alpha}} = E_P^*$$

so that the proportionate change in energy use, $\dot{E}$, is:

(8) $$\dot{E}_P = \dot{n}\left[\frac{\alpha}{1-\alpha}\right] > 0$$

Note that the growth in energy use is positively related to the growth in energy efficiency. More especially, from equations (6) and (8), the growth in output will be equal the growth of energy inputs, so that the energy productivity will remain unchanged, though energy efficiency has improved.

This result is illustrated in Figure 3a. The reaction to the increase in energy efficiency is an equal proportionate expansion in output and energy use, with the profit (and consumption) maximising energy use increasing from $E_{P,1}^*$ to $E_{P,2}^*$. The supply of local resources is fixed and fully employed.

5. Price changes within the model

As has been argued in Section 2, the limitations to using improved energy efficiency to achieve sustainability targets stem from the increased choice presented by such improvements. The increase in energy efficiency allows greater consumption and encourages greater energy use, measured in efficiency units. One response to this is that the government could use tax or subsidy policy to change the prices faced by producers so as to bring about a more appropriate allocation of resources. That is to say, if energy use is too high after the introduction of improvements in energy efficiency, the government could place an appropriate tax on energy to improve the allocative efficiency of the market mechanism in the attainment of sustainability goals.[8]

At present the model has only one price, p_E, which is the price of energy relative to the domestically produced good. This price is fixed in international markets. However, it will be useful to introduce the post-tax price of energy, p_T, defined as:

(9) $$p_T = tp_E$$

where t is the ratio of the post- to pre-tax energy price. Where t is unity, there is no tax. Values of t less than one imply a subsidy and greater than one imply a tax. In this model the tax is raised simply to adjust for externalities and not in order to finance public goods. The revenues would therefore be redistributed to the local population.

The use of tax policy together with energy efficiency improvements is shown in Figure 4. It is perhaps appropriate here to discuss in a bit more detail the maximising procedure involved. With no taxes, the surplus (income) paid to local resources from production is given by equation (1). Rearranging equation (1) implies that the combinations of production and energy inputs that would generate any specific local resource income, $\overline{S}$, are given by the a positively sloped straight line:

(10) $$Q = p_E E_P + \overline{S}$$

These are iso-income curves. They have a slope equal to the energy price level and the constant term, which is the intercept on the Q axis, equals the value of the income. In Figures 1a and 2a the consumption maximising output, which is also the competitive equilibrium, is identified as the point on the relevant production frontier just tangent to the highest iso-income curve.

Where the government introduces a tax on energy this has two implications. First the iso-income curves that determine production choice in a market economy change to:

$$Q = p_T E_P + \overline{S}_T \tag{11}$$

With the introduction of a tax, the slope is now steeper and equals the post-tax price, p_T. The income earned by local resources, $\overline{S}_T$, is net of tax. Some of the income generated in production now goes to the government in tax revenue for redistribution. This tax income equals $E_P p_E (t-1)$.

The presence of rebound effects reduces the effectiveness of energy efficiency improvements in meeting energy saving targets. In Figure 4, as in Figure 3a, energy efficiency improvements shift the production frontier outwards from $Q_1(E_P)$ to $Q_2(E_P)$. The energy use is initially at the consumption maximising point $E_{P,1}$, producing output Q_1. With no tax, the figure is constructed such that energy use will rise in line with energy efficiency. To reduce rebound effects to zero, energy taxes should be introduced so that the income maximising output remains constant. This implies that the input of energy in efficiency units remains constant, so that the reduction in energy use in natural units is the full extent of the improvement in energy efficiency.

The necessary tax adjustment can be derived using equation (7), but using the post-tax price of energy, as given in equation (9). The international price for energy, p_E, and the production function parameters α and B are taken to be fixed, so that:

(12) $$\frac{\alpha}{1-\alpha}\dot{n}-\frac{\dot{t}}{1-\alpha}=\dot{E}_P$$

where the dot notation again represents proportionate changes. For no rebound effects, the fall in the energy demand is to equal the improvement in energy efficiency, so that:

(13) $$\dot{E}_P=-\dot{n}$$

Substituting equation (13) into (12) gives the result that:

(14) $$\dot{t}=\dot{n}$$

This no rebound result is illustrated in Figure 4 by pivoting the highest iso-income curve, S_2^{NR}, around the point on the Q axis, $\bar{S}_1$, until it is tangent to the new production frontier at G_2. The income maximising output remains unchanged at Q_1, but energy use falls from $E_{P,1}$ to $E_{P,2}$.

FIGURE 4 SOMEWHERE HERE

Although equation (14) has been derived for the particular Cobb-Douglas production function, the result is general. To totally neutralise any rebound effects in production from increased energy efficiency, the proportionate increase in the post-tax price of energy must be the same as the proportionate increase in energy efficiency. There are two practical problems with this result. The first is that as energy efficiency increases, the tax on energy required to remove totally rebound effects has to increase monotonically. The absolute level of present consumption forgone to prevent any rebound effects will increase over time.

A second problem is that the post-tax income received by local productive resources remains unchanged after the efficiency improvement. The output is unaffected, as is the post-tax price of energy in efficiency units. There is an increase in consumption, but this is generated solely by the redistributed increase in tax revenue. However, improvements in energy efficiency will generally require the commitment of resources by the production sector, in the form of investment in R and D, for example. In order to motivate firms to introduce the required efficiency improvements in the face of positive costs of innovation, the government must be able to credibly commit to continuously increasing energy taxation at the appropriate rate. There are clear credibility problems in implementing such a strategy (Leicester, 2005)

It is of interest also to consider, in the Cobb-Douglas case, what the tax policy should be if backfire is to be avoided. Again using equations (12) but in this case setting $\dot{E}$ equal to zero gives:

(15) $$\dot{t} = \alpha \dot{n}$$

This lower change in the tax rate means that the income maximising position shifts from G_2 to H_2 along the production frontier $Q_2(E_P)$. The no backfire highest iso-income curve is now S_2^{NB}, with a production income rising to $\overline{S}_{T,2}^{NB}$.

The result given in equation (15) is specific for the Cobb-Douglas production function. However, they show that even where a competitive market outcome would otherwise generate backfire, an appropriate tax policy can engineer an outcome where consumption rises, income to local resources increases but energy use in production falls. Again, in the Cobb-Douglas case this requires tax changes in the range:

$$\dot{n} \geq \dot{t} \geq \alpha\dot{n} \tag{16}$$

6. Modifications to the model

The model at present imposes values for three key elasticities: the elasticity of demand for exports, η; the elasticity of substitution in the production of the domestic good, σ; and the elasticity of supply of the local resource, λ. In this section we investigate the effect of varying these parameters. In the case of an open economy, the both the elasticity of export demand and the elasticity of substitution are shown to be key determinants of the size of rebound effects. These effects are also magnified where the supply of local resources is more elastic.

6.1 Elasticity of demand

It is common to focus on the elasticity of substitution as being the key parameter in the analysis of the impact of changes in energy efficiency. However we begin here by considering the elasticity of demand for the commodity. At present the small country assumption is made: that is, that the economy faces the law of one price in international markets. However, this is an extreme assumption, which effectively imposes a demand curve that is infinitely elastic. But if the output price has to fall to sell higher levels of output, the price ratio between energy inputs and local output becomes endogenous. Specifically, the price of energy relative to the numeraire good, the locally produced output, will rise. This price change restricts the increase in energy demand.

The general relationship between proportionate changes in the product price and quantity demanded is given as:

(17) $$\dot{Q} = \eta \dot{p}_E$$

where η is the price elasticity of demand, given a positive sign here, and noting that the product price is the numeraire. Using equation (17), together with equations (6) and (7), produces the result:

(18) $$\dot{E}_P = \frac{\alpha(\eta - 1)\dot{n}}{(1-\alpha)\eta + \alpha}$$

The relationship between employment change and the demand elasticity is given in Figure 5. Where demand is completely inelastic, so that $\eta = 0$, there is no rebound: energy demand in production falls the full amount of the efficiency change: $\dot{E}_P = -\dot{n}$. Where product demand is inelastic, so that $1 \geq \eta \geq 0$, there is a reduction in energy demand, but by less than the increase in energy efficiency. Some rebound occurs. Finally where demand for the product is elastic, with the price elasticity of demand taking a value greater than unity, energy use increases with an improvement in energy efficiency. Backfire occurs in this elasticity range, and $\dot{E}_P \rightarrow \frac{\alpha}{1-\alpha}\dot{n}$ as $\eta \rightarrow \infty$.

FIGURE 5 SOMEWHERE HERE

Clearly the elasticity of export demand is important for determining the way that energy use in production responds to an increase in energy efficiency. The more elastic the demand, the greater is the output response to the efficiency improvement and the higher the probability of getting backfire.

6.2 Elasticity of substitution in production

There is a very large literature relating to the relationship between energy efficiency, energy use and the elasticity of substitution in production (Broadstock *et al*, 2007; Saunders, 2008). In a two-factor production function, the elasticity of substitution, σ, is the responsiveness of the ratio of the inputs to changes in the relative input prices. If the elasticity of substitution is high, it is relatively easy to substitute one input for the other, whereas if the elasticity of substitution is low, substitution is difficult.

The previous sections of this chapter have used the Cobb Douglas production function, which has an elasticity of substitution equal to unity. Greater analytical scope is available with a Constant Elasticity of Substitution (CES) production function, where the impact of varying the substitution elasticity in production can be investigated (Varian, 1992). Such a production function has a constant elasticity of substitution between inputs but this elasticity figure can take any non-negative value.

A side relationship of the CES function gives the input intensity as:

(19) $$\left[\frac{p_N}{p_F}\frac{\phi}{1-\phi}\right]^{\sigma} = \frac{F_P}{N}$$

where p_N and p_F are the prices of local resources and energy, measured in efficiency units, ϕ is a distribution parameter and σ is the elasticity of substitution, where $0 \le \sigma \le \infty$.

The increase in the energy efficiency generates a reduction in the price of energy in efficiency units of $\dot{n}$. The price of output is constant so that, for small changes, an improvement in energy efficiency generates a proportionate increase in the price of the local resources is given by:

$$(20) \qquad \dot{p}_N = \frac{\alpha \dot{n}}{(1-\alpha)}$$

In our standard model, local resources, N, are fixed and ϕ is a parameter, so that using equations (19) and (20) produces:

$$(21) \qquad \dot{F}_P = \sigma\left[\dot{p}_N - \dot{p}_F\right] = \frac{\sigma \dot{n}}{(1-\alpha)}$$

Equation (21) gives the demand for energy in efficiency units. In order to convert this to the change in energy demand in natural units, we subtract the percentage increase in energy efficiency, so that:

$$(22) \qquad \dot{E}_P = \dot{F}_P - \dot{n} = \frac{(\sigma + \alpha - 1)\dot{n}}{(1-\alpha)}$$

From equation (22) it is clear that the extent of the change in energy use will depend on the value of the elasticity of substitution, σ. If the value of the elasticity of substitution is zero, there is no rebound effect in production: the fall in energy use is equal to the increase in energy efficiency, $\dot{n}$. Where the elasticity of substitution lies within the range: $1-\alpha > \sigma \geq 0$, then there is rebound but not backfire: energy use will fall but by less than the extent of the efficiency improvement. For values of $\sigma \geq 1-\alpha$, energy use does not fall as energy efficiency increases. This is the parameter range over which backfire occurs. Note that the Cobb Douglas function, with an elasticity equal to unity, always lies within this range.

In the literature there is often the implicit assumption that a lower elasticity of substitution in production is desirable, in that this reduces rebound effects. However, it does so only by offering policy makers more restricted options. Figure 6 presents the zero elasticity of substitution production frontier. This is derived from a fixed coefficients production function, where the inputs per unit of output for both the local resource and energy, measured in efficiency units, are fixed. There is no flexibility concerning the resource intensity of production. The input intensities are therefore invariant to changes in input prices. The line $Q_1^{FC}(E_P)$ represents the initial production frontier, where the superscript identifies the production function as having fixed coefficients.

FIGURE 6 SOMEWHERE AROUND HERE

This initial fixed coefficients (zero elasticity) production frontier comprises two linear segments: the straight line from the origin to the point A_1, which is associated with the maximum output, Q^*, and the horizontal line as subsequent increases in energy fail to increase output.[9] Compare the fixed coefficients $Q_1^{FC}(E_P)$ and Cobb-Douglas (unitary elasticity) production function $Q_1^{CD}(E_P)$ that goes through the same optimal point A_1. Also assume that A_1 is initially the profit maximising point in the Cobb-Douglas case. A key observation is that the Cobb-Douglas production function gives a greater range of production options. At no level of energy input does the output from the fixed coefficient, zero elasticity, production function generate a greater output than that from the Cobb-Douglas production function.

If there is an increase in energy efficiency, the fixed coefficients production function shifts to the left, with the maximum output still at Q^*, but with energy inputs reduced by $\dot{n}$. The profit-maximising point has moved to A_2. However, note that where the same efficiency improvement is imposed in the Cobb-Douglas production function the frontier also goes through point A_2. However, in the Cobb-Douglas case, this will not be the allocation chosen in a free market as it is not the new profit maximising point. The point can be made more generally: a conventional well-behaved production function offers possibilities that are technically ruled out with the fixed coefficients production function. However, as we argued in Section 5, with a Cobb-Douglas (or any other well-behaved production function), the government can bring the economic system back to the zero rebound state at A_2 through the appropriate tax policy. Further, if the government wished to reduce energy use below $E^*_{P,2}$, it can do so with a smaller reduction in output and consumption, the higher the elasticity of substitution.

6.3 Supply of other factors

One comment concerning the standard economic approach is that the changes generated by improvements in energy efficiency are small. As noted in Section 2, in a competitive economy, the parameter α is the share of energy costs in total output. From equations (6) and (8), with our standard open economy model, the increase in energy use and output resulting from even a large increase in energy efficiency is modest. If energy costs are 5% of total inputs, a 50% increase in energy efficiency will generate a 2.6% increase in output and energy use.

However in the model at present, all non-energy inputs are provided locally with a supply elasticity of zero. But if non-energy inputs have a positive supply elasticity, the impact of increased energy efficiency can be much greater. For example, in analysing energy use, it is very common to use the KLEM production function, where inputs of capital, labour and materials are identified as K, L and M. For the constant returns to scale Cobb Douglas case, equation (6) is amended to:

(23) $$Q = AF_P^{\alpha} K^{\beta} M^{\gamma}, L^{1-\alpha-\beta-\gamma}$$

Imagine that in this case, the capital and materials inputs, like energy, are supplied in international markets at fixed prices. Setting the marginal products of these inputs equal to their prices generates:

(24) $$\frac{K}{Q} = \frac{\beta}{p_K}, \quad \frac{M}{Q} = \frac{\gamma}{p_M}$$

where p_K and p_M are the prices of capital and materials respectively. Substituting these results into equation (23) and rearranging produces:

(25) $$Q = DE_P^{\frac{\alpha}{1-\beta-\gamma}} n^{\frac{\alpha}{1-\beta-\gamma}}$$

Equation (25) is in a form comparable to equation (6) except that the coefficients on the energy and energy efficiency terms E_P and n are increased.[10] This means that in the standard Cobb-Douglas case discussed in Section 4, an energy improvement of $\dot{n}$ now generates an increase in energy use given by:

(26)
$$\dot{E}_P = \frac{\alpha \dot{n}}{1-\alpha-\beta-\gamma}$$

Using the same numerical example of a 50% improvement in efficiency, if the combined energy, capital and material costs made up 75% of the total costs, the increase in energy use would now be 10%.

A second consideration concerns the supply of the local input. If we again take the KLEM production function with labour as the only non-imported input, the proportionate increase in the price of labour as the price of energy, measured in efficiency units, falls is:

(27)
$$\dot{p}_L = \frac{\alpha \dot{n}}{1-\alpha-\beta-\gamma}$$

With an elasticity of labour supply equal to λ, the increase in employment is given as:

(28)
$$\dot{L} = \frac{\alpha \lambda \dot{n}}{1-\alpha-\beta-\gamma}$$

The change in the energy demand identified in equation (26) is driven by the increase in energy use per unit of the local resource (now labour). Incorporating the increase in labour supply implies summing the expressions in equation (26) and (28) producing:

(29)
$$\dot{E}_P = \frac{\alpha (1+\lambda) \dot{n}}{1-\alpha-\beta-\gamma}$$

From equations (29) and (8), the effect of incorporating the elasticity of supply of non-energy inputs multiplies the proportionate increase in energy use by a factor of $\frac{(1+\lambda)(1-\alpha)}{1-\alpha-\beta-\gamma} > 1$. The supply augmented increase in energy use could be substantially higher than the unadjusted calculation.

7. Consumption

Up to this point we have only considered energy use in production. That is to say, we have analysed the impact of an improvement in energy efficiency in production on subsequent energy use in production. We now incorporate the impact of changes in energy efficiency on the consumption demand for energy. We consider two cases. In the first, the improvement in energy efficiency occurs only in the production sector. In the second, the improvement in energy efficiency occurs only in the consumption sector.

7.1 The impact of improvements in energy efficiency in production on energy use in consumption

In those variants of the small open economy model where all commodity prices remain constant, it is straightforward to analyse the impact on consumption of an increase in energy efficiency in production.[11] Real local income increases as a result of the rise in the return to the local resource. The proportionate change in energy use in consumption, $\dot{E}_C$, is then given as the product of the proportionate change in real income and the income elasticity of demand for energy, ψ.

In the most basic model outlined in Section 4, with a Cobb-Douglas production function and no augmented supply effects, the increase in income is given by equation (20). The proportionate increase in consumption demand for energy, $\dot{E}_C$, is then:

$$(30) \qquad \dot{E}_C = \dot{n}\left[\frac{\alpha}{1-\alpha}\right]\psi = \dot{E}_P\psi$$

This increase in consumption demand does not come from higher energy expenditure in consumption partly or wholly replacing lower expenditure on energy in production. Rather it stems from the expansion in output, and the subsequent increase in the price for the services of the fixed local input, that the increase in productive efficiency creates. This increases local income and energy use for local consumption.

The proportionate rise in total energy use, $\dot{E}_T$, is the weighted sum of the proportionate changes in consumption and production demand. The weights are the initial production and consumption energy use. For an initial output, Q, these initial absolute energy use levels are:

$$(31) \qquad E_P = \frac{\alpha Q}{p_E}, \quad E_C = \frac{(1-\alpha)\mu Q}{p_E}$$

where μ is the share of energy in total consumer expenditure. Using equations (6), (20), (30) and (31):

$$(32) \qquad \dot{E}_T = \dot{E}_P\left[\frac{\alpha+(1-\alpha)\mu\psi}{\alpha+(1-\alpha)\mu}\right] = \frac{\alpha\left[\alpha+(1-\alpha)\mu\psi\right]\dot{n}}{(1-\alpha)(\alpha+(1-\alpha)\mu)}$$

In this case, equation (32) implies that whether the proportionate change in total energy use is greater or less than the proportionate change in energy use in production depends solely on the value of the income elasticity of demand for energy in consumption. Where the income elasticity of demand, ψ, is greater than unity, total energy use grows faster than energy use in production. Also if the absolute change in total energy use in the production sector, ΔE_P, is compared to the corresponding change in use in consumption, ΔE_C, the consumption impacts are relatively large, with the absolute increase in energy use in consumption being the greater if:

(33) $$(1-\alpha)\mu\psi > \alpha$$

This would be feasible with a relatively high share of energy in consumption ($\mu > \alpha$) and/or a relatively high income elasticity of demand for energy ($\psi > 1$). Essentially, in the basic open economy case with the Cobb-Douglas production function, changes in consumption demand for energy substantially reinforce the backfire effect identified in the production demand for energy.

It is also of interest to investigate the effect of incorporating the consumption demand for energy where the elasticity of substitution in production differs from unity, as in Section 6.2. Here equations (20), (22) (30) and (31) generate the following result:

(34) $$\dot{E}_T = \frac{\alpha\dot{n}\left[(\sigma+\alpha-1)+(1-\alpha)\mu\psi\right]}{(1-\alpha)(\alpha+(1-\alpha)\mu)}$$

First, even where the elasticity of substitution in production, σ, is zero, so that there is no rebound in production, with energy use falling in production by $\dot{n}$, there is some rebound as a result of increased consumption. This is represented by the second term in the brackets in equation (34).

Second, the range of values of σ over which the change in aggregate energy use is positive with an increase in energy efficiency in production is given by:

$$\sigma > (1-\alpha)(1-\mu\psi) \tag{35}$$

Therefore, using equations (22) and (35), there are a range of substitution elasticities:

$$1-\alpha > \sigma > (1-\alpha)(1-\mu\psi) \tag{36}$$

where there is no backfire when the impact on energy use in production is considered on its own but where there is backfire once the impact on the demand for energy in consumption is incorporated.

Where the other adjustments to the standard model introduced in Sections 6.1 and 6.3 are considered, these have implications for the extent of the increase in energy use in consumption. First, the more expansionary supply-side implications of relaxing the fixity of non-energy inputs will have a positive impact on consumption demand, as well as energy demand in production. Second, where the price of the commodity falls as output expands, there will be a more complex reaction, especially if the locally produced domestic product is a major part of the local consumption bundle. However, even here, there will be a rise in real income associated with introduction of the improvement in energy efficiency that should stimulate consumer energy demand.

7.2 The impact of energy efficiency improvements in consumption

In this section the assumption is made that the improvement in efficiency occurs solely in the use of energy for consumption purposes. In the standard version of the model, where local resources are fixed and fully employed, there is no feedback running from changes in consumption to changes in production: locally produced commodities that are not sold on the local market are exported at the same, internationally determined price.

In that case an improvement in energy efficiency in consumption corresponds to a reduction in the price of energy measured in efficiency units. The change in consumption can be decomposed into the standard substitution and income effects. The consumption of energy, in efficiency units is expected to rise as the price falls, so that rebound is expected.

(37) $$\dot{F}_C = \tau \dot{n}$$

where τ is the price elasticity of consumption demand, given a positive sign. To convert to the change in electricity use in natural units, subtract the efficiency gain, so that:

(38) $$\dot{E}_C = \tau \dot{n} - \dot{n} = (\tau - 1)\dot{n}$$

If the price elasticity of demand for energy is greater than unity, the proportionate increase in demand is greater than the proportionate reduction in price so that the total expenditure on energy will rise as the price, measured in efficiency units, falls. Where this occurs, backfire takes place for energy use in consumption.

Where the supply of non-energy inputs is not fixed, as discussed in Section 6.3, changes in energy efficiency in consumption will increase the real wage, increase the supply of labour and therefore affect energy use in both the production and consumption spheres. A proportionate rise in energy efficiency in consumption of $\dot{n}$ generates a similar proportionate fall in the price of energy to consumers, measured in efficiency units. This increase in the real wage equals $\dot{n}\mu$, and the corresponding increase in labour supply equals $\dot{n}\mu\lambda$. The impact of the expansion in supply of the local resource is an equal proportionate increase in energy use in both production and consumption. The proportionate change in total energy use, incorporating supply-side impacts, from an increase in energy efficiency in consumption is therefore:

(39) $$\dot{E}_{T,CS} = \dot{n}\mu\left[\lambda + \frac{(1-\alpha)(\tau-1)}{\alpha}\right]$$

The labour-supply effects always add to the demand for energy, even where the direct effect of the energy efficiency improvement in consumption identified in equation (38) is negative (that is where consumer demand for electricity is inelastic). Also the value the resource supply elasticity can be large if migration effects are important.

8. Conclusion

In this chapter we identify the impact of changes in energy efficiency in a stylised small open economy model. The aim is pedagogic. We have four main themes. The first is that an improvement in energy efficiency will have system-wide effects. This means that in order to analyse the impact on energy use we need to model all the key interactions within the economy.

Second, a change in efficiency in the use of energy inputs increases the options open to the economy. The actual outcome will depend upon which of those options is chosen. Therefore in considering the effect of an improvement in energy efficiency, allocative efficiency, as well as technical efficiency, is under scrutiny.

Third, the existence of an important export sector in small open economies means that the conditions facing this sector are crucial in determining the subsequent energy use that follows from an improvement in energy efficiency. The impact of increased efficiency on competitiveness is an important stimulus to the aggregate economy and therefore to energy use. In particular, we show that the elasticity of demand for the export sector's output is as important as the elasticity of substitution in production in the analysis of the impact of improvements in energy efficiency on energy use.

Fourth, whilst analysing an economy in which energy inputs are assumed to be freely available at the existing international price, we identify the implication of varying the elasticities of supply of the other inputs. Any increase in the ease of supply of other inputs generally increases the impact of improved energy efficiency in production and also leads to an interaction between improved efficiency in energy use in consumption and the level of energy used in production.

Captions for diagrams

Figure 1: Output, consumption and energy use levels in the standard open economy model.

Figure 2: Energy productivity.

Figure 3: The impact of an increase in energy efficiency on output, consumption and energy use.

Figure 4: The effect of energy taxation on output and energy use.

Figure 5: The relationship between profit maximising energy use and the elasticity of export demand.

Figure 6: An increase in energy efficiency with the fixed coefficients production function.

Appendix

The production frontier zero elasticity of substitution

The rationale for the linear jointed production frontier where elasticity of substitution is zero is straightforward. If the required input of local resources per unit of output is θ_N then the maximum output, Q^*, is given by:

(A1) $$Q^* = \frac{\bar{N}}{\theta_N}$$

To attain the maximum output, the energy supply must lie in the range:

(A2) $$E_P \geq E_P^* = Q^* \theta_E = \frac{\bar{N}\theta_E}{\theta_N}$$

If total energy inputs are below E_P^*, production is constrained. Any increase in energy, ΔE_P, that relaxes that constraint generates additional output equal to $\frac{\Delta E_P}{\theta_E}$. The slope of the production frontier between the origin and A_1 is therefore $\frac{1}{\theta_E}$. However, once the total energy input reaches E_P^*, further increases in energy inputs have no impact on output as the level of local resources forms the binding constraint on production. Finally, if production is profitable at all, maximum profitability will be attained initially at A_1.

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Figure 1(a)

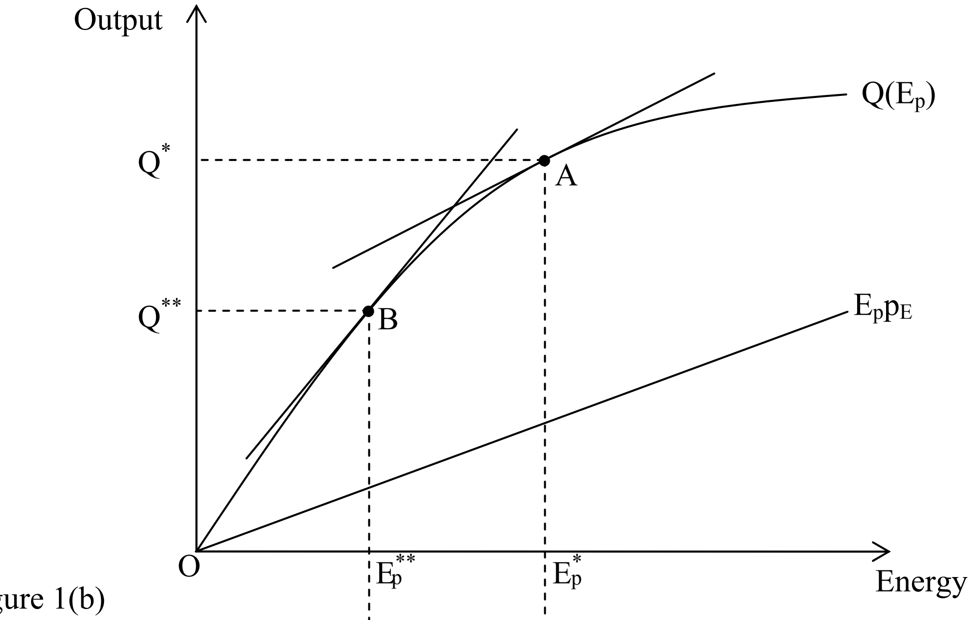


Figure 1(b)

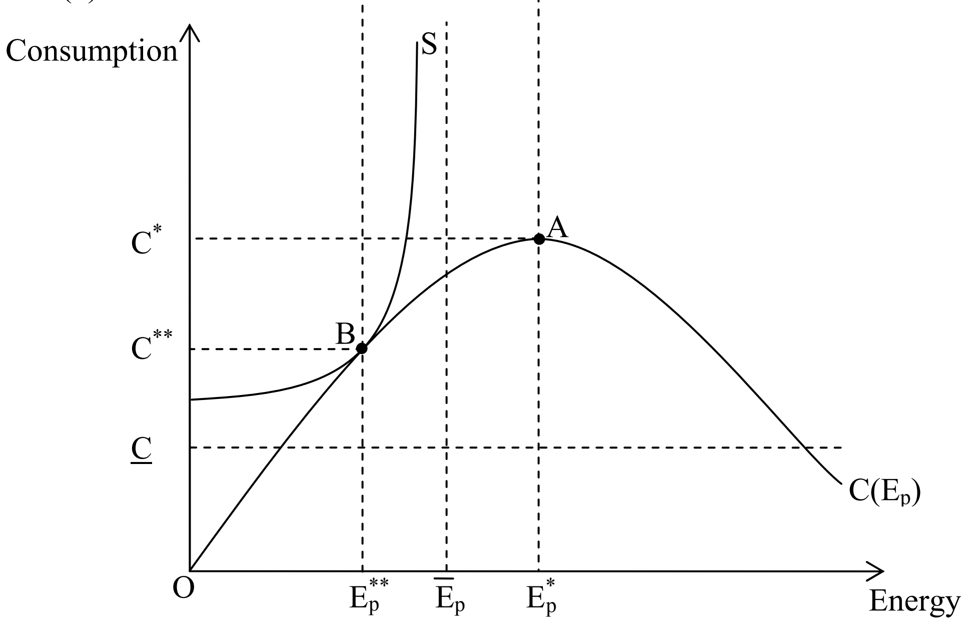


Figure 2

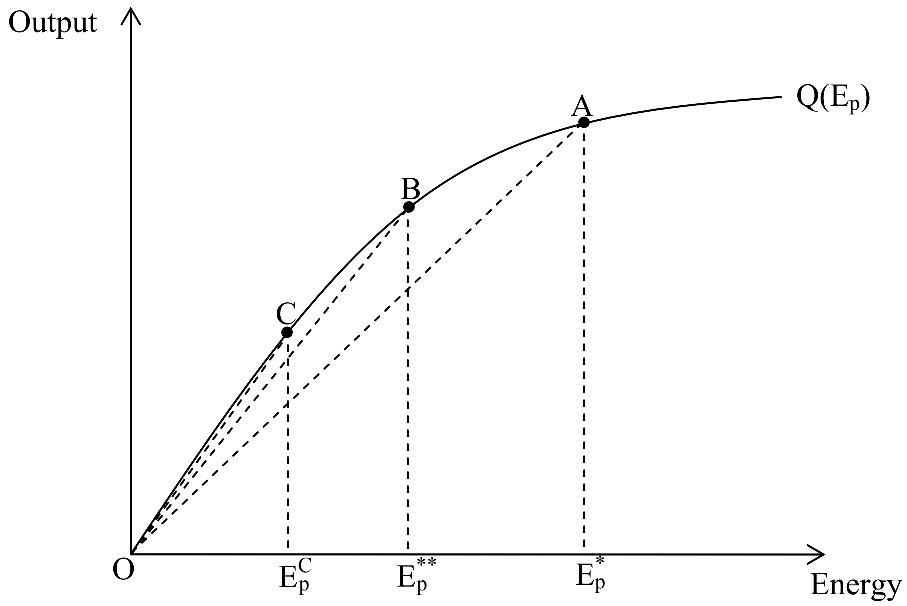


Figure 3(a)

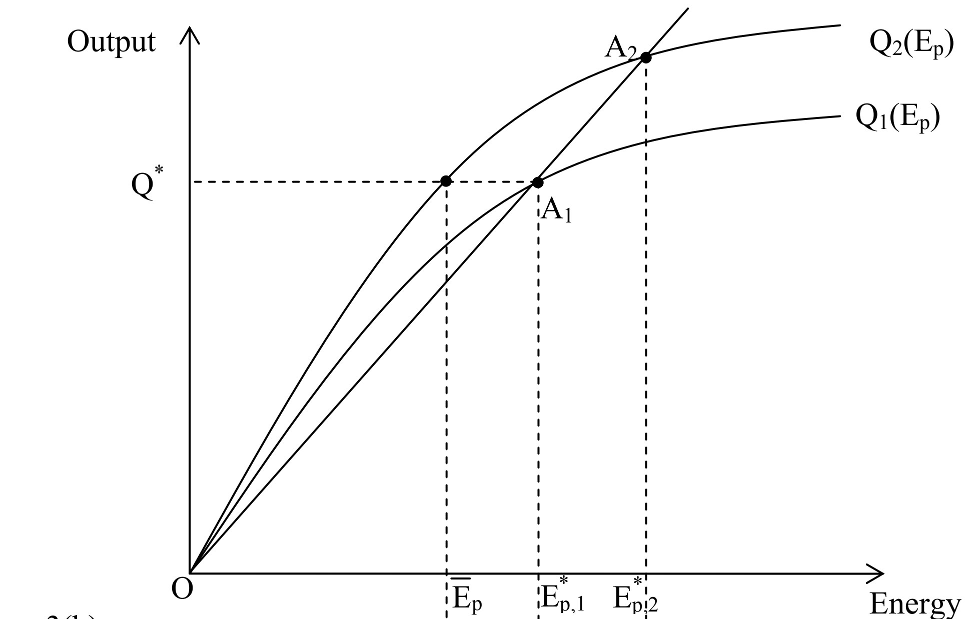


Figure 3(b)

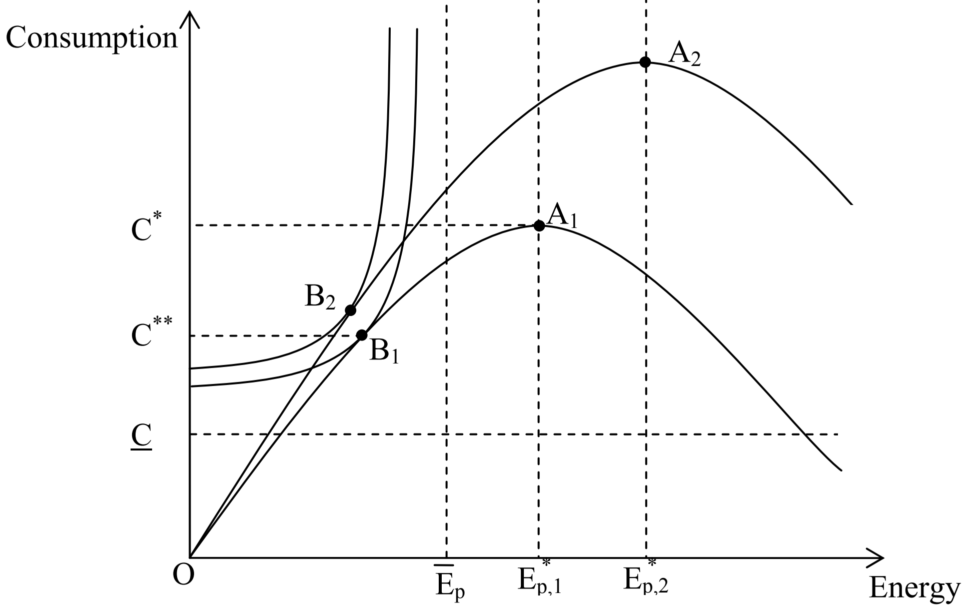
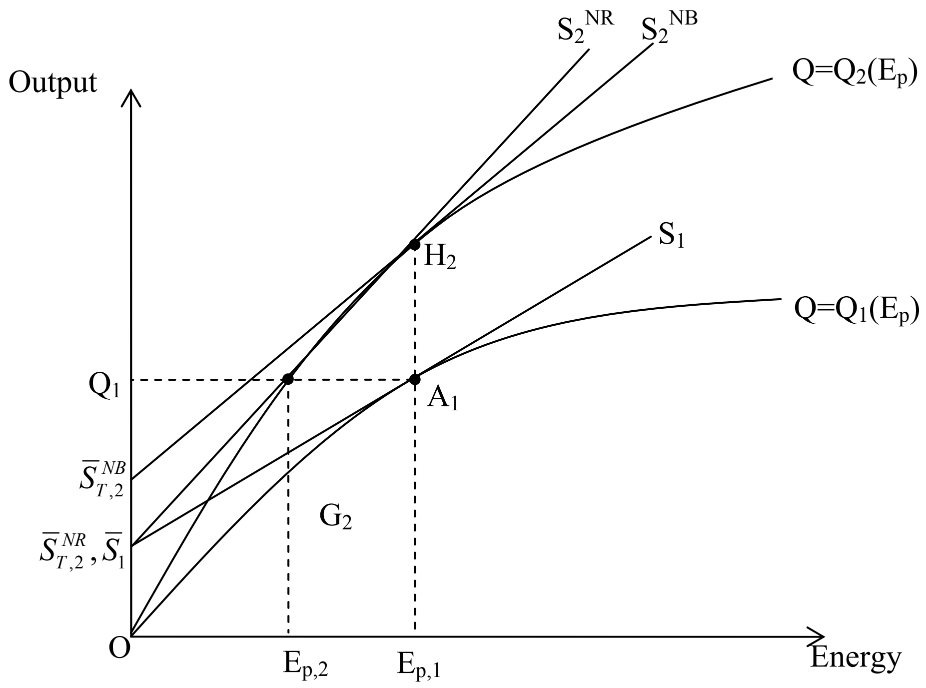


Figure 4



S_2^{NR}
S_2^{NB}
Output
Q=Q2(Ep)
S1
H2
Q=Q1(Ep)
Q1
A1
$\overline{S}_{T,2}^{NB}$
G2
$\overline{S}_{T,2}^{NR}, \overline{S}_1$
O
Ep,2
Ep,1
Energy

Figure 5

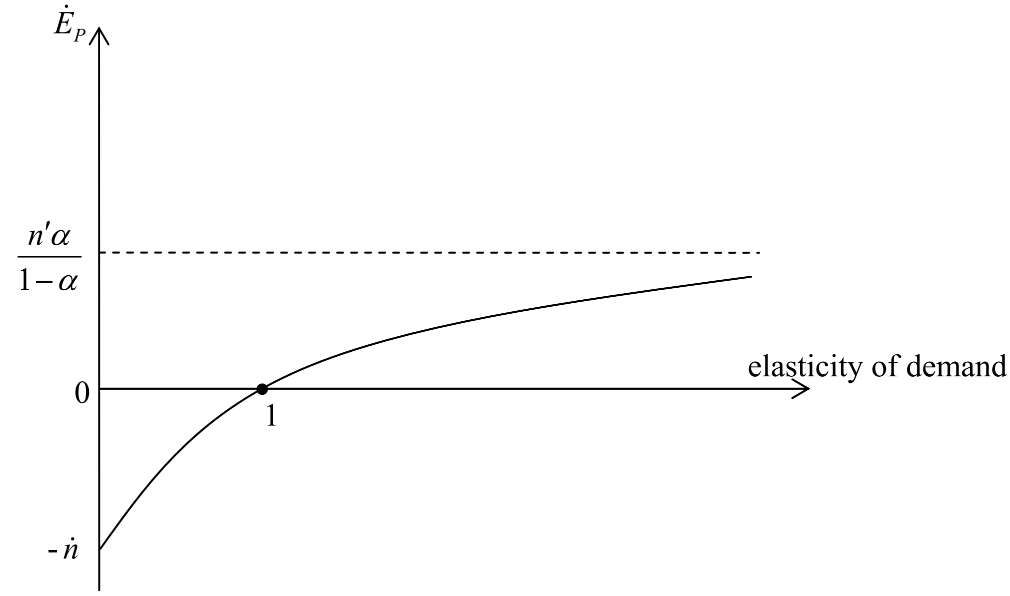
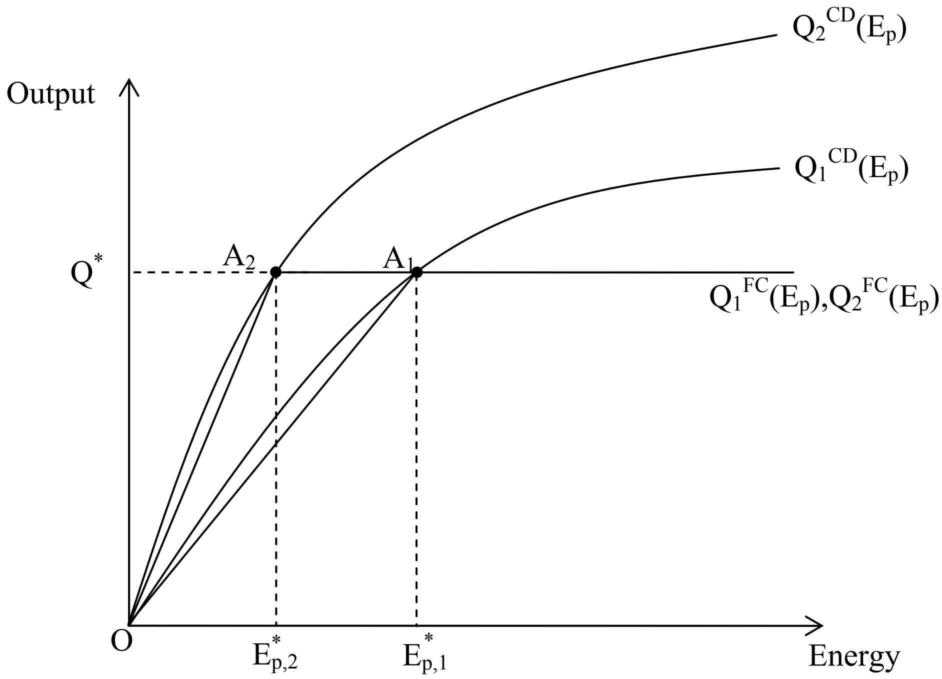


Figure 6



FOOTNOTES

[1] Specifically, there are no issues concerning the distribution of income amongst individuals.

[2] Essentially lower present energy use represents increased welfare for future generations.

[3] Allocative efficiency involves making the best choice of inputs and scale of production among the technically efficient alternatives. To reach overall economic efficiency, the outcome must be both technically and allocatively efficient.

[4] In this case the tax is simply to change the prices facing producers and would be distributed back to consumers.

[5] The Cobb-Douglas production function has the characteristic that the elasticity of substitution between the inputs equals unity. This is discussed in greater detail in Section 6.2.

[6] $B = AN^{1-\alpha}$, so that where the general productivity and the natural resource input is fixed, B is a constant.

[7] Although as drawn in Figure 3b social welfare would be maximised with a small drop in consumption.

[8] Similar goals could be played by physical restrictions, though in this simple model these would be expected to act very much in the same way as price signals.

[9] The form of the production frontier with zero elasticity of substitution is explained in greater detail in the Appendix.

[10] $D = \left[AL^{1-\alpha-\beta-\gamma} \left[\frac{\beta}{p_K}\right]^{\beta} \left[\frac{\gamma}{p_M}\right]^{\lambda} \right]^{\frac{1}{1-\beta-\gamma}}$. Where the domestic labour supply and the prices of capital and materials are fixed in international markets, D is a constant.

[11] These are the variants where the law of one price holds in the export sector.