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HARMFUL COMPETITION IN THE INSURANCE MARKETS

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Harmful competition in the insurance markets

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Abstract

There is a general presumption that competition is a good thing. In this paper we show that competition in the insurance markets can be bad and that adverse selection is in general worse under competition than under monopoly. The reason is that monopoly can exploit its market power to relax incentive constraints by cross-subsidization between different risk types. Cream-skimming behavior, on the contrary, prevents competitive firms from using implicit transfers. In effect monopoly is shown to provide better coverage to those buying insurance but at the cost of limiting participation to insurance. Performing simulation for different distributions of risk, we find that monopoly in general performs (much) better than competition in terms of the realization of the gains from trade across all traders in equilibrium. However, most of the surplus is retained by the firm and, as a result, most individuals prefer competitive markets notwithstanding their performance is generally poorer than monopoly.

Keywords: monopoly, competition, insurance, adverse selection.

JEL Classification: G22, G82, H20

1 Introduction

In this paper we address the critical question: how and how well do competition on the markets handle the fundamental problems of information. With imperfect information, market actions or choices convey information and we know from earlier work (e.g. Rothschild and Stiglitz, 1976) that inefficiency and even existence problems can arise in competitive markets because the slight change in the action of the informed side of the market discretely changes beliefs of the other side of the market. While information asymmetries inevitably arise, the extent to which they do so and their consequences on the realization of the gains from trade depend on the how the market is structured. This raises the fundamental question of the interplay between two forms of market imperfections: imperfect information and imperfect competition. There is no particular reason why competition should be better in the presence of imperfect information. The simplest way by which this would not be true is when the firm could exploit its market power to relax the incentive constraints.

The aim of this paper is to evaluate the efficiency of competition on the insurance market in the presence of adverse selection. Using the benchmark model of Rothschild and Stiglitz (1976), we contrast the competitive equilibrium outcome with the monopoly equilibrium outcome à la Stiglitz (1977) and we compare their relative efficiency. Following Rustichini et al. (1994), the (expected) efficiency of an equilibrium is the fraction whose numerator is the expected gains from trade across all traders in the equilibrium and whose denominator is the gains from trade across all traders with full information. Using this criterion we compare the monopoly outcome with one seller of insurance contracts and many potential buyers with different risks against the competitive outcome imposing zero profit on each contract that might be offered in equilibrium. With a continuum of types the competitive equilibrium à la Rothschild-Stiglitz does not exist, but since then various alternative equilibrium concepts have been proposed, based either on game theoretic refinements or on a Walrasian approach, that prove the existence of competitive equilibria. Even though no general agreement has been reached so far about the equilibrium concept, the general intuition in the classical as well as more recent literature is that competition typically results in the provision of a set of contracts that fully separate types (Chiappori, 2006). In this paper we refer to the concept of reactive equilibrium developed by Riley (1979) and Engers and Fernandez (1987) for which the Pareto-dominant full separating zero-profit outcome is the unique reactive equilibrium.[1]

The key finding is that the monopoly outcome, in general, is more efficient than the competitive outcome (according to our expected efficiency criterion). The reason why monopoly performs better than competition is that the monopolist can *exploit its market power* to offer contracts that better satisfy the incentive constraints. More precisely, the monopolist can offer contracts with implicit transfers across agent types that can relax the incentive constraints and implement a larger set of allocations. This is one of many examples of the interplay between market imperfections (see Jaffe and Stiglitz, 1990; Stiglitz, 1975). The economy, in

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[1]see also Hellwig (1987), Bisin and Gottardi (2006), and Dubey and Geanakoplos (2002) for other equilibrium concepts leading to the same outcome.

effect has to trade off between two different imperfections: imperfections of information or imperfections of competition, with no particular reason that these imperfections will be balanced optimally. As we shall see, competition provides all risk types with an insurance contract, but coverage is only partial. On the contrary, under monopoly low risk types are forced to quit the market, but coverage increases for the participating risk types. Even though monopoly performs better than competition in terms of the realization of the gains from trade, most of the surplus is retained by the firm. As a result, there is always a majoritarian support for competitive markets among individuals, notwithstanding their performance is generally poorer than monopoly.

Our paper continues a line of research begun by Stiglitz (1977), who analyzed monopoly insurance markets, and compared the equilibrium outcome with the (two-type) competitive outcome. Dahlby (1987) studied the same issue in the same simplified two-type framework, but without using the expected efficiency criterion to compare competition and monopoly. His result is that monopoly may provide higher coverage than competition but only when the proportion of low types is much larger than high types, and so less important from the social viewpoint since there are little gains from insurance. We claim that, while the simplified framework with just two types is able to provide general intuitions and qualitative results, it is no longer justified when quantitative comparison is performed, as in the present paper. Furthermore, it is fairly intuitive that this simplification skews the results in favour of competition: by reducing the number of types screening becomes less costly because fewer risk types have to be separated, while fewer cross-subsidizing opportunities are available to monopoly. As a result, using the more realistic framework of a continuum of types, monopoly performs better than competition for almost any possible distribution of risk types, except for the extreme cases in which the mode of the distribution is on the highest risk.[2]

We perform this analysis in a non-expected utility framework using the *dual theory approach* to choice under risk developed by Yaari (1987). It turns out that by using this specification of individual preferences we are able to provide a clear-cut comparison between monopoly and competition. The dual theory has the property that utility is linear in income, and risk aversion is expressed entirely by a transformation of probabilities in which bad outcomes are given relatively higher weights and good outcomes are given relatively lower weights. In our simple two-state model the probability of bad outcome is weighted up by a loading factor. It would be absurd to suggest that the dual theory provides a better model than the expected utility. The latter has obvious appeal and has provided so many useful results in insurance theory. Nonetheless, we feel there is some gain from studying the properties of our simple non-expected utility model, even if only to derive some clear insights on the efficiency of competition in the presence of adverse selection.[3]

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[2]For an illustration of the effect of the number of risk types on the outcome of the comparison, see De Feo and Hindriks (2005) in which we analyze both the two-type and the continuum-of-risk-type case.

[3]Indeed another distinctive property of insurance under dual theory is that the demand of insurance cannot decrease with wealth. In contrast the expected utility model makes the comparison between competition and monopoly difficult since by charging a higher premium (relative to competition) for a given coverage the

The paper is organized as follows. In Section 2 we present the model. Section 3 contains the monopolist and competitive equilibria with a continuum of types. In Section 4 simulation results are provided about efficiency of competitive and monopoly markets. Section 5 concludes.

2   The model

There are two possible states of the world: the "no accident" state and the "accident" state. Individuals differ only by their probability of accident, in which case they face a (fixed) damage $d = 1$. There is no moral hazard since individuals cannot affect their probability of accident which is fixed. There is a continuum of risk in the economy distributed according to a cumulative probability function $F(\theta)$ with density $f(\theta) > 0$ on a closed and compact interval $\theta \in [\underline{\theta}, \overline{\theta}]$ (with $0 \leq \underline{\theta} < \overline{\theta} < 1$).

Adverse selection is introduced by assuming that individual risk is private information, while the distribution of risks is common knowledge. We model individuals' risk preferences using Yaari (1987)'s dual theory (DT). We first give a general description of this approach before applying it to our model. Let wealth X be a random variable distributed over $[\underline{x}, \overline{x}]$ according to the distribution function $\Psi(x)$. Yaari's representation of preferences is dual to the expected utility theory (EU) in the sense that it is linear in wealth but non linear in probabilities. Probabilities are transformed by a function Φ defined on the distribution function $\Psi(x)$.[4] More precisely, DT preferences over X are given by

$$V(X) = \int x \ \Phi'(\Psi(x)) \ d\Psi(x)$$

where $\Phi(0) = 0$, $\Phi(1) = 1$ and $\Phi'(.) > 0$. $\Phi'(.)$ are non-negative weights adding up to one. Attitude towards risk is conveyed entirely by the shape of $\Phi(.)$. Risk aversion is characterized by the concavity of $\Phi(.)$, i.e. $\Phi''(.) < 0$. In this case, bad outcomes (with low $\Psi(X)$) receive higher weights than good outcomes (with high $\Psi(X)$). In other words, $V(X)$ is the certainty-equivalent of X computed as a weighted average of outcomes in which bad outcomes are given high weight while good outcomes are given low weight. Since $V(X)$ is linear in wealth, this approach separates attitude towards risk from attitude towards wealth.

We now apply DT to our simple two-state setting. For an individual with wealth w facing a damage $d = 1$ with probability θ, an insurance contract $\{\delta, \pi\}$ with coverage rate $\delta \in [0, 1]$ and premium $\pi > 0$ yields the random variable $X = (w - \pi - (1 - \delta), \theta; w - \pi, 1 - \theta)$. We thus define the utility associated to this insurance contract as

$$
\begin{aligned}
V(\theta; \delta, \pi) &= \phi(\theta)(w - \pi - (1 - \delta)) + (1 - \phi(\theta))(w - \pi) \\
&= w - \pi - \phi(\theta)(1 - \delta)
\end{aligned}
$$

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monopoly increases the marginal willingness to pay for insurance.

[4]Alternatively this probability transformation function could be defined on the decumulative distribution function $1 - \Psi$ such as in Yaari (1987).

where risk aversion is represented by $\phi(\theta) > \theta$ (and $1 - \phi(\theta) < 1 - \theta$).[5] In this paper, we further assume that $\phi(\theta) = (1 + \alpha)\theta$, with $0 \leq \alpha \leq \frac{1-\bar{\theta}}{\bar{\theta}}$ (the upper bound guaranteeing that $\phi(\theta) \leq 1 \; \forall \theta$). Making α independent of w accords with our desire to disentangle risk aversion from income and will greatly simplify the analysis. Using this formulation, type-θ utility function from insurance contract $\{\delta, \pi\}$ is

$$V(\theta; \delta, \pi) = \omega - \pi - (1 + \alpha)(1 - \delta)\theta$$

where the utility loss from the residual risk $(1 - \delta)\theta$ is inflated by the markup factor $1 + \alpha$. Now, comparing the utility with insurance against the utility without insurance we can define the reservation premium for each type. This is the premium $\tilde{\pi}(\theta)$ that solves

$$
\begin{aligned}
V(\theta; \delta, \pi) &= V(\theta; 0, 0) \\
\omega - \pi - (1 + \alpha)(1 - \delta)\theta &= \omega - (1 + \alpha)\theta
\end{aligned}
$$

so that the reservation premium of type θ for coverage δ is:

$$\tilde{\pi}(\theta; \delta) = (1 + \alpha)\delta\theta$$

Moreover the surplus of the agent is defined as the difference between the reservation price and the price effectively paid:

$$
\begin{aligned}
S(\theta; \delta, \pi) &= \tilde{\pi}(\delta; \theta) - \pi \\
&= (1 + \alpha)\delta\theta - \pi
\end{aligned}
$$

Assuming $\pi > 0$, with free participation, those agents for which $\tilde{\pi}(\theta; \delta) < \pi$ will drop out of the market.

It is straightforward to see that the functions V and S have the *Single-Crossing property* in the contract space-(δ, π), because the marginal value of coverage is increasing in θ.

3 Monopoly versus competition equilibrium outcomes

In this section we study the equilibrium outcomes of monopoly and competition in an insurance market with a continuum of risks.

The optimization problem of the monopolist is:

$$\max_{\pi(\theta), \delta(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\pi(\theta) - \delta(\theta)\theta\right] dF(\theta)$$

---

[5]Note that in our model with a *discrete* random variable, risk aversion translates into the transformation of the discrete density function $\phi(\theta) > \theta$ rather than the concave transformation of the distribution function $\Phi''(\Psi) < 0$ as for *continuous* random variable. In both cases risk aversion implies that bad outcomes are given higher weight and good outcomes lower weight.

subject to

$$V\left(\theta;\delta\left(\theta\right),\pi\left(\theta\right)\right) \geq V\left(\theta;0,0\right) \qquad \forall\theta\in[\underline{\theta},\overline{\theta}] \tag{1}$$

$$V\left(\theta;\delta\left(\theta\right),\pi\left(\theta\right)\right) \geq V\left(\theta;\delta\left(\widehat{\theta}\right),\pi\left(\widehat{\theta}\right)\right) \qquad \forall\theta,\widehat{\theta}\in[\underline{\theta},\overline{\theta}] \tag{2}$$

where (1) is the set of participation constraints and (2) denotes the set of incentive constraints. Analyzing the set (1) we can see that

$$V\left(\underline{\theta};\delta\left(\underline{\theta}\right),\pi\left(\underline{\theta}\right)\right) \geq V\left(\underline{\theta};0,0\right)$$

must be binding, for otherwise it would be possible to increase $\pi(\theta)$ $\forall\theta > \underline{\theta}$. This is the classical monopoly result of full rent extraction at the bottom.

In the following Proposition the monopolist outcome is summarized.

Proposition 1 *In a monopoly insurance market with a continuum of risk, there exists*

$$\theta^* = \frac{1+\alpha}{\alpha h\left(\theta^*\right)}$$

with $h(.)$ the non-decreasing hazard rate function, such that the equilibrium contracts are

$$\{\delta^m\left(\theta\right),\pi^m\left(\theta\right)\} = \{0,0\} \qquad \forall\theta\in[\underline{\theta},\theta^*]$$

$$\{\delta^m\left(\theta\right),\pi^m\left(\theta\right)\} = \{1,(1+\alpha)\,\theta^*\} \qquad \forall\theta\in\left[\theta^*,\overline{\theta}\right]$$

Proof. See Appendix. ∎

Therefore the solution is characterized by a (pooling) contract that offers full coverage to all $\theta \geq \theta^*$ with a premium extracting the entire surplus from type θ^* and no insurance to all $\theta < \theta^*$. The equilibrium payoff of type θ under monopoly is:

$$V\left(\theta;\delta^m\left(\theta\right),\pi^m\left(\theta\right)\right) = \omega-(1+\alpha)\,\theta \qquad \forall\theta\in[\underline{\theta},\theta^*)$$

$$V\left(\theta;\delta^m\left(\theta\right),\pi^m\left(\theta\right)\right) = \omega-(1+\alpha)\,\theta^* \qquad \forall\theta\in\left[\theta^*,\overline{\theta}\right] \tag{3}$$

Monopolist (per capita) profit is

$$\Pi^m = (1+\alpha)\,\theta^*\left[1-F\left(\theta^*\right)\right] - \int_{\theta^*}^{\overline{\theta}}\theta dF\left(\theta\right) \tag{4}$$

Rewriting $h\left(\theta^*\right) = \frac{f(\theta^*)}{1-F(\theta^*)}$ the pivotal type solves

$$\alpha\theta^* f\left(\theta^*\right) = (1+\alpha)\left(1-F\left(\theta^*\right)\right)$$

where the LHS is the revenue loss of an increase in θ^* due to the non-participation of pivotal type and the RHS is the revenue gain from charging a higher price on all agents above the pivotal type θ^*.

The pooling contract performs cross-subsidization among types. In fact, while θ^*-type individuals are extracted the whole surplus, higher types are left with some rent and possibly pay a premium lower than the fair price.

Shifting to the analysis of competition, it is well known that with a continuum of types a competitive equilibrium may fail to exist. In fact Riley (2001) showed the general non-existence of the Rothschild-Stiglitz equilibrium. This existence problem can be circumvented by resorting to the *reactive equilibrium* concept introduced by Riley (1979) and developed further by Engers and Fernandez (1987). A reactive equilibrium is a set of offers such that there is no profitable deviation by any firm given that other firms can optimally react to this deviation by offering new contracts. Engers and Fernandez (1987) provide general conditions, for which the Pareto-dominant full-separating zero-profit set of contracts is the unique reactive equilibrium outcome. It turns out that those conditions hold true in our framework.[6] The key element is that firms are deterred to deviate from the full separating equilibrium by the belief that other firms will react to "skim the cream" and make such initial deviation unprofitable.

The Pareto-dominant fully separating zero-profit competitive equilibrium solves

$$\max_{\substack{\pi(.) \geq 0 \\ \delta(.) \in [0,1]}} V\left(\theta; \delta\left(\theta\right), \pi\left(\theta\right)\right) \qquad \forall \theta = \left[\underline{\theta}, \overline{\theta}\right]$$

subject to (1), (2) and the additional zero profit constraint:

$$\pi\left(\theta\right) - \delta\left(\theta\right)\theta = 0 \qquad\qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right] \qquad (5)$$

The following Proposition summarizes the result due to Hindriks and De Donder (2003).

Proposition 2 *In a competitive insurance market with a continuum of risk, the reactive equilibrium is characterized by the following set of contracts:*

$$\{\delta^c\left(\theta\right), \pi^c\left(\theta\right)\} = \left\{ \left(\theta/\overline{\theta}\right)^{\frac{1}{\alpha}}, \theta\left(\theta/\overline{\theta}\right)^{\frac{1}{\alpha}} \right\} \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]$$

Proof. See Appendix. ∎

The equilibrium payoff of type θ under competition is then:

$$V\left(\theta; \delta^c\left(\theta\right), \pi^c\left(\theta\right)\right) = \omega - (1 + \alpha)\theta + \alpha\theta\left(\theta/\overline{\theta}\right)^{\frac{1}{\alpha}} \qquad \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right] \qquad (6)$$

So, while in the monopoly every type $\theta \in \left[\theta^*, \overline{\theta}\right]$ gets full insurance, with competition only the highest-risk individuals obtain full coverage and all the other individuals with lower risk obtain partial coverage. On the other hand, every $\theta \in [\underline{\theta}, \theta^*)$ gets no insurance with monopoly, while they are provided at least with partial coverage in the competitive case. Figure 1 compares equilibrium coverage with monopoly and competition for a given distribution of risks.

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[6]The conditions for existence and uniqueness of a reactive equilibrium in our model are: (1) a continuous probability distribution $F(\theta)$; (2) the profit function of insurance firms is continuous, bounded and non increasing in θ and δ; (3) $V(\theta; \delta, \pi)$ is continuous on $\Theta \times \Delta \times \Pi$ where $\Delta = [0, 1]$ and $\Pi = [\underline{\pi}, \bar{\pi}]$ with $\underline{\pi} = \inf\{\tilde{\pi}(\theta; \delta) : \theta \in \Theta, \delta \in \Delta\}$ and $\bar{\pi} = \sup\{\tilde{\pi}(\theta; \delta) : \theta \in \Theta, \delta \in \Delta\}$, is strictly decreasing in π and satisfies the Single-Crossing property; (4) the contract space is a closed set $\Delta \times \Pi$.
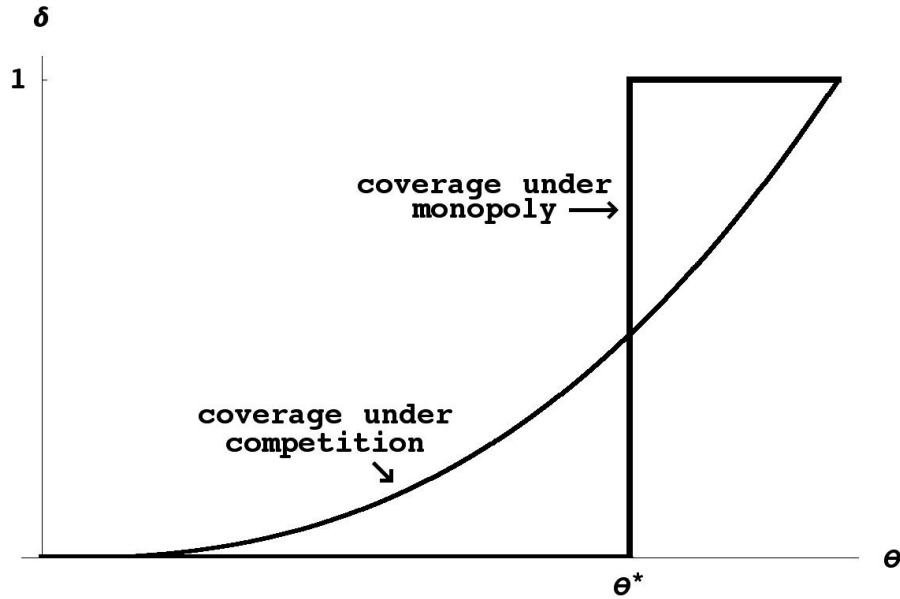
Figure 1: Relative coverage rates under competition and monopoly.

The crucial feature of competitive equilibrium is that the set of implementable contracts is smaller than under monopoly. Since each contract must break even – by the constraint (5) – no cross-subsidization can be performed among types. As a consequence, the distribution of risks in the population does not influence the equilibrium outcome. In fact, there is a unique solution to the problem of maximizing the utility of each type given that every contract must break even and must be incentive compatible. In Figure 2 the effect of a change in distribution is depicted: the left panel shows a positively skewed distribution, while the right panel shows a negatively skewed distribution. While the coverage function under competition is unchanged, the level of θ^* in the left panel is lower than in the right panel. This means that when the distribution of risks is positively skewed, the monopolist charges a lower premium then when it is negatively skewed. The gain for the monopolist of including more types in the pooling exceeds the profit loss due to the reduction in premium.
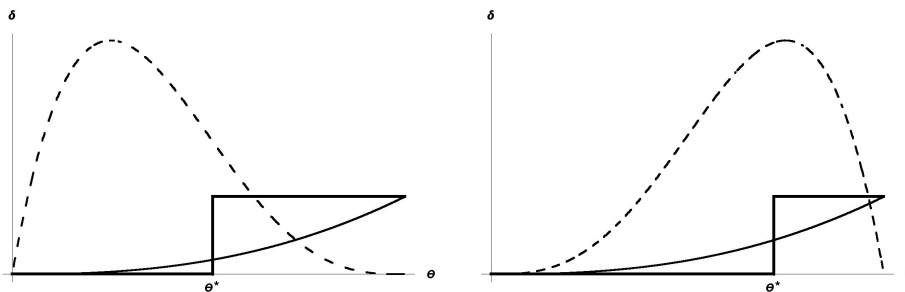

Figure 2: The effect of different distributions on coverage and participation under competition and monopoly.

	b				
a	1	2	3	5	10
1	20.00	11.11	7.87	5.29	3.46
2	33.33	22.09	17.05	12.41	8.59
3	42.86	31.35	25.54	19.64	14.22
5	55.56	45.19	39.20	32.32	24.98
10	71.43	64.24	59.49	53.20	44.98

Table 1: Participation rate under monopoly for various distributions over risk Beta(a,b).

There is a clear trade-off between participation and coverage in the insurance markets under monopoly and competition. Monopoly, in fact, ensures full coverage to the individuals that buy insurance contracts, but this is obtained at the expenses of low risk types that do not participate. Under competition, on the other hand, each individual is provided with an insurance contract, but coverage is only partial. As a consequence, there is no clear analytical result when we compare monopoly and competition. We need to perform numerical simulations on the distribution of individuals and the next Section provides the results.

4 Relative efficiency

In this section we perform some numerical simulations using Beta distribution of risks with non-decreasing hazard rate.
We have seen that the risk distribution affects the monopoly outcome by changing the critical level θ^*, while it does not affect the competition equilibrium outcome.

The effect of changing the distribution on the equilibrium monopoly participation rate is illustrated in Table 1.[7] In this Table a Beta distribution (a, b) is used to show that the participation rate increases with the concentration of the distribution (i.e., simultaneous increase in a and b). Moreover, participation is higher with negatively skewed distributions (i.e., $a - b > 0$). The intuition of the latter result is simple. Consider a small decrease in the skewness of the distribution such that the effect on $f(\theta^*)$ (a second order effect) is negligible. The first order effect of this change is an increase in $1 - F(\theta^*)$. Then, at θ^* the monopoly is no longer in equilibrium since

$$\theta^* h(\theta^*) < \frac{1 + \alpha}{\alpha}$$

and the firm can increase its profits by increasing the premium charged and, as a consequence, the risk level of the critical type. At the new equilibrium

$$\theta^{**} h(\theta^{**}) = \frac{1 + \alpha}{\alpha}$$

where $\theta^{**} > \theta^*$, that implies $h(\theta^{**}) < h(\theta^*)$. Given that the effect on $f(\theta^*)$ is negligible, it follows that the participation rate has increased; i.e., $[1 - F(\theta^{**})] > [1 - F(\theta^*)]$.

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[7]Table 1 and the following tables are built assuming $\alpha = 1/3$, $\underline{\theta} = 0$ and $\bar{\theta} = 0.7$.

Following Rustichini et al. (1994) we measure efficiency in terms of total surplus generated in the market as a fraction of the first best (full information) surplus.

Table 2 shows the total surplus realized under competition and monopoly as a percentage of the total surplus under full information. Fixing the degree of risk aversion and the spread of risks we can compare competition and monopoly for different Beta distributions. The key result is that except for the uniform distribution ($a = b = 1$) and distributions for which the highest risk is the mode ($b = 1$), monopoly performs better than competition.

| | b | | | | | | | | | |
| | 1 | | 2 | | 3 | | 5 | | 10 | |
a	mon	comp	mon	comp	mon	comp	mon	comp	mon	comp
1	36.00	40.00	25.92	20.00	21.37	11.43	17.05	4.76	13.33	1.10
2	45.57	50.00	35.51	28.57	30.43	17.86	25.20	8.33	20.23	2.20
3	52.58	57.14	43.06	35.71	37.90	23.81	32.22	12.12	26.44	3.57
5	62.21	66.67	54.01	46.67	49.15	33.94	43.34	19.58	36.73	6.86
10	74.79	78.57	69.08	62.86	65.33	51.07	60.34	35.05	53.71	16.15

Table 2: Surplus under monopoly and competition as a percentage of the First Best surplus for various distributions over risk Beta(a,b).

The difference in performance increases with the concentration and with the skewness (i.e., as $b - a$ increases). The level of the parameters may marginally affect the ranking and only in favor of monopoly. An increase in the risk aversion increases the surplus both under competition and monopoly. Under competition the coverage function $\delta^c(\theta) = (\theta/\overline{\theta})^{1/\alpha}$ is increasing in α; the intuition is that separation becomes easier and a lower difference in coverage suffices.

Under monopoly, in the equilibrium condition

$$\theta^* h(\theta^*) = \frac{1+\alpha}{\alpha}$$

the RHS is decreasing in α and so the equilibrium entails a lower θ^* and a higher participation rate. The intuition is that the willingness to pay increases with the risk aversion and the increase in profit of including lower-risk types in the pool is larger than the loss of reducing slightly the premium.

It is possible to show with simulations that the ranking between monopoly and competition is not affected by any value of $\alpha \in (0,1)$.[8]

On the contrary, it is possible to show that a change in the spread of risks $[\underline{\theta}, \overline{\theta}]$ may alter the ranking in favor of monopoly. Consider, for example, the uniform distribution case. In this case $h(\theta) = \frac{1}{\overline{\theta} - \underline{\theta}}$ and the equilibrium condition becomes:

$$\frac{\theta^*}{\overline{\theta} - \theta^*} = \frac{1+\alpha}{\alpha}$$

---

[8]It is worth recalling that $\alpha > 1$ means that $\phi(\theta) > 2\theta$.

So, if $\underline{\theta} \geq \frac{1+\alpha}{1+2\alpha}\overline{\theta}$, then $\theta^* \equiv \underline{\theta}$ and all the individuals are provided with full insurance under monopoly – the first best solution; this result is not possible under competition.

4.1 Distribution and political support

It is worth emphasizing that both monopoly and competition do not implement neither constrained Pareto-optimal nor second-best allocations in terms of total surplus. On the one hand, implementable allocations under competition are reduced by the additional set of zero-profit constraints; on the other hand, monopolist firm maximizes profits rather than consumers or total surplus. A Pareto-dominant allocation with respect to the monopolist outcome is simply the allocation that increases the pool of types buying full insurance up to the point in which the premium paid is equal to the average cost; the allocation obviously increase total surplus, too. A Pareto-dominant allocation with respect to the competitive outcome is the one that pools the closest-to-the-highest risk types with the highest risk type in a full coverage contract up to the point in which the lowest risk in the pool is indifferent between the new pooling contract and the old competitive equilibrium contract intended for him, and leaves the same competitive equilibrium contracts to all the other (lower) risk types.[9] This new allocation also increases the total surplus generated on the market.

Surplus obtained by individuals under monopoly and competition can be computed from equations (3) and (6):

$$
\begin{aligned}
S\left(\theta ; \delta^m\left(\theta\right), \pi^m\left(\theta\right)\right) &= 0 & \forall \theta \in [\underline{\theta}, \theta^*] \\
&= (1+\alpha)\left(\theta - \theta^*\right) & \forall \theta \in \left(\theta^*, \overline{\theta}\right] \\
S\left(\theta ; \delta^c\left(\theta\right), \pi^c\left(\theta\right)\right) &= \alpha\theta\left(\theta/\overline{\theta}\right)^{\frac{1}{\alpha}} & \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]
\end{aligned}
$$

In Figure 3 the benefit from insurance for each type under monopoly and competition is depicted. It is easy to see that most types are better off under competition than monopoly.

In fact, even though monopoly is in general more efficient in terms of total surplus, most of the gains from trade are retained by the firm. In Table 3 we show that, whatever is the distribution, most individuals prefer competition to monopoly. So, there is always a majoritarian support for competitive insurance markets even when it is less efficient.

5 Conclusions

Using the benchmark model of Rothschild and Stiglitz (1976), we contrast the competitive equilibrium outcome with the monopoly equilibrium outcome à la Stiglitz (1977) and we compare their relative efficiency. The main change is that we adopt the dual theory of risk so that the comparison comes out neatly when dealing with a continuum of types. The dual theory has the property that utility is linear in income, and risk aversion is expressed entirely

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[9]This allocation is always feasible; Riley (2001) shows how reducing the premium of the full coverage contract (and so attracting more risk types) is always profitable for any continuous distribution.
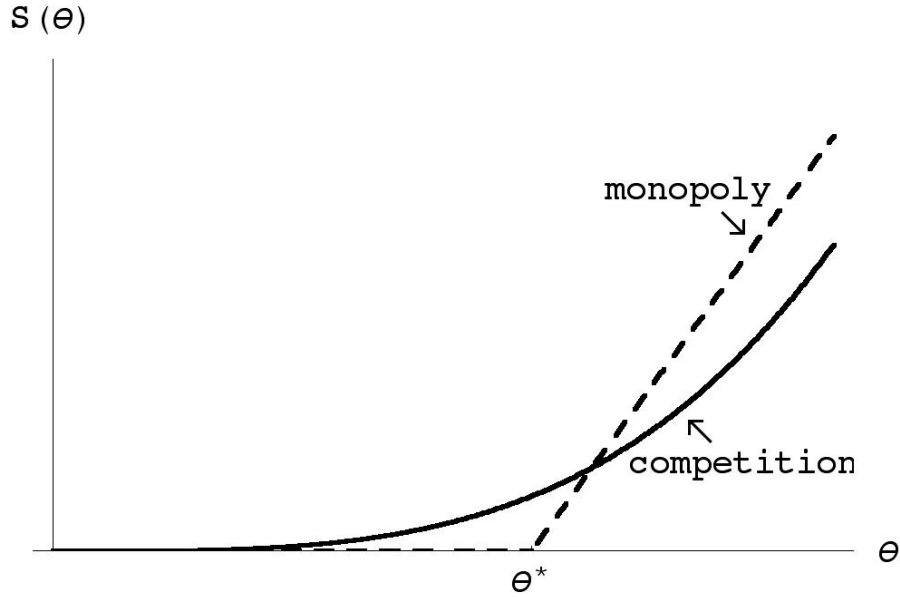
Figure 3: Benefit from insurance of the types under competition and monopoly.

	b				
a	1	2	3	5	10
1	0	6.62	6.17	4.80	3.38
2	0	10.69	12.35	10.87	8.30
3	0	10.84	16.87	16.56	13.56
5	0	0	20.05	24.98	23.14
10	0	0	0	29.19	37.94

Table 3: Percentage of the population that prefers monopoly to competition for various distributions over risk Beta(a,b).

by a transformation of probabilities in which bad outcomes are given relatively higher weights and good outcomes are given relatively lower weights.

Our main finding is that competition is bad and that the monopoly outcome in general is more efficient than the competitive outcome (according to our expected efficiency criterion defined as the fraction of the total surplus that is realized by the market). The reason why monopoly performs better than competition is that the monopolists can exploit its market power to relax the incentive constraints. This is one of many examples of the interplay between market imperfections. The economy, in effect has to trade off between two different imperfections: imperfections of information or imperfections of competition, with no particular reason that these imperfections will be balanced optimally.

Even though monopoly performs better than competition in terms of the realization of the gains from trade, most of the surplus is retained by the firm. As a result, most individuals always prefer competitive markets notwithstanding their performance is generally poorer than

11

monopoly.

We expect our result about the inefficiency of competition in insurance markets with adverse selection to carry over on other markets with adverse selection like the capital market or the job market.

There is a final remark about the use of the dual theory of risk. With this specification there is no income effect on the demand of insurance. In contrast, the expected utility approach will raise the demand for insurance in the monopoly market relative to the competitive market if the absolute risk aversion is decreasing. This is because monopoly price is higher than competitive price which reduces income and thus raises the marginal willingness to pay for insurance. It is then expected that moving to the expect utility will further increase the amount of insurance in the monopoly market relative to the competitive market, thereby reinforcing our conclusion about the inefficiency of competition.

6   Appendix

Proof of Proposition 1.   Because the unique binding IR constraint is for the lowest type:

$$V\left(\underline{\theta}; \delta\left(\underline{\theta}\right), \pi\left(\underline{\theta}\right)\right) = V\left(\underline{\theta}; 0, 0\right)$$

The set of incentive constraints implies that

$$\theta \in \arg\max_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta})) \qquad \forall\, \theta, \hat{\theta} \in [\underline{\theta}, \overline{\theta}]$$

the first order condition for the type θ is:

$$
\begin{aligned}
\frac{\partial V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta}))}{\partial \hat{\theta}} &= \frac{\partial}{\partial \hat{\theta}}[\omega - (1+\alpha)(1 - \delta(\hat{\theta}))\theta - \pi(\hat{\theta})] \\
&= (1+\alpha)\delta'(\hat{\theta})\theta - \pi'(\hat{\theta}) = 0
\end{aligned}
$$

which evaluated at $\hat{\theta} = \theta$ gives the local incentive compatibility conditions (LIC)

$$(1+\alpha)\delta'(\theta)\theta - \pi'(\theta) = 0 \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}] \tag{7}$$

Moreover, the necessary LIC is also sufficient condition when the utility function satisfies the increasing differences property,

$$\frac{\partial^2 V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta}))}{\partial \hat{\theta} \partial \theta} = (1+\alpha)\delta'(\hat{\theta}) \geq 0$$

which requires the coverage to be monotonically increasing $\delta'(\theta) \geq 0$.

Define the value function of the maximization problem evaluated at the truth-telling equilibrium:

$$U(\theta) = V(\theta; \delta(\theta), \pi(\theta)) = \omega - (1+\alpha)(1 - \delta(\theta))\theta - \pi(\theta)$$

differentiating with respect to θ

$$
\begin{aligned}
U'(\theta) &= -(1+\alpha)(1-\delta(\theta)) + (1+\alpha)\delta'(\theta)\theta - \pi'(\theta) \\
&= -(1+\alpha)(1-\delta(\theta)) < 0
\end{aligned}
\tag{8}
$$

where the second equality follows from (7).

Using these results we can rewrite the maximization programme of the monopolist as follows:

$$
\max_{\pi(\theta),\delta(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [\pi(\theta) - \delta(\theta)\theta]dF(\theta)
$$

subject to

$$
\begin{aligned}
\delta'(\theta) &\geq 0 \\
(1+\alpha)\delta'(\theta)\theta - \pi'(\theta) &= 0 \\
V(\underline{\theta};\delta(\underline{\theta});\pi(\underline{\theta})) &\geq V(\underline{\theta};0,0)
\end{aligned}
$$

Ignoring for the moment the monotonicity constraint that will be checked later, we can rewrite the objective function after substituting the constraints in it. From the definition of the value function

$$
\pi(\theta) = \omega - (1+\alpha)(1-\delta(\theta))\theta - U(\theta)
\tag{9}
$$

and by equation 8

$$
U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} -(1+\alpha)(1-\delta(s))ds
$$

By using the binding constraint for the low type,

$$
\begin{aligned}
&= \omega - (1+\alpha)\underline{\theta} - (1+\alpha)(\theta - \underline{\theta}) + \int_{\underline{\theta}}^{\theta}(1+\alpha)\delta(s)ds \\
&= \omega - (1+\alpha)\theta + \int_{\underline{\theta}}^{\theta}(1+\alpha)\delta(s)ds
\end{aligned}
\tag{10}
$$

Substituting this expression into (9):

$$
\begin{aligned}
\pi(\theta) &= \omega - (1+\alpha)(1-\delta(\theta))\theta - \omega + (1+\alpha)\theta - \int_{\underline{\theta}}^{\theta}(1+\alpha)\delta(s)ds \\
&= (1+\alpha)\delta(\theta)\theta - \int_{\underline{\theta}}^{\theta}(1+\alpha)\delta(s)ds
\end{aligned}
\tag{11}
$$

This expression for the insurance premium captures the incentive and participation constraints. Plugging this premium in the objective function we get the reduced problem:

$$
\max_{\delta(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left[(1+\alpha)\delta(\theta)\theta - \delta(\theta)\theta - \int_{\underline{\theta}}^{\theta}(1+\alpha)\delta(s)ds \right] dF(\theta)
$$

13

The second term is the aggregate informational rent which integrating by parts is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} (1+\alpha)\delta(s)ds dF(\theta) = \left| \int_{\underline{\theta}}^{\theta} (1+\alpha)\delta(s)ds F(\theta) \right|_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} (1+\alpha)\delta(\theta)F(\theta)d\theta$$

with

$$\int_{\underline{\theta}}^{\underline{\theta}} (1+\alpha)\delta(s)ds = 0$$
$$F(\underline{\theta}) = 0$$
$$F(\overline{\theta}) = 1$$

Hence:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} (1+\alpha)\delta(s)ds dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} (1+\alpha)\delta(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} (1+\alpha)\delta(\theta)F(\theta)d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \frac{1-F(\theta)}{f(\theta)} (1+\alpha)\delta(\theta)dF(\theta)$$

Plugging the solution for the informational rent into the objective function

$$\max_{\delta(\theta)\in[0,1]} \int_{\underline{\theta}}^{\overline{\theta}} \left[\alpha\delta(\theta)\theta - \frac{1-F(\theta)}{f(\theta)}(1+\alpha)\delta(\theta) \right] dF(\theta)$$

Let $h(\theta) = \frac{f(\theta)}{1-F(\theta)}$ be the hazard rate, then the monopoly programme is

$$\max_{\delta(\theta)\in[0,1]} \int_{\underline{\theta}}^{\overline{\theta}} \left[\alpha\delta(\theta)\theta - \frac{(1+\alpha)}{h(\theta)}\delta(\theta) \right] dF(\theta)$$

Because the objective is maximized when the argument of the integral is maximized $\forall \theta \in [\underline{\theta}, \overline{\theta}]$ the result of Proposition 4 is obtained.

It remains to check the monotonicity constraint. To be verified, it requires that

$$\frac{\partial^2}{\partial\delta(\theta)\partial\theta} \left[\alpha\delta(\theta)\theta - \frac{(1+\alpha)}{h(\theta)}\delta(\theta) \right] \geq 0$$

This condition can be expressed in terms of the hazard rate

$$\frac{h'(\theta)}{h^2(\theta)} \geq -\frac{\alpha}{1+\alpha}$$

A sufficient condition is that the hazard rate is non-decreasing in the interval $[\underline{\theta}, \overline{\theta}]$ which completes the proof. ∎

Proof of Proposition 2. From the zero profit conditions (5) it follows that the participation constraints of all the types (1) are not binding and can be disregarded.

Following the incentive compatibility approach of Mailath (1987) the set (2) can be rewritten as the maximization programme of agents

$$\theta \in \arg\max_{\hat{\theta}\in[\underline{\theta},\overline{\theta}]} V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta})) \qquad \forall\, \theta, \hat{\theta} \in [\underline{\theta},\overline{\theta}]$$

By the zero profit condition (5), every agent with risk θ is facing actuarially fair premium: $\pi(\theta) = \delta(\theta)\theta$. This can be incorporated in the first order condition for the type θ that is:

$$
\begin{aligned}
\frac{\partial V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta}))}{\partial \hat{\theta}} &= \frac{\partial}{\partial \hat{\theta}}[\omega - (1+\alpha)(1 - \delta(\hat{\theta}))\theta - \delta(\hat{\theta})\hat{\theta}] \\
&= (1+\alpha)\delta'(\hat{\theta})\theta - \delta'(\hat{\theta})\hat{\theta} - \delta(\hat{\theta}) = 0
\end{aligned}
$$

which evaluated at $\hat{\theta} = \theta$ gives the local incentive compatibility condition

$$(1+\alpha)\delta'(\theta)\theta - \delta'(\theta)\theta - \delta(\theta) = \alpha\delta'(\theta)\theta - \delta(\theta) = 0 \tag{12}$$

This first order condition is also sufficient when the second order condition is respected, i.e.

$$\frac{\partial^2 V(\theta; \delta(\hat{\theta}), \pi(\hat{\theta}))}{\partial \hat{\theta}\partial \theta} = (1+\alpha)\delta'(\hat{\theta}) \geq 0$$

that requires the coverage function to be monotonically increasing. Hence the equilibrium coverage rate function $\delta(\theta)$ is the solution to the following differential equation, coming from the first order condition (12),

$$\frac{\delta'(\theta)}{\delta(\theta)} = \frac{1}{\alpha\theta} \qquad \forall \theta \in [\underline{\theta},\overline{\theta}] \tag{13}$$

Fortunately, the differential equation can be solved directly; the primitive of the LHS is

$$\int \frac{\delta'(\theta)}{\delta(\theta)}d\theta = \log\delta(\theta) + c_1$$

and the for the RHS it is

$$\int \frac{1}{\alpha\theta}d\theta = \log\theta^{\frac{1}{\alpha}} + c_2$$

So, taking exponential on both sides

$$\delta(\theta) = \theta^{\frac{1}{\alpha}}k$$

By using the usual no distortion at the top argument we have a terminal condition, $\delta(\overline{\theta}) = 1$, by which

$$\overline{\theta}^{\frac{1}{\alpha}}k = 1 \implies k = \left(\frac{1}{\overline{\theta}}\right)^{\frac{1}{\alpha}}$$

So, the solution is the following coverage function

$$\delta^c(\theta) = \left(\theta/\overline{\theta}\right)^{\frac{1}{\alpha}} \in [0,1] \qquad \forall\, \theta \in [\underline{\theta},\overline{\theta}]$$

By the zero profit condition the premium function is

$$\pi^c(\theta) = \delta^c(\theta)\,\theta = \left(\frac{\theta^{1+\alpha}}{\overline{\theta}}\right)^{\frac{1}{\alpha}} \qquad \forall\,\theta \in [\underline{\theta}, \overline{\theta}]$$

Since the coverage function is increasing in θ, the second order condition is respected, which completes the proof.[10] ∎

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[10]In fact, $\delta^{c\prime}(\theta) = \frac{1}{\alpha}\left(\frac{\theta^{1-\alpha}}{\overline{\theta}}\right)^{\frac{1}{\alpha}} \geq 0 \qquad \forall\theta \in [\underline{\theta}, \overline{\theta}]$.

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