Point Process Model for Reliability Analysis of Evolutionary Designs

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Abstract

A point process model is developed to assess the reliability of new designs which are variants of an existing product. The model is Bayesian and makes use of existing expert judgement. Prior distributions are used to assess the number of inherent weaknesses introduced through evolution of the design. The enhancement of the design will be experienced through a growth process where failure information is gained about the new design and corrective actions implemented in order to mature reliability. The general model formulation and mathematical underpinnings are presented and an illustrative example of the model application is described.

1. Introduction

Typically models used to predict system reliability based on linear combinations of component failure rates are used in isolation and do not inform the design process. This is because the impact of environmental and operating conditions on base reliabilities are difficult to assess, the assumption of statistical independence between components is unrealistic and the lack of involvement of designers. Therefore predictions are not trusted to provide useful forecasts or to provide insight into ways product reliability might be enhanced.

For the case where designs are evolutionary we propose to formulate a model which captures information about novel features between design variants. Two sources of data are required: historical data concerning the performance of the earlier designs to be used as the base reliability; expert engineering judgement to assess differences between the base design and the variant currently under review.

We propose a point process model. We model the reliability of the new item as a superimposition of two types of failure process. First, the failure modes within the item not removed through the evolutionary design process and so inherited from earlier designs. Second, the failure modes that have been introduced through design changes made to the existing design.

The model is formulated within a Bayesian framework and is described in the following section. In part, our model has been motivated by earlier growth modelling reported in Meinhold and Singpurwalla (1983). We present an example of the model application and conclude by reflecting upon its use in design decision-making.

2. Formulation of Model

2.1 Assumptions

We assume a new design has a fixed but unknown number of faults, N, which will be realised as failures in operation. A failure taxonomy is defined a priori and contains C+1 classes, C of which categorise the fault according to root cause. The additional class corresponds to those failures where no underlying fault is found and is labelled 'no fault found (NFF)' and represents noise in the inherent hazard rate of the design. We denote the number of faults in class i as N_i .

The distribution function of the operational time, t, to realise a particular fault classified in category i is denoted by $F_i(t)$.

For class i, we have a prior distribution describing the experts belief in the number of faults, N_i, likely to be inherent in the design. We denote this as $\pi_i(N_i=n_i)$. For each prior distribution we denote its associated Probability Generating Function as $A_i(z)$. We make use of the relationship between the theoretical distributions and the Probability Generating Functions as this provides a suitable summary of the expert judgement to use within the model.

2.2 Model Derivation

The density function of the time to realise the j^{th} fault within a particular class can be expressed as:

$$
\begin{aligned}
f_{(j)}\left(t_{(j)}=t\right) &= E_{N_i|N_i\geq j}\left[\frac{N_i!}{(j-1)!(N_i-j)!}\left[F_i(t)\right]^{j-1} f_i(t)\left[1-F_i(t)\right]^{N_i-j}\right] \\
&= \frac{\left[F_i(t)\right]^{j-1} f_i(t)}{(j-1)!} \times \frac{\left[\left.\dfrac{d^j A(Z)}{dZ^j}\right|_{Z=1-F_i(t)}\right]}{\left[1-\sum_{n_i=0}^{j-1} p_i(N_i=n_i)\right]}
\end{aligned}
$$

Assuming the realisation of faults is assumed independent, the distribution function of the time to first failure of the item is:

$$
\begin{aligned}
F(t) &= 1-\Pr\{\text{all faults detected after time t and time to next NFF is after time t}\} \\
&= 1-\left(1-F_{NFF}(t)\right)\prod_{i=1}^{C} E_{N_i}\left[1-F_i(t)\right]^{N_i} \\
&= 1-\left(1-F_{NFF}(t)\right)\left\{\prod_{i=1}^{C} A_i\left(1-F_i(t)\right)\right\}
\end{aligned}
$$

The distribution of time until all faults have been realised for the C classes is constructed by a similar argument giving:

$$
\begin{aligned}
F(t) &= \prod_{i=1}^{C} E\left[F_i(t)^{N_i}\right] \\
&= \prod_{i=1}^{C} A_i\left(F_i(t)\right)
\end{aligned}
$$

2.3 Intensity Function

The realisation of faults through failure can be conceptualised as a point process. Using the same assumptions as before the expected number of faults that have been realised by time t are:

$$
\begin{aligned}
E\left[N(t)\right] &= \sum_{i=1}^{C} E\left[N_i(t)\right] \\
&= \sum_{i=1}^{C} E_{N_i} E_{N_i(t)}\left[N_i(t)\middle| N_i\right] \\
&= \sum_{i=1}^{C} E_{N_i}\left[N_i\right]\times F_i(t)
\end{aligned}
$$

Assuming the no fault found failures occur according to a Homogeneous Poisson Process at rate μ then the intensity function can be simplified to:

$$
\begin{aligned}
\boldsymbol{i}_{\text{item}}(t) &= m + \sum_{i=1}^{C} \frac{dN_i(t)}{dt} \\
&= m + \sum_{i=1}^{C} E[N_i] \times f_i(t)
\end{aligned}
$$

2.4 Inference

As the failure process for each class is assumed independent we can develop inference procedures for a particular class and ignore the indexing. Therefore the likelihood function can be expressed as:

$$
L\left(N, \Theta \middle| \underset{\sim}{t}\right) \propto \frac{N!}{(N\text{-}j)!}\left[1\text{-}F\left(t_j\right)\right]^{N\text{-}j} \prod_{k=1}^{j} f\left(t_k \middle| \Theta\right)
$$

and the posterior distribution of the number of faults in the design given j faults have been realised at times $\underset{\sim}{t} = \left(t_1, t_2, \ldots, t_j\right)$

$$
\begin{aligned}
p\left(N{=}n, \Theta \middle| \underset{\sim}{t}\right) &= \frac{\dfrac{n!}{(n\text{-}j)!}\left[1\text{-}F\left(t_j \middle| \Theta\right)\right]^{n\text{-}j} \displaystyle\prod_{k=1}^{j} f\left(t_k \middle| \Theta\right) p\,(N{=}n)\, p\,(\Theta)}{\displaystyle\int_{\forall\Theta} \sum_{n=j}^{\infty} \frac{n!}{(n\text{-}j)!}\left[1\text{-}F\left(t_j \middle| \Theta\right)\right]^{n\text{-}j} p\,(N{=}n) \prod_{k=1}^{j} f\left(t_k \middle| \Theta\right) p\,(\Theta)\, d\Theta} \\
&= \frac{\dfrac{n!}{(n\text{-}j)!}\left[1\text{-}F\left(t_j \middle| \Theta\right)\right]^{n\text{-}j} \displaystyle\prod_{k=1}^{j} f\left(t_k \middle| \Theta\right) p\,(N{=}n)\, p\,(\Theta)}{\displaystyle\int_{\forall\Theta} \left.\frac{d^j A(Z)}{dZ^j}\right|_{Z=1\text{-}F_i\left(t \middle| \Theta\right)} \prod_{k=1}^{j} f\left(t_k \middle| \Theta\right) d\Theta}
\end{aligned}
$$

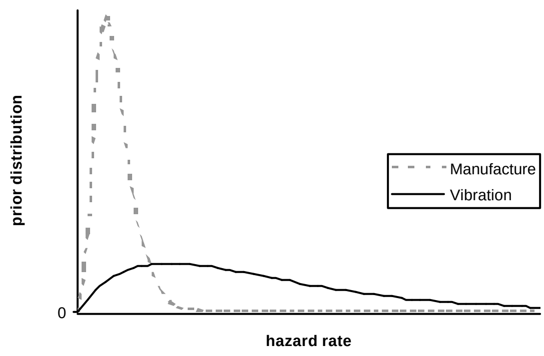
3. Illustrative Example

Consider the evaluation of a new design where, for the sake of simplicity, we consider only two classes, namely vibration and build, without loss of generality.

An elicitation exercise allows us to construct prior distributions for the number of potential faults within the design. In this case both priors are Poisson with mean 4 and 8 for vibration and manufacture respectively.

We also have extracted failure data for the related items from historical records. From these we develop empirical prior distributions for the hazard rate for each class. The hazard rate is associated with the distribution describing the length of time a fault remains undetected within a design. For simplicity we assume the time to realise failures within a particular class are independently and identically exponentially distributed random variables.

Figures 1 and 2 illustrate these priors. The computed distribution functions of the time to first and last fault detected are illustrated in Figures 3 and 4 respectively.



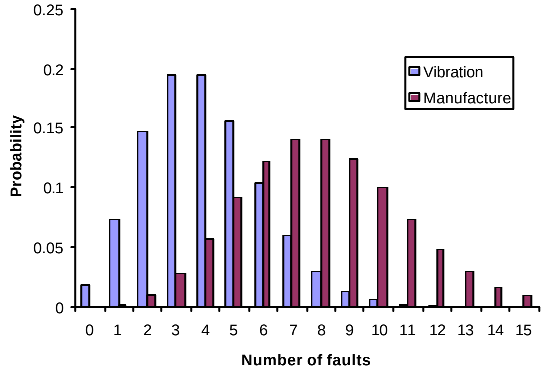
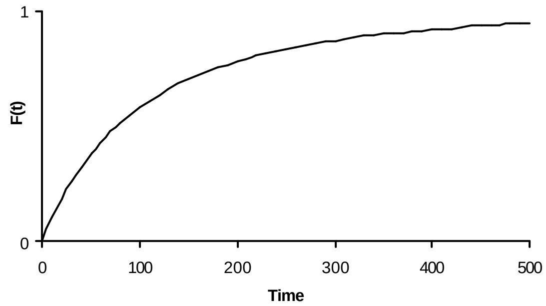


Figure 1: Prior distributions for hazard rate

Figure 2: Prior distribution for number of faults



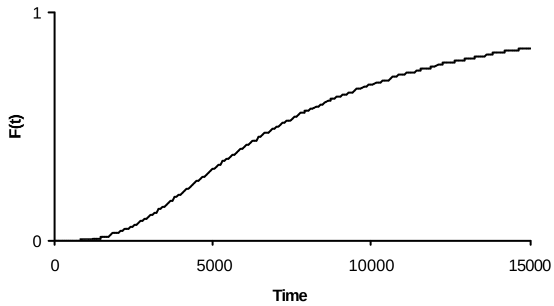


Figure 3: CDF for time to first failure

Figure 4: CDF of time to detect last fault

4. Conclusion

The proposed model requires the involvement of design, and other, engineers to assess the design weaknesses and analyse historical failure data. Involvement in the model specification and set-up has allowed engineers to accept the model and thus use it to inform design decisions. Further details are reported in Hodge et al (2001).

The general model is presented here along with results for particular distributional assumptions. Further details about the statistical inference procedures and their properties for the case where we have Poisson priors and exponential times to fault realisation are described in Quigley and Walls (2002). However the model supports a wider class of distributions and results have been derived for other parametric models as well as nonparametric approaches.

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