Design by Analysis of Ductile Failure and Buckling in Torispherical Pressure Vessel Heads

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Abstract:

Thin shell torispherical pressure vessel heads are known to exhibit complex elastic-plastic deformation and buckling behaviour under static pressure. In pressure vessel Design by Analysis, the designer is required to assess both of these behaviour modes when specifying the allowable static load. The EN and ASME Boiler and Pressure Vessel Codes permit the use of inelastic analysis in design by analysis, known as the direct route in the EN Code. In this paper, plastic collapse or gross plastic deformation loads are evaluated for two sample torispherical heads by 2D and 3D FEA based on an elastic-perfectly plastic material model. Small and large deformation effects are considered in the 2D analyses and the effect of geometry and load perturbation are considered in the 3D analysis. The plastic load is determined by applying the ASME Twice Elastic Slope Criterion of plastic collapse and an alternative plastic criterion, the Plastic Work Curvature criterion. The formation of the gross plastic deformation mechanism in the models is considered in relation to the elastic-plastic buckling response of the vessels. It is concluded that in both cases, design is limited by formation of an axisymmetric gross plastic deformation in the knuckle of the vessels prior to formation of non-axisymmetric buckling modes.

Keywords: Gross plastic deformation, plastic load, criterion of plastic collapse, axisymmetric torispherical pressure vessel heads, buckling.

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1. INTRODUCTION

Pressure vessels Design by Analysis requires the designer to demonstrate that a proposed design satisfies a number of criteria associated with specific failure modes. In most designs the fundamental failure mechanism associated with static loading is Gross Plastic Deformation (GPD) and the designer is required to demonstrate a specified margin of safety against GPD under the specified mechanical design loads. Codes and Standards such as PD5500 *Unfired fusion welded pressure vessels* [1], ASME *Boiler & Pressure Vessel Code* Sections III and VIII [2] and EN 13445-*3:2002 Unfired pressure vessels* [3] specify two distinct approaches to design by analysis. The most widely used approach in current practice is that based on linear elastic stress analysis of the vessel. Elastic analysis has the advantage that the stress analysis part of the design procedure is relatively straightforward. However, the procedure is complicated by the need to relate the elastic stress to the inelastic GPD failure mechanism. This is done in practice by applying a stress classification procedure to determine specific classes of stress for which allowable maximum values are defined. GPD failure is related to the primary stress category, which is yield-limited to preclude failure due to this mechanism. In the alternative approach, the designer performs an inelastic analysis incorporating post-yield stress redistribution, simulating the formation of the GPD mechanism. The GPD load is defined directly from the simulated structural response, through application of a criterion of an appropriate criterion. In EN13445, this design methodology is referred to as “the direct route”. The perceived disadvantage of the direct route is that it requires more advanced non-linear stress analysis but it has the advantage that it avoids the requirement for stress categorisation. This significant advantage, coupled with the availability of user-friendly inelastic Finite Element Analysis (FEA) programs and relatively inexpensive but powerful computers, has led to increased use of the direct route in design.

The type of inelastic analysis permissible varies between the Design Codes. PD5500 implies the use of an elastic-perfectly plastic material model and small (first order) deformation theory; in effect, traditional “limit analysis”. EN13445 specifies an elastic-perfectly plastic material model but requires large (second order) deformation effects to be considered for vessels or components exhibiting geometric weakening. The ASME code is less prescriptive, permitting the use of elastic-perfectly plastic *or* strain hardening material models and small *or* large deformation theory. The inelastic analysis method used determines how the GPD load is defined. In a traditional FEA based “limit analysis” (small deformation, elastic perfectly plastic), the GPD load is specified as the “limit load” of the vessel, the greatest load that the vessel can support before equilibrium between internal and external forces is violated. This is often regarded as a conservative load for design purposes but in geometrically weakening structures changes in geometry lead to earlier onset of lack of equilibrium and the limit load may not be conservative. This is recognised in EN13445, which requires second order effects to be considered for geometrically weakening structures. EN13445 B.4 *Failure modes and limit states* specifies “A limit state is classified as either an ultimate or a serviceability limit state.” Thus, the maximum load at which equilibrium is assured assuming an elastic-plastic material and large deformation theory is viewed as a limit state.

The ASME Code provides procedures for design based on limit analysis and on *plastic analysis*, which may include strain hardening and/or large deformation effects.

In plastic analysis, the GPD load is defined by applying a *criterion of plastic collapse* to a characteristic load-deformation curve calculated for the vessel. The ASME Twice Elastic Slope (TES), criterion is based on an empirical procedure for calculating collapse loads in experimental stress analysis of pressure vessels and is illustrated in Figure 1a. The plastic load, P_{ϕ}, is the load corresponding to the intersection of the load-deformation curve and a straight line called the *collapse limit line,* emanating from the origin of the load-deformation curve at angle $\varphi = \tan^{-1}(2\tan\theta)$. Several practical problems that can occur when applying the TES criterion have been identified in the literature [4-7].

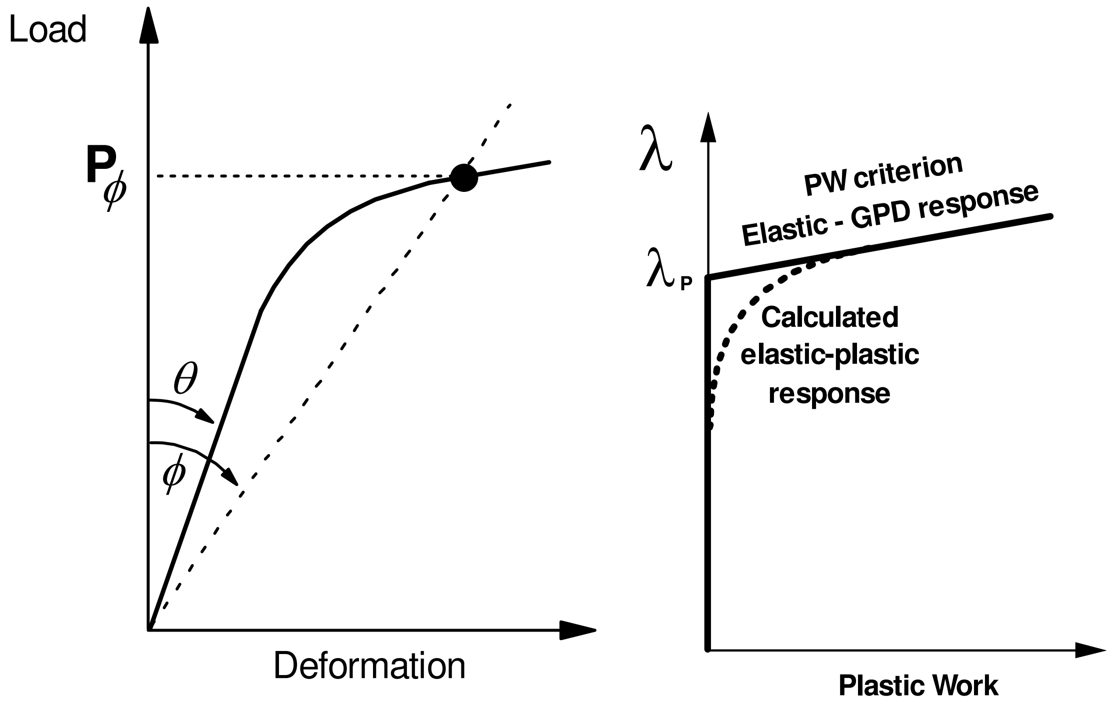


Figure 1: (a) Twice elastic slope criterion of plastic collapse (b) plastic work criterion.

In addition to performing a check against GPD under static loads, the Codes also require the designer to consider the possibility of a buckling instability failure mode occurring prior to the formation of a full GPD mechanism. Buckling analysis may be carried out independently of the GPD check to determine the allowable buckling load. EN13445 *B.8.4 Instability (I)* states that the static design load shall not be greater than the buckling strength of the vessel, (subject to a maximum strain limitation of 5%), based on a model "incorporating pre-deformations according to the critical (classical/bifurcation) buckling shapes and deviations according to the allowed ones as per EN 13445-4:2002". However, this approach may not identify situations in which the buckling modes and gross plastic deformation interact, leading to failure at loads less than that predicted for each mode individually. The object of this paper is to investigate interaction between elastic-plastic buckling and the formation of the GPD mechanism in a vessel configuration known to be susceptible to buckling failure: thin internally pressurised 3D torispherical pressure vessel heads. Torispherical heads may exhibit buckling failure as internal pressure increases due to compressive hoop stress in the knuckle. Two head geometries known to exhibit this response are considered in the investigation. The formation of the gross plastic deformation mechanism with and without the presence of initial shape imperfection and perturbation loads applied to the knuckle of the vessel are determined using the ASME TES criterion and a recently proposed criterion, the Plastic Work Curvature criterion.

2. PLASTIC WORK CURVATURE CRITERION

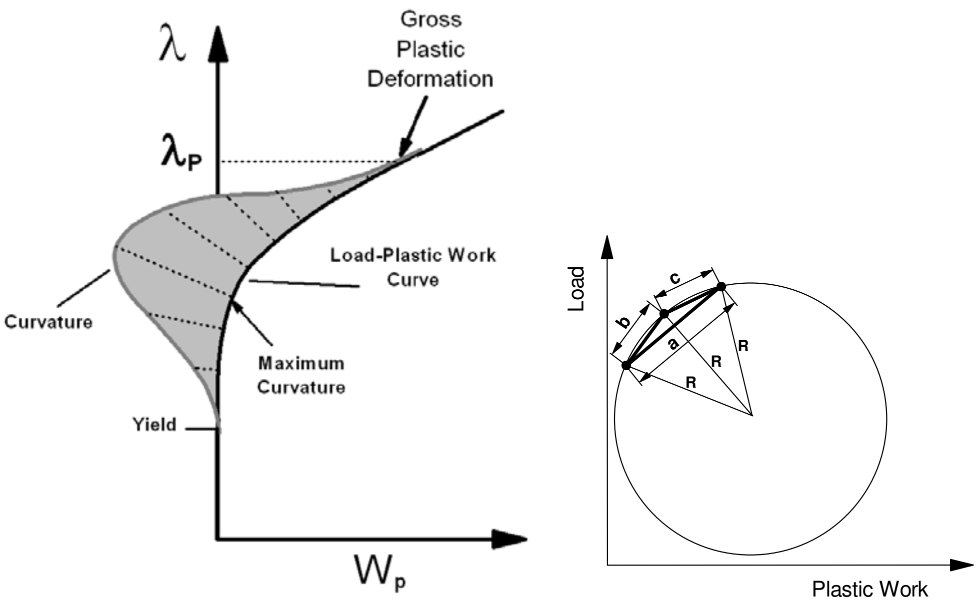
When a vessel is loaded beyond yield a measure of work done on the structure is dissipated internally as plastic work. Gerdeen proposed that plastic dissipation could provide an improved failure criterion of plastic collapse in reference [8], in which he presented a collapse criterion based on the relationship between plastic dissipation and elastic strain energy in the vessel in a general form. More recently, Muscat *et al* proposed a plastic collapse criterion based on a characteristic plot of a global load parameter, λ, representing all applied loads, against plastic work dissipation in the vessel, as illustrated in Figure 1b [9]. The criterion applies a geometric construction to define the GPD load. A more detailed investigation of the transition from elastic to gross plastic response was presented by Li & Mackenzie [10, 11], in which it was proposed that the curvature of the characteristic load-plastic work curve could be used to define the GPD load, as illustrated in Figure 2a. In the plot, the PWC is normalised with respect to the *maximum* value of PWC calculated in the analysis. In the elastic region, the curvature is zero indicating zero plastic deformation. Post yield, plastic stress redistribution occurs and the Plastic Work Curvature, PWC, increases to a maximum as the plastic deformation mechanism develops. The maximum stress redistribution occurs at the load corresponding to the maximum PWC, where after it begins to decrease as the plastic deformation mechanism is established. When the PWC reaches a minimum constant or zero value, relatively little or no further plastic stress redistribution occurs in the vessel unless a second plastic deformation mechanism is initiated in a formerly elastic region. At this stage the structure exhibits constant or gross plastic deformation with increased loading and the corresponding load is therefore designated as the plastic load for DBA. This criterion was applied to determine the GPD load for benchmark torispherical heads in reference [**12**]. Torispherical ends experience complex plastic deformation prior to failure, with the formation of plastic-hinge bending mechanisms in the knuckle and membrane plastic deformation in the crown and cylinder. It was found that thin-wall torispherical heads exhibited complex load-PWC response, with several local peaks in the curvature associated with the formation of plastic zones in different regions of the vessel. The response was found to be dependent on the material model and deformation theory used in the analysis. It was concluded that the plastic pressure should be determined with respect to the first local maxima or peak, as this represented the formation of a local gross deformation mechanism.

The PWC criterion requires a plot of load against normalised load-plastic work curvature. The load-PWC plot may be created from the numerical results of the FE analysis and plotted against applied pressure using a simple technique based on the circumradius of three points [13]. The plastic work corresponding to the applied load is calculated by the FE program for each load step. The results are written to a data file as a series of load-plastic work points. The curvature of a sector of curve defined by three consecutive points is the inverse of the circumradius of the three points. The circumradius R of a triangle of sides length a, b and c, as shown in Figure 2b, is given by:

$$R = \frac{abc}{4\sqrt{s(a+b-s)(a+c-s)(b+c-s)}} \quad (1)$$

where s is the semiperimeter given by:

$$s = \frac{a+b+c}{2} \quad (2)$$



(a) (b)

Figure 2: Plastic work curvature criterion and circumradius evaluation of curvature.

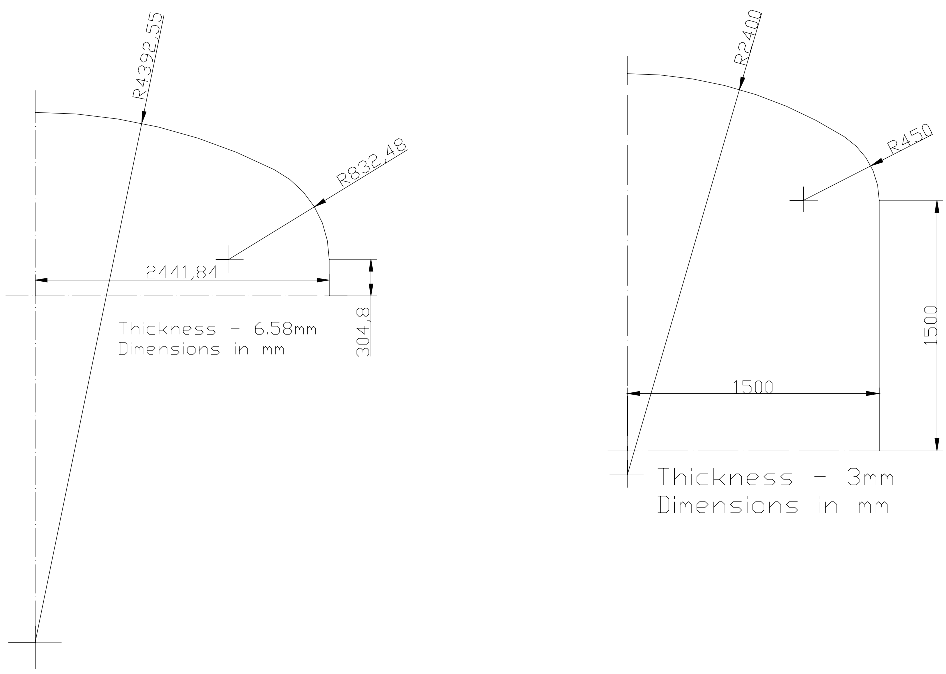
3. EXAMPLE TORISPHERICAL VESSELS

The GPD loads of two thin wall torispherical heads previously investigated by Miller et. al [14] and Galletly et. al. [15] were considered in the investigation, denoted Head 1 and Head 2 respectively. Head 1 was analysed experimentally to determine its buckling and rupture strength. Head 2 was analysed by elastic-plastic finite deflection analysis using the BOSOR 5 program [16].

The geometry of the vessels is defined in Figure 3 and the material properties given in Table 1.

Material Property	Head 1	Head 2
Young's modulus (GPa)	200	207
Yield strength (MPa)	353	310

Table 1: Material properties of pressure vessels



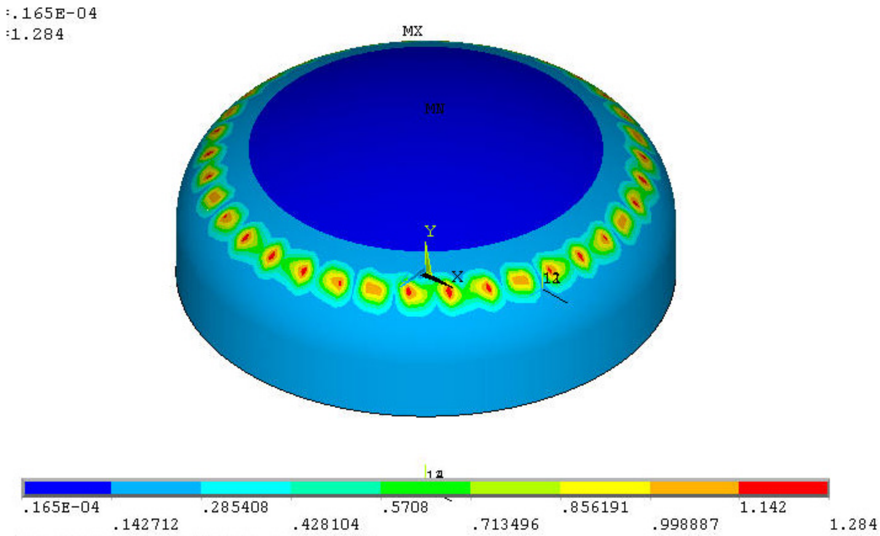
a) Head 1 b) Head 2

Figure 3: Torispherical head geometry.

The ideal geometry, loading and boundary conditions of the two vessels are axisymmetric and if buckling deformation is not considered, the vessels can be analysed by axisymmetric Finite Element Analysis (FEA). However, it is known that as these vessels are loaded compressive hoop stress is established in the knuckle region and the vessels may experience non-axisymmetric buckling, as local buckles form around the knuckle. To simulate this failure mode, it is necessary to model the structure in 3D.

Finite element analysis was performed using the ANSYS program [17]. The heads were initially investigated using ANSYS 8 noded axisymmetric Plane 82 elements. The mesh of the heads consisted of a total of 2760 elements having 6 elements through thickness and refined at the knuckle and crown region. The models are capable of examining axisymetric yielding. Three general forms of plastic collapse mechanisms may occur in an axisymmetric torispherical head: a bending or hinge mechanism located at the knuckle or membrane deformation in the cylinder or in the domed end. Previous work performed by the authors [12] on relatively thick torispherical heads showed that GPD occurred in the knuckle. However, it is known that the two head geometries considered here are also subject to local circumferential elastic-plastic buckling of the knuckle [14, 15].

In order to model the evolution of buckling deformation, the heads were analysed with 3D ANSYS 4-noded Shell 181 models. The mesh of the three-dimensional models consisted of 8504 and 10004 elements for Head 1 and Head 2 respectively. Three different types of analysis were performed for these models. In the first type of analysis the head was modelled with an ideal shape, within the levels of approximation of the shell elements. In the second analysis, initial geometric perturbation corresponding to the first non-axisymmetric eigen buckling mode, shown in Figure 4a for Head 1, was applied, with maximum displacement corresponding to half the shell thickness. In the third analysis, 2kN perturbation forces of were applied normal to the mid-section of the knuckle region of in each quadrant, as shown in Figure 4b.



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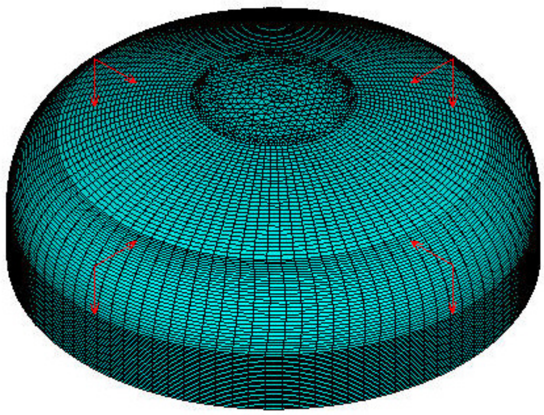
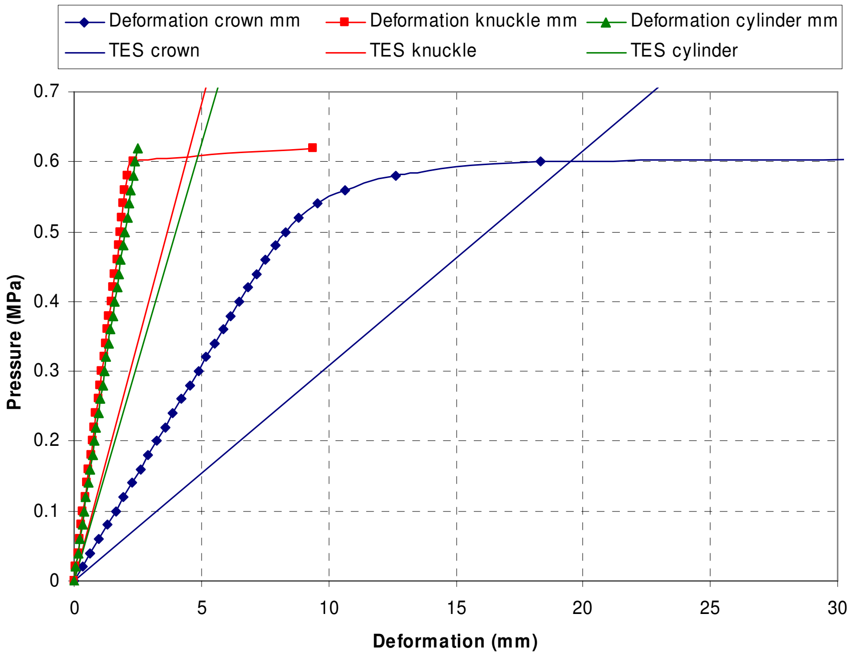


(b)

Figure 4: (a) applied perturbation geometry (b) applied perturbation forces.

4. RESULTS

When applying the TES criterion, it in necessary to specify a deformation parameter at a point on the vessel. Torispherical heads experience membrane deformation in the crown and cylindrical region and plastic hinge deformation in the knuckle region and the choice of location of deformation parameter should be made according to which region first experiences GPD. Three deformation parameters were considered in the investigation: normal displacement at the crown, knuckle and cylinder.



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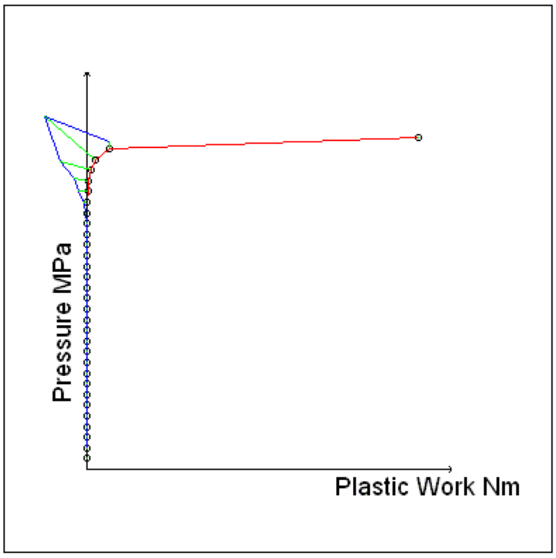

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Figure 5: Head 1 2D small deformation theory (a) TES load-deformation plots (b) PWC load-curvature plot.

Two-dimensional small deformation theory analysis TES pressure-deformation plots for Head 1 are shown in Figure 5a and the PWC pressure-plastic work plot in Figure 5b. The calculated plastic loads for the model and corresponding numerical instability load are given in Table 2. In small deformation analysis, numerical instability occurred at a pressure of 0.62MPa. In this type of analysis, the numerical instability load is the limit load of the vessel. The TES criterion plastic pressure based on crown and knuckle deformation parameters is slightly lower than the limit pressure, at 0.60MPa. In the case of the cylinder deformation parameter, the collapse limit line and load-deformation plot do not intersect and the plastic load is undefined. The GPD load predicted by the PWC criterion is equal to the limit load. The equivalent plastic strain distribution and (scaled) deformed geometry at the limit or GPD pressure is shown in Figure 6. This illustrates that GPD occurs in the knuckle due to formation of a hinge mechanism.

Model	**Plastic Pressure (MPa)**				
	PWC	**TES**			**Instability**
		Crown	Knuckle	Cylinder	
2D small defn	0.62	0.60	0.60	n/a	0.62
2D large defn	1.08	1.03	0.90	n/a	1.49
3D large defn No perturbation	0.87	n/a	0.87	n/a	0.91
3D large defn Geom. perturbation	0.81	0.91	0.82	0.92	1.50
3D large defn Load perturbation	0.84	0.90	0.84	0.88	0.96
Nominal buckling load [15]	0.73				

Table 2: Head 1 calculated failure loads.

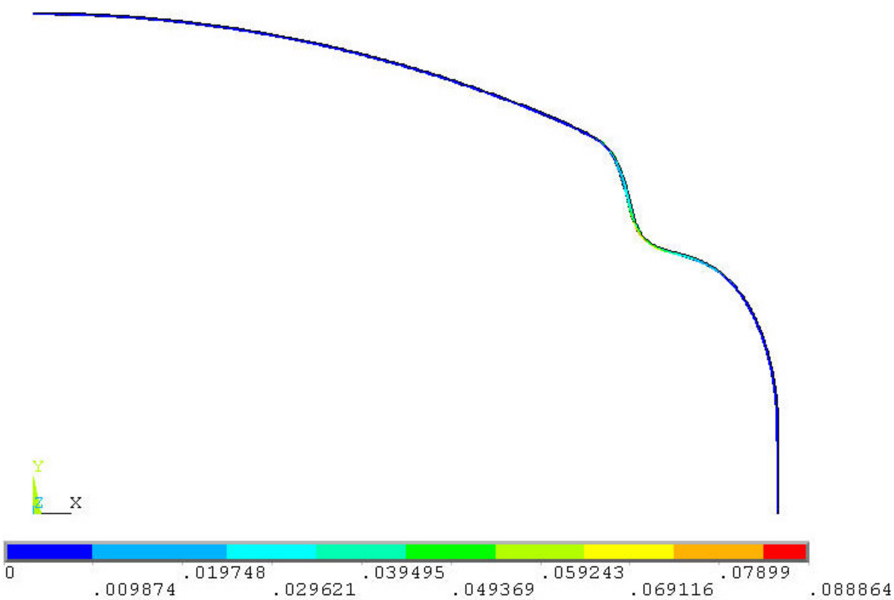
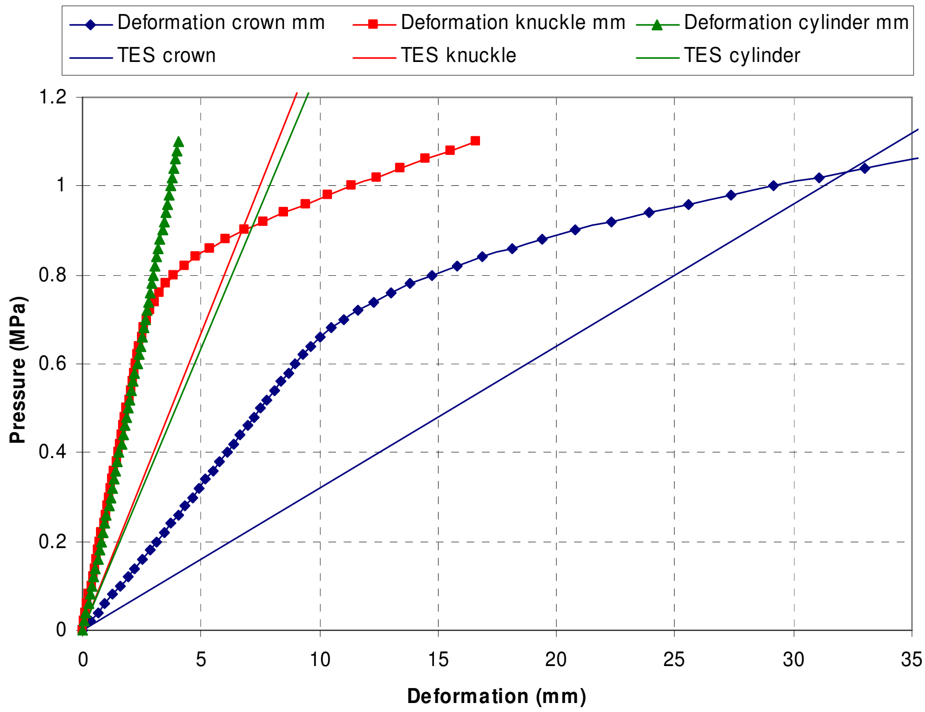


Figure 6: Head 1 2D small deformation theory equivalent plastic strain distribution and deformed geometry at 2D limit load/PWC criterion GPD load.

Two-dimensional large deformation theory analysis TES pressure-deformation plots for Head 1 are shown in Figure 7a and the PWC pressure-plastic work plot in Figure 7b. Comparison with the plots for small deformation analysis in Figure 5 shows that including non-linear geometry in the analysis significantly affects the simulated response. Numerical instability occurred at a pressure of 1.49MPa. This is greatly in excess of the limit load, indicating geometric strengthening occurs when large deformation effects are considered. The TES criterion plastic pressure based on crown and knuckle deformation parameters are 1.03MPa and 0.9MPa respectively, approximately 50% greater than the limit load. As in the small deformation case, the cylinder deformation parameter does not define a plastic load. The GPD load predicted by the PWC criterion, 1.08MPa, is similar to the pressures obtained by the

TES criterion. The equivalent plastic strain distribution and (scaled) deformed geometry at the GPD pressure is shown in Figure 8. The shape of the head is seen to have changed from the original torispherical geometry, tending towards an elliptic shape. The GPD mechanism occurs in the knuckle region of the vessel but there is a distinct change in the form of the predicted mechanism compared with the small deformation analysis.



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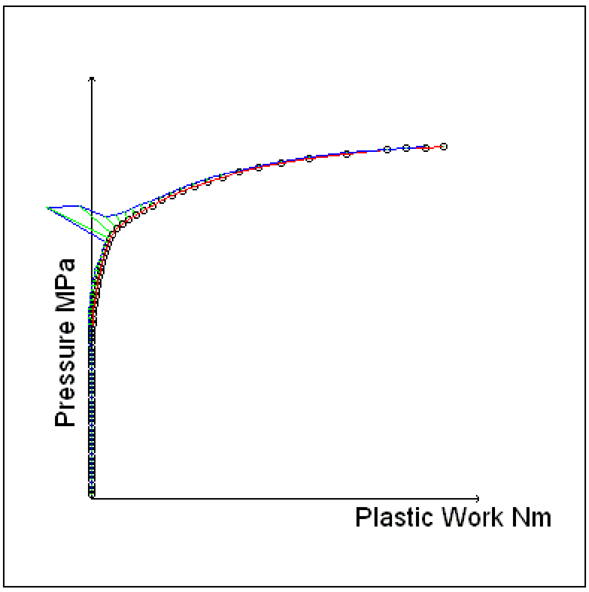

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Figure 7: Head 1 2D large deformation theory (a) TES load-deformation plots (b) PWC load-curvature plot.

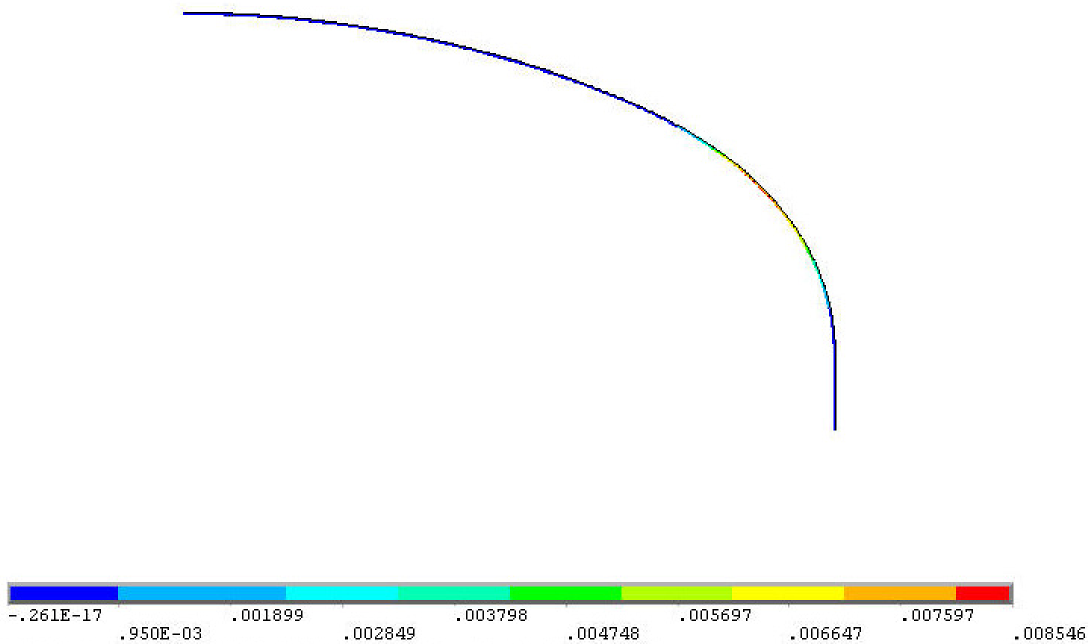
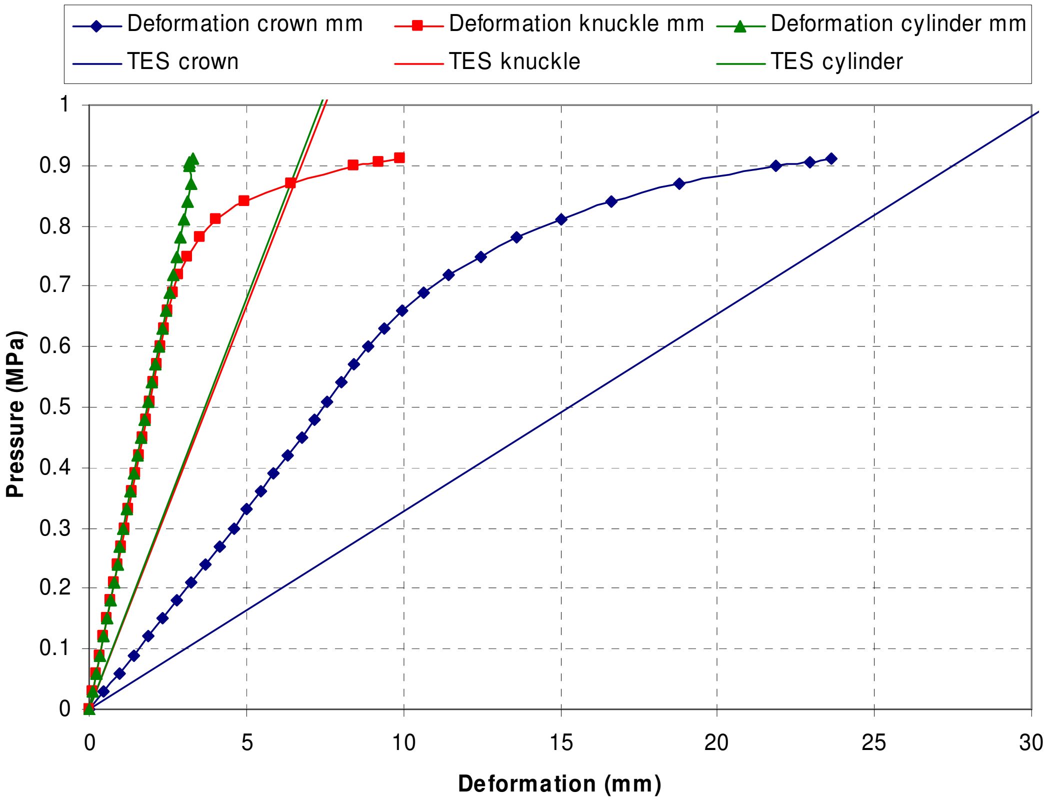


Figure 8: Head 1 2D large deformation theory equivalent plastic strain distribution and deformed geometry at PWC criterion GPD load.

Three-dimensional large deformation theory analysis TES pressure-deformation plots for Head 1 based on the original head geometry are shown in Figure 9a and the PWC pressure-plastic work plot in Figure 9b. The TES deformation parameters used for the 3D model were defined at the same location as those used in the 2D model. When the TES construction is applied to the 3D results, the corresponding collapse limit lines and load-deformation curves do not intersect for the crown and cylinder parameters. The knuckle parameter indicates a plastic pressure of 0.87MPa, slightly lower than numerical instability load of 0.91MPa. The GPD pressure defined by the PWC criterion for the unperturbed 3D model is 0.81MPa. The equivalent plastic strain distribution at the GPD pressure is shown in Figure 10a. The GPD mechanism occurs in the knuckle region and is axisymmetric. When the load is increased beyond the GPD pressure, the plastic deformation in the knuckle becomes non axisymmetric as local buckling occurs around the head. The plastic strain distribution prior to instability is shown in Figure 10b. Figures 10a and 10b therefore show that the PWC criterion identifies a GPD mechanism forms in the knuckle prior to circumferential buckling occurring.



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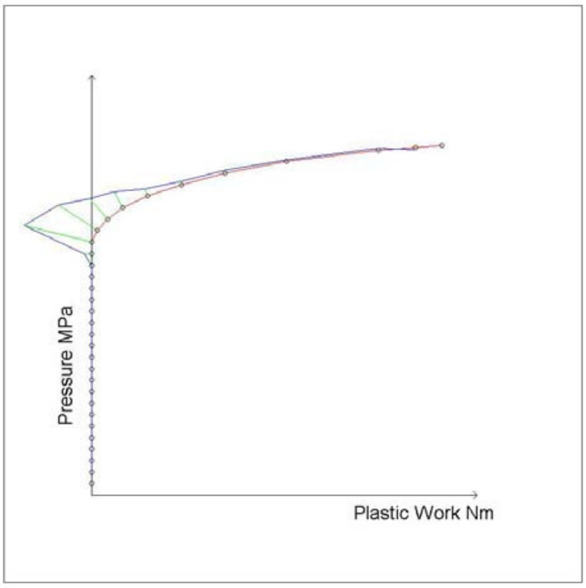

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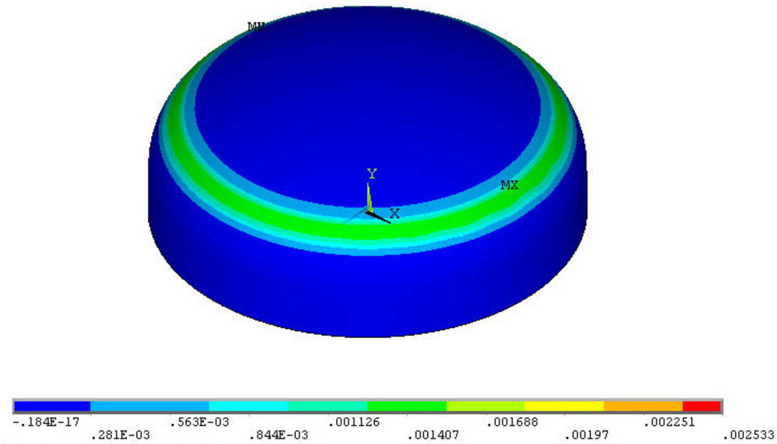
Figure 9: Head 1 3D large deformation theory (a) TES load-deformation plots (b) PWC load-curvature plot.



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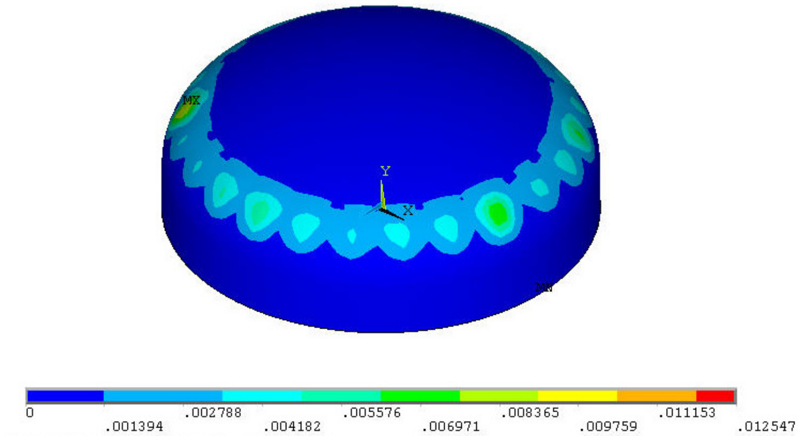
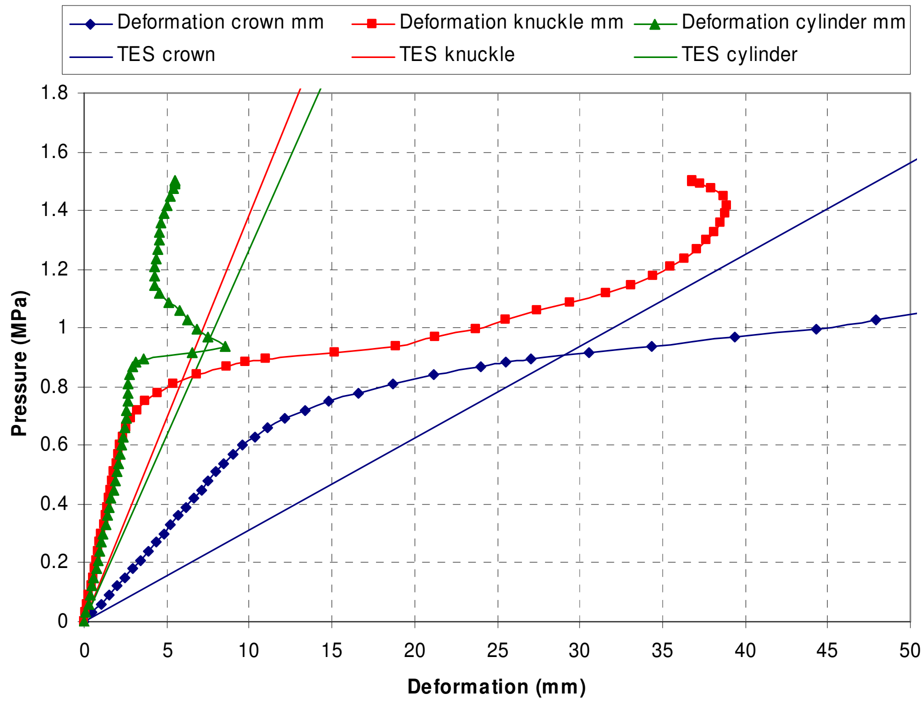


(b)

Figure 10: Head 1 - 3D large deformation theory equivalent plastic strain distribution at (a) PWC criterion GPD pressure 0.81MPa (b) numerical instability pressure 0.91MPa.

The TES and PWC plots for the 3D model with initial deformation perturbation are shown in Figure 11a and 11b respectively. The response curves are seen to be more complex than their axisymmetric analysis equivalents due to the formation of the buckling mechanism, which is not modelled in the 2D analysis. In this case, the collapse limit lines and load-deformation curves for all three deformation parameters intersect before numerical instability, which occurs at 1.50MPa. The TES criterion plastic pressure based on crown, knuckle and cylinder deformation parameters are 0.91MPa, 0.82MPa and 0.92MPa respectively. The form of the PWC pressure-plastic work curve shown in Figure 11b is more complex than that obtained for the model without perturbation. Following the elastic response, the curvature increases indicating plastic deformation. As load increases, the curvature reaches a peak then reverses at a pressure 0.81MPa. The equivalent plastic strain distribution at this pressure is shown in Figure 12a. This shows that the knuckle region is experiencing GPD. The plot shows slight circumferential variation in plastic straining, due to the initial geometry perturbation, but the GPD failure mechanism identified is essentially similar to the axisymmetric mechanism identified in the vessel without initial perturbation at the same pressure. Beyond this GPD pressure of 0.81MPa, local buckling starts to occur around the circumference in the knuckle. This is followed by membrane plastic straining in the crown of the vessel. However, for design purposes the critical mechanism is the initial GPD mechanism.



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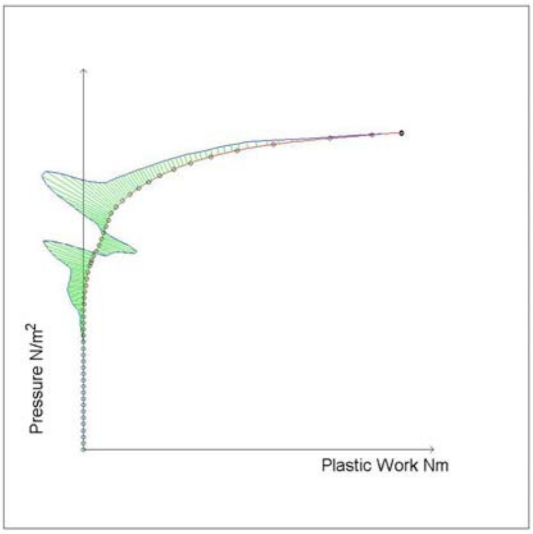

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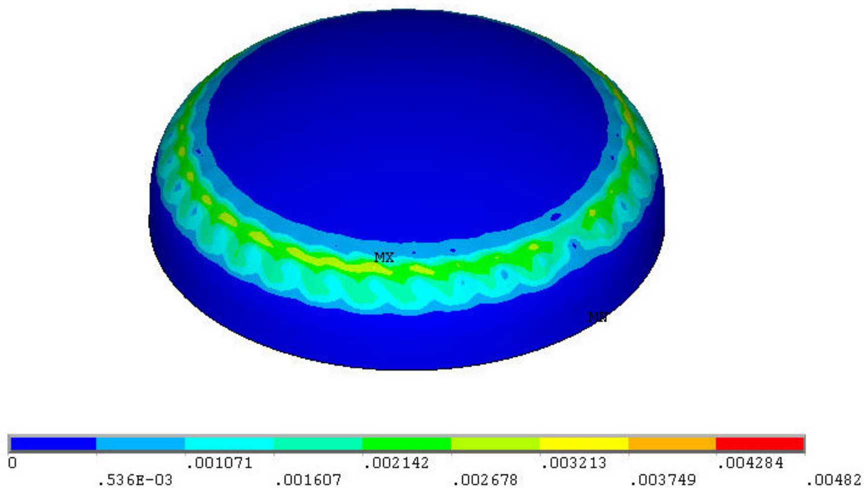
Figure 11: Head 1-3D large deformation theory with geometric perturbation (a) TES load-deformation plots (b) PWC load-curvature plot.

The 3D model with load perturbation exhibited a similar overall response to the vessel with geometry perturbation, with TES plastic pressure based on crown, knuckle and cylinder deformation parameters are 0.90MPa, 0.84MPa and 0.88MPa respectively. The PWC criterion GPD pressure is 0.84MPa and the equivalent plastic strain distribution at this pressure, shown in Figure 12b, indicates a similar GPD mechanism to the model with geometric perturbation.



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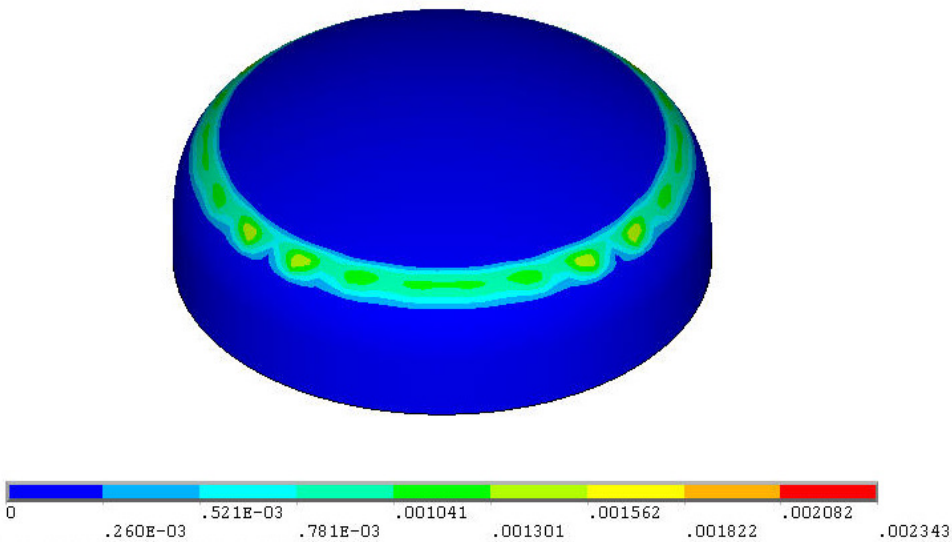

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Figure 12: Head 1 - 3D large deformation theory equivalent plastic strain distribution at PWC criterion GPD load (a) geometric perturbation (b) load perturbation.

The results of the analyses of Head 2 are summarised in Table 3. Equivalent plastic strain distribution plots at the PWC criterion GPD pressure for no perturbation, geometric perturbation and load perturbation are shown in Figures 13a, 13b and 13c respectively. The plots show that the PWC criterion indicates an essentially axisymmetric GPD mechanism for the no perturbation model, as in Head 1. In the model with geometric perturbation the PWC criterion GPD mechanism is non-axisymmetric, with distinct regions of high plastic strain around the knuckle. The GPD mechanism in the model with perturbed load also exhibits variation in plastic strain with circumferential position but the variation is less than in the model with perturbed geometry.

Model	Plastic Pressure MPa				
	PWC	TES			Instability
		Crown	Knuckle	Cylinder	
2D small defn	0.44	0.37	0.38	0.72	1.48
2D large defn	0.54	0.58	0.51	n/a	0.72
3D large defn No perturbation	0.44	0.49	0.45	0.49	0.49
3D large defn Geom. perturbation	0.45	0.45	0.45	0.46	0.71
3D large defn Load perturbation	0.42	n/a	0.45	n/a	0.46
Critical buckling load [16]	0.41				

Table 3. Head 2 calculated failure loads.

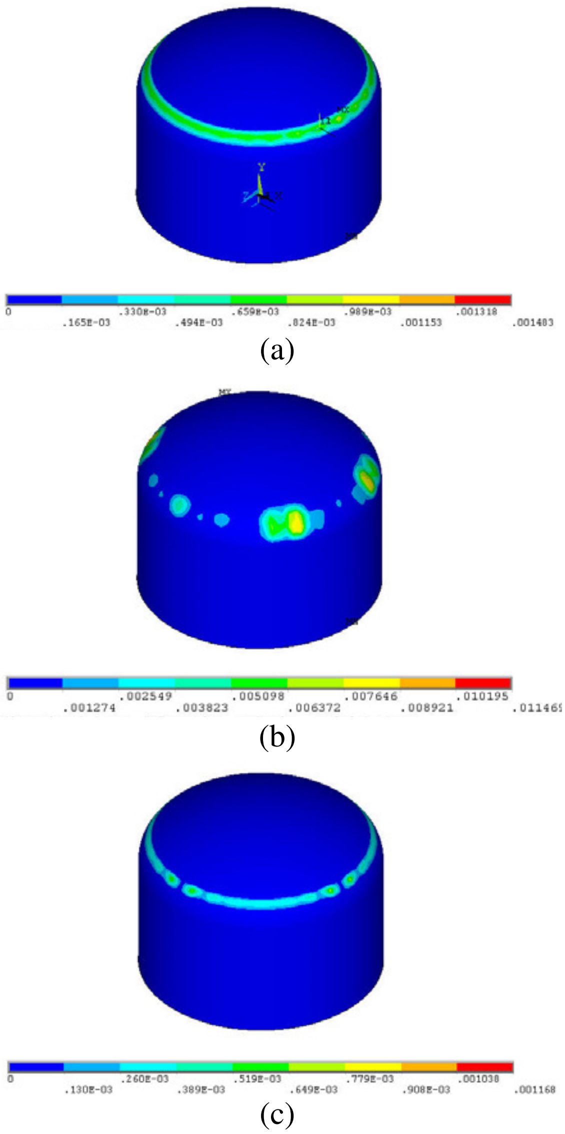

Figure 13: Head 2 3D large deformation theory equivalent plastic strain distribution at PWC criterion GPD load (a) unperturbed model (b) geometric perturbation (c) load perturbation.

5. Discussion and Conclusions

Comparing the results of the small and large deformation analyses of the example heads shows that they experience geometric strengthening, as expected. Geometric strengthening is not considered in EN13445 and in a direct route design the allowable static load would be based on the small deformation elastic-perfectly plastic limit analysis results, subject to assessment of the critical buckling load. In Head 1, the nominal buckling load given in the literature [14] indicates the formation of local buckles in the knuckle. Local elastic-plastic local buckles in the knuckle were seen to form gradually over a pressure range from 0.73MPa to 1.6MPa, rather than form rapidly upon reaching a critical load. If the nominal buckling load of 0.73MPa is considered in DBA, the allowable load would be determined with respect to the limit load. In Head 2, the critical buckling load given by the BOSOR program [15] is slightly lower than the limit load and this load would therefore be used to determine the allowable pressure in DBA.

In the ASME DBA procedure, the calculated plastic load may include large deformation effects causing geometric strengthening. The plastic load, or GPD pressure, calculated in the present investigation using FEA and the TES and PWC criteria for the model with initial shape imperfection, 0.81MPa is slightly higher than the nominal buckling load of 0.73MPa specified in [14], which actually designates the load at which local buckles began to form gradually. In Head 2, the buckling loads evaluated by FEA are similar to the load calculated using the BOSOR program in [15].

The plastic load calculated using the TES criterion is dependent on the location of the deformation parameter used. Three deformation parameters were considered: normal displacement in the crown, knuckle and cylinder regions of the vessel. The knuckle parameter was found to give the most conservative value of plastic pressure. The PWC criterion indicated that GPD failure occurred in the knuckle region of the vessel prior to the formation of non-axisymmetric buckling and gross plastic membrane deformation of the crown or shell. This finding indicates that use of a knuckle deformation parameter is appropriate in the TES criterion.

In the present investigation, the TES criterion has the advantage that it is simple to apply and interpret, and gives plastic pressures consistent with the requirements of the DBA procedure provided the deformation parameter used is chosen correctly. The PWC criterion does not require the designer to select a deformation parameter as it is a global indicator of gross plastic deformation. The form of the PWC pressure-plastic work plot also helps the designer identify the evolution of distinct plasticity mechanisms in different regions of the vessel as they occur with increasing load. However, the PWC curves must be interpreted with care. The criterion identifies the formation of a GPD mechanism in the knuckle region as the limiting plastic mechanism in design. However, as load is increased beyond the GPD load, extensive plastic deformation occurs in the crown of the vessel. This can have the effect of dominating the form of the curve to the extent that the initial GPD response may appear less significant. This is a weakness in the PWC approach: it may introduce subjectivity into the design process. Further work is required to establish if this criterion, which is otherwise more consistent and less arbitrary than the TES criterion, can be defined in a form suitable for design application.

In the analyses of the two vessels considered, applying the PWC criterion to 3D elastic-plastic large deformation theory finite element models indicated that an axisymmetric gross plastic deformation mechanism occurs in the knuckle region of the vessels prior to the occurrence of non-axisymmetric elastic-plastic buckling of the knuckle. On the basis of this analysis, the design is limited by the calculated GPD pressure.

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