Design and Real Time Implementation of Nonlinear Minimum Variance Filter

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Abstract: In this paper, the design and real time implementation of a Nonlinear Minimum Variance (NMV) estimator is presented using a laboratory based ball and beam system. The real time implementation employs a LabVIEW based tool. The novelty of this work lies in the design steps and the practical implementation of the NMV estimation technique which up till now only investigated using simulation studies. The paper also discusses the advantages and limitations of the NMV estimator based on the real time application results. These are compared with results obtained using an extended Kalman filter.

Keywords: Polynomial, Estimator, Kalman Filter and Weiner Filter.

1. INTRODUCTION

In control and signal processing community the linear filtering and estimation problem using least squares methods are well known. There are well established linear estimators like Wiener (Wiener, 1949) and Kalman filters (Kalman 1960, 1961). Up till now there are no well established simple and practical techniques for nonlinear estimation. The solution of the *Nonlinear Minimum Variance* estimation problem is an attempt to provide a simple nonlinear estimator suitable for systems with nonlinear channel dynamics.

In Nonlinear Minimum Variance estimation the nonlinear operator is used to represent the channel dynamics and derive the estimator. This is instead of the use of linearization for deriving an approximate nonlinear estimator algorithm. The nonlinear operator may be a set of nonlinear equations or could be a black box model containing unknown code and look-up tables. There may only be a limited number of applications but it is a powerful technique for systems that can be represented in the assumed structure. The cost-function to be minimized involves the variance of the estimation error (Åström,, 1979) and a relatively simple optimization procedure and solution follows.

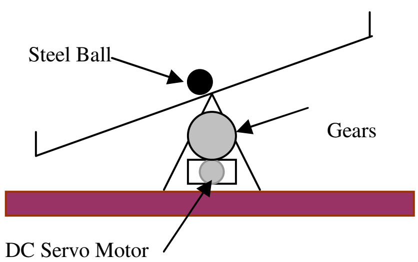

Fig. 1: Ball and Beam System

The system selected for this study is a ball and beam system which is one of the popular test rigs used in laboratories for feedback control studies and education. The system itself is very simple, a ball is placed on a straight beam and rolls back and forth as the beam is raised and lowered by a gear system, as illustrated in Fig. 1. The position of the ball is controlled by changing the angular position of the beam regulated by a DC servo motor. The position of the ball can be measured by using a potentiometer embedded in the beam. The system is open loop unstable (Virseda, 2004 and Olfati-Saber et al, 1998), because the output that represents the ball position tends to infinity for a fixed input. The system is also highly non-linear for large changes in the magnitude of the angle and rate.

In the following study, the position of ball will be estimated using the *Nonlinear Minimum Variance* estimation algorithm. The paper is organized as follows. In section 2, the theory of *Nonlinear Minimum Variance* estimation algorithm is discussed. Section 3 illustrates the experimental framework and results. Conclusions are presented in section 4.

2. NONLINEAR MINMIUM VARIANCE ESTIMATION

The Nonlinear Minimum Variance Estimation technique involves the estimation of a signal that passes through a communications channel having nonlinearities and communication/transport delays (*Grimble* 1995, 2006). The measurements are assumed to be corrupted by a noise signal, which is correlated with the signal to be estimated. Signal and noise models are assumed to be linear and time-invariant. The NMV derivation is based on the minimization of the error variance criterion. Consider the system shown in Fig.2, which includes the *nonlinear* signal channel model and linear measurement noise and signal models.

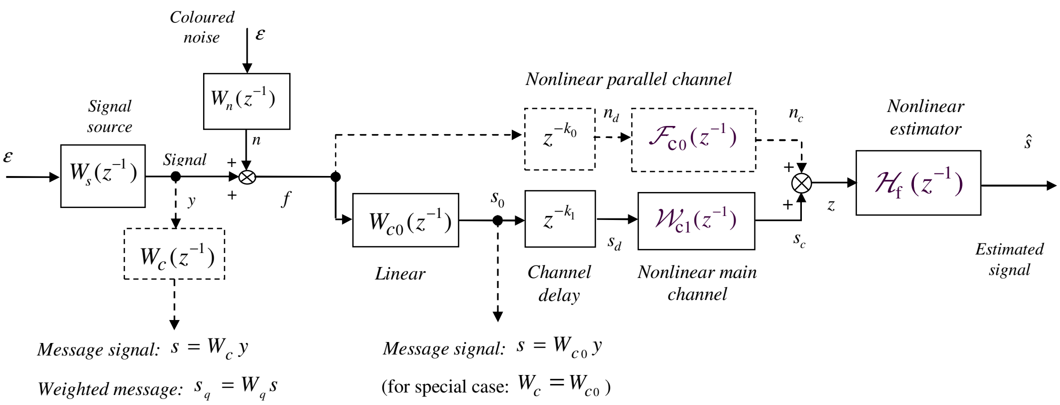

Fig. 2: Signal & Noise Model and Communication Channel Dynamics

The signal channel model includes the nonlinearities that may involve both linear and nonlinear dynamics. The signal channel dynamics with a delay can be expressed as:

$$\left(\mathcal{W}_c f\right)(t) = \left(\mathcal{W}_{c1} z^{-\Lambda_0} W_{c0} f\right)(t) \tag{1}$$

The parallel path dynamics shown in Fig. 2, by a dotted line, can be expressed as.

$$\mathcal{F}_c(z^{-1}) = \mathcal{F}_{c0}(z^{-1}) z^{-\Lambda_0} \tag{2}$$

This is a fictitious channel that can be used to represent uncertainties in channel knowledge, which provides additional design freedom. The combined signal source and noise signal $f(t) \in R^r$ is given as:

$$f(t) = y(t) + n(t) \tag{3}$$

The signals shown in the closed-loop system model of Fig. 2 may be listed as follows:

Noise:	$n(t) = W_n \varepsilon(t)$
Input signal:	$y(t) = W_s \varepsilon(t)$
Channel input:	$f(t) = y(t) + n(t)$
Linear channel subsystem:	$s_o(t) = (W_{co} f)(t)$
Channel interference:	$n_c(t) = (\mathcal{F}_c \varepsilon)(t)$
Nonlinear channel:	$s_c(t) = \left(\mathcal{W}_{c1} s_d\right)(t)$
Nonlinear channel input:	$s_d(t) = z^{-\Lambda_0} s_0(t)$
Observations signal:	$z(t) = n_c(t) + s_c(t)$
Message signal to be estimated:	$s(t) = W_c\, y(t) = W_c W_s \varepsilon(t)$
Weighted message signal:	$s_q(t) = W_q W_c\, y(t)$
Estimation error signal:	$\tilde{s}(t\vert t) = s(t) - \hat{s}(t\vert t)$
Estimation error:	$e(t\,\vert\, t-l) = s(t) - \tilde{s}(t\,\vert\, t-l)$

where $\hat{s}(t\,|\,t-1)$ denotes the estimate of the signal s(t) at time t, given observations z(t) up to time t-1. Value of l may be positive or negative according to the following conditions: $l = 0$, for estimation; $l > 0$, for prediction and $l < 0$, for fixed-lag smoothing. The optimality criterion of minimum variance is given below:

$$J = trace\{E\{W_q \tilde{s}(t\,|\,t-\ell)(W_q \tilde{s}(t\,|\,t-\ell))^T\}\} \tag{4}$$

where $E\{.\}$ denotes the expectation operator and W_q (Grimble, 2005) denotes a linear strictly minimum-phase dynamic cost-function weighting function matrix which is assumed to be strictly minimum phase, square and invertible. The estimate $\hat{s}(t\,|\,t-\ell)$ is assumed to be generated from a nonlinear estimator of the form:

$$\hat{s}(t\,|\,t-\ell) = H_f(t,\, z^{-1}) z(t-\ell) \tag{5}$$

where

$$\mathcal{H}_f(t, z^{-1}) = W_q^{-1} G_0 (\mathcal{F}_{c0} A + \mathcal{W}_{c1}\; W_{c0} D_f)^{-1} \tag{6}$$

While $H_f(t,\, z^{-1})$ denotes a minimal realisation of the optimal nonlinear estimator. Since an infinite-time $(t_o = -\infty)$ problem is of interest therefore no initial condition term is required. The block diagram representation of $H_f(t,\, z^{-1})$ will be as shown in Fig.3.

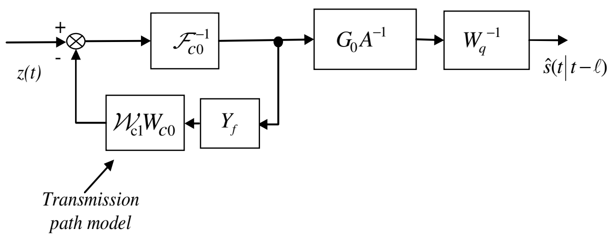

Fig.3: Implementation of the Nonlinear Estimator

The terms G_o, A & D_f used in equation (6) can be calculated by using the concept of *power spectrum* for the combined linear models using: $\Phi_{ff} = (W_s + W_n)(W_s^* + W_n^*)$, where the notation for the adjoint of W_s implies: $W_s^*(z^{-1}) = W_s^T(z)$, and in this case the z denotes the z-domain complex number. The *generalized spectral-factor:* Y_f may be computed using: $Y_f Y_f^* = \Phi_{ff}$, where $Y_f = A_0^{-1} D_{f0} = D_f A^{-1}$. The system models are assumed such that D_{f0} is a *strictly Schur* polynomial matrix (*Kucera* 1979, 1980) satisfying:

$$D_{f0} D_{f0}^* = (C_s + C_n)(C_s^* + C_n^*) \tag{7}$$

The *right-coprime* polynomial matrix model can be defined as:

$$[C_f \quad D_f] A^{-1} = [W_q W_c W_s \quad Y_f] \tag{8}$$

The polynomial operators G_0 now may be obtained from the *minimal degree* solution (G_0, F_0), with respect to F_0, of the following Diophantine equation:

$$F_0 A + G_0 z^{-\Lambda_0 - \ell} = C_f \tag{9}$$

The estimation error can be penalised in a particular frequency range by using a dynamic asymptotically stable weighting function: $W_\Omega = A_\Omega^{-1} B_\Omega$, where A_Ω and B_Ω are polynomial matrices. The modified cost function now will be as follows:

$$J = trace\{E(W_\Omega e(t \mid t-l))(W_\Omega e(t \mid t-l))^T\}$$

$$= trace\{1/(2\pi j) \oint_{|z|=1} (W_\Omega \Phi_{ee} W_\Omega^*) dz / z\} \tag{10}$$

3. IMPLEMENTION & EXPERIMENTAL RESULTS

The experiment was conducted using the equipment shown in Fig. 3. The signal to be estimated by NMV estimation algorithm is the position of ball on the beam.

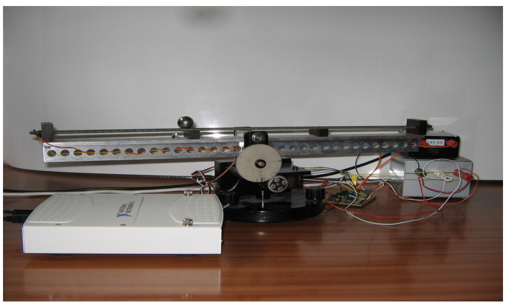

Fig. 3 Ball and Beam Setup

The nonlinearities faced by the estimated signal in the transmitting channel path will be friction, saturation and quantization error as shown in Fig.4. The mathematical models used, during this implementation, are given below.

$$W_s = \frac{0.0047(z^{-1}+z^{-2})}{1-2z^{-1}+z^{-2}} \quad ; \quad W_n = \frac{0.02788\,(z^{-1}-z^{-2})}{1-1.971z^{-1}+0.9721z^{-2}}$$

$$G_o = 0.0051\text{-}0.014z^{-1}+0.013z^{-2}\text{-}0.0039z^{-3}$$

$$A = 0.12\text{-}0.48z^{-1}+0.72z^{-2}\text{-}0.47z^{-3}+0.12z^{-4}$$

$$Y_f = \frac{0.03257z^{-1}\text{-}0.08818z^{-2}+0.07894z^{-3}\text{-}0.02332z^{-4}}{1\text{-}3.971z^{-1}+5.915z^{-2}\text{-}3.916z^{-3}+0.9721z^{-4}}$$

$$\mathcal{F}_{c0}^{-1} = \frac{191\text{-}191z^{-1}\text{-}2.35e\text{-}13z^{-2}+2.9e\text{-}13z^{-3}\text{-}1.4e\text{-}13z^{-4}+1.7e\text{-}14z^{-5}}{1\text{-}4.24z^{-1}+6.98z^{-2}\text{-}5.502z^{-3}+2.023z^{-4}\text{-}0.2608z^{-5}}$$

Sampling Time = 0.125sec

The real time implementation was carried out in LabVIEW while the LabVIEW code developed is shown in Fig.5.

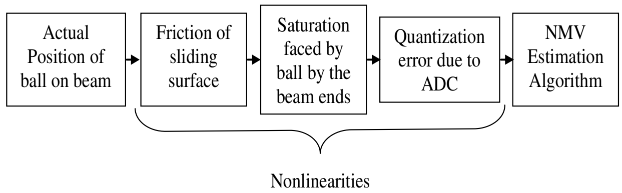

Fig.4. Nonlinearities in measurement channel

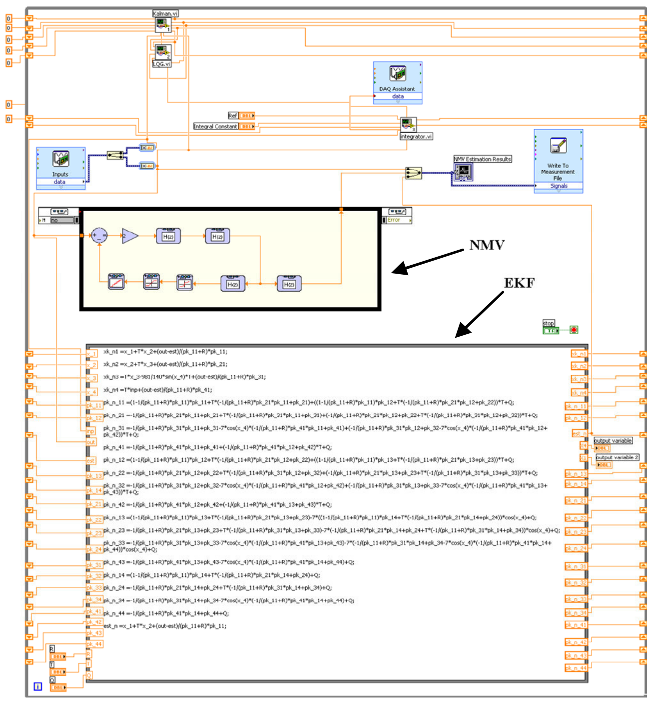

Fig. 5 Block Diagram of LabVIEW Code

The measured and estimated signals are as shown in Fig.6 and Fig. 7, respectively. The minimum variance for the NMV estimator is 8.73e-05 and for the extended Kalman filter (EKF) is 3.36e-02. The NMV filter does not involve linearization around a trajectory but does require a model which represents the global behaviour (at least within the operating regime) of the system. It is not therefore surprising that if the system model matches the real situation that the NMV filter has the possibility of providing improved results relative to the extended Kalman filter. From Figures.6 and.7 we can also see that the performance of NMV estimation is good in comparison to the extended Kalman filter.

There are few advantages which NMV estimation may have over the EKF. There is no formal mechanism within the EKF to allow for uncertainty within the system model. The NMV filter does of course have a parallel path to represent interference or output measurement noise and this may be used to compensate for uncertainties. Secondly one has to calculate the Kalman gain at each iteration, which is computationally quite costly for embedded systems whilst for the NMV filter there is no such requirement. It is true that the spectral factorisation which occurs within the NMV filter can be solved by a Riccati equation. This might suggest that the computations are about the same for both types of estimator. However, this is not correct. The spectral factorisation which is involved is that for the constant coefficient linear models and it would therefore require an off-line (one off) calculation involving a steady state Riccati equation. The EKF, on the other hand, involves what is equivalent to a time-varying Riccati equation solution which has to be performed at each sample instant on-line. The conclusion is that the EKF is computationally more intensive than the NMV filter. Moreover the NMV estimator does not seem so sensitive to parameter tuning on for example the measurement noise covariance Q and process noise covariance R matrices as in the extended Kalman filter (Brown et al, 1992 and Grewal, 1993).

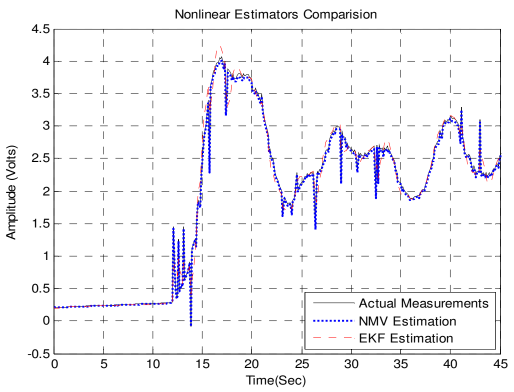

Fig. 6 Comparison Results Between NMV & EKF

4. CONCULSION

In this paper, the theory and practical implementation of NMV estimation has been investigated. It was shown that the NMV filter is relatively better than extended Kalman filter in three respects i.e. it requires less computational cost, easy to implement and requires no special tuning. The results obtained from these observations are encouraging which will lead us to investigate the real time implementation of the multi input and multi output (MIMO) NMV estimator.

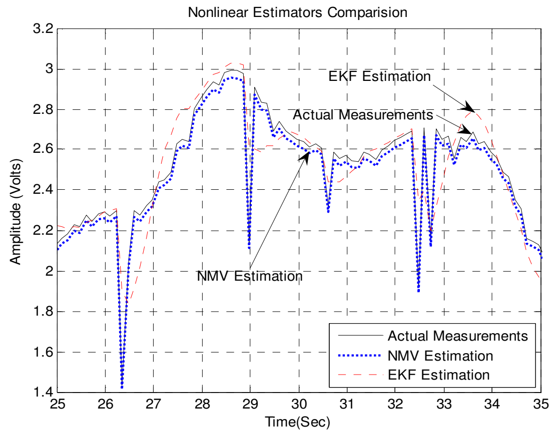

Fig. 7 Comparison Results between NMV & EKF

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