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Introduction to Plasma Accelerators: the Basics

R.Bingham[1,2] *and R. Trines*[1]
[1]Central Laser Facility, Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire, UK
[2]Physics Department, University of Strathclyde, Glasgow, UK

Abstract
In this article, we concentrate on the basic physics of relativistic plasma wave accelerators. The generation of relativistic plasma waves by intense lasers or electron beams in low-density plasmas is important in the quest for producing ultra-high acceleration gradients for accelerators. A number of methods are being pursued vigorously to achieve ultra-high acceleration gradients using various plasma wave drivers; these include wakefield accelerators driven by photon, electron, and ion beams. We describe the basic equations and show how intense beams can generate a large-amplitude relativistic plasma wave capable of accelerating particles to high energies. We also demonstrate how these same relativistic electron waves can accelerate photons in plasmas.

Keywords
Laser; accelerators; wakefields; nonlinear theory; photon acceleration.

1 Introduction

Particle accelerators have led to remarkable discoveries about the nature of fundamental particles, providing the information that enabled scientists to develop and test the Standard Model of particle physics. The most recent milestone is the discovery of the Higgs boson using the Large Hadron Collider—the 27 km circumference 7 TeV proton accelerator at CERN. On a different scale, accelerators have many applications in science and technology, material science, biology, medicine, including cancer therapy, fusion research, and industry. These machines accelerate electrons or ions to energies in the range of tens of megaelectronvolts to tens of gigaelectronvolts. Electron beams with energies from several gigaelectronvolts to tens of gigaelectronvolts are used to generate intense X-rays in either synchrotrons or free electron lasers, such as the Linear Collider Light Source at Stanford or the European XFEL in Hamburg, for a range of applications. Particle accelerators developed in the last century are approaching the energy frontier. Today, at the terascale, the machines needed are extremely large and costly; even the smaller-scale lower energy accelerators are not small. The size of a conventional accelerator is set by the technology used to accelerate the particle and the final energy required. In conventional accelerators, radio-frequency microwave cavities support the electric fields responsible for accelerating charged particles. In these accelerators, owing to electrical breakdown of the walls, the electric field is limited to about $100\ \mathrm{MV\,m^{-1}}$. For more than 30 years, plasma-based particle accelerators driven by either lasers or particle beams have shown great promise, primarily because of the extremely large accelerating electric fields that they can support, about a thousand times greater than conventional accelerators, leading to the possibility of compact structures. These fields are supported by the collective motion of plasma electrons, forming a space charge disturbance moving at a speed slightly below c, the speed of light in a vacuum. This method of particle acceleration is commonly known as plasma wakefield acceleration.

Plasma-based accelerators are the brainchild of the late John Dawson and his colleagues at the University of California, Los Angeles, and are being investigated worldwide with a great deal of success. Will they be a serious competitor and displace the conventional 'dinosaur' variety? The impressive results that have so far been achieved show considerable promise for future plasma accelerators at the energy frontier, as well as providing much smaller 'table-top' ion and electron accelerators. Research on plasma-based accelerators is based on the seminal work by the late John Dawson and his collaborator

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Toshi Tajima [1]. The main advantage of a plasma-based accelerator is that it can support accelerating electric fields many orders of magnitude greater than conventional devices that suffer from breakdown of the waveguide structure, since the plasma is already 'broken down'. The collective electric field E supported by the plasma is determined by the electron density $E \propto n^{1/2}$, where n is the electron density, and is known as an electron plasma wave; the collective electric fields are created by a drive beam that may be either a laser or a charged particle beam. These electron plasma waves travel with a phase speed close to the speed of the drive beam. The electric field strength E of the electron plasma wave is approximately determined by the electron density, $E \propto n^{1/2}$, where n is the density in cm^{-3}; for example, a plasma with density 10^{18} cm^{-3} can support a field of about 10^9 $\mathrm{V\,cm}^{-1}$, a thousand times greater than a radio-frequency accelerator. This translates to a reduction in size of the accelerator and a reduction in cost.

The original plasma accelerator schemes investigated in the 1980s and 1990s were based on a long-pulse laser. Short-pulse lasers did not exist because chirped pulse amplification had not yet been demonstrated in the optical regime, only in the microwave regime. Experiments used the beat-wave mechanism of Tajima and Dawson [1], where two laser beams with a frequency difference equal to the plasma frequency drive a large-amplitude plasma wave. This changed when the process of chirped pulse amplification was ported from microwaves to laser beams by Strickland and Mourou [2, 3]. Suddenly, laser pulses could be produced that were shorter than the plasma wavelength (or skin depth) c/ω_{p}, where ω_{p} is the electron plasma frequency. This led to a dramatic change in the shape of the wakefield, from a 'density ripple' with many periods to a one- or two-period 'bubble-shaped' wakefield [4, 5]. The regime where the pulse length of the driving laser or particle beam is of the order of the plasma wavelength is commonly called the 'bubble' or 'blowout' regime. Most laser-driven and particle-driven particle accelerator experiments today are in this regime, and are commonly known as laser wakefield or beam-driven plasma wakefield accelerators. In the laser wakefield accelerator, the radiation pressure of a short, intense laser beam pushes plasma electrons forward and aside, creating a positively charged ion column. As the laser beam passes the displaced electrons snap back, owing to the restoring force of the ions, and overshoot, setting up a plasma density modulation behind the laser pulse. Similar plasma wakefields are set up by relativistic charged particle beams propagating through uniform plasma. A number of reviews on electron acceleration by laser-driven or beam-driven plasma waves have been published [6–10].

Early experiments produced beams with large energy spread, but in 2004 three independent groups in three different countries demonstrated laser wakefield acceleration producing mono-energetic electron beams with good emittance using short-pulse lasers [11–13], a result predicted by Pukhov and Meyer-ter-Vehn [14] and Tsung *et al.* [15]. Many groups worldwide now routinely produce electron beams at gigaelectronvolt energies using this scheme [16–18]. Similar plasma wakefields are set up by relativistic charged particle beams propagating through uniform plasma [19, 20]. In 2007, Joshi's group at the University of California demonstrated acceleration of electrons in metre-long plasma columns using a SLAC charged particle beam as a driver. This resulted in particles near the back of the electron beam doubling their energy from 42 GeV to 85 GeV in a 1 m long lithium plasma [21]. This is a remarkable result, since it takes 3 km of the SLAC linac to accelerate electrons to 42 GeV. The plasma beam-driven wakefield is incorporated into the latest round of experiments at SLAC by a consortium called the Facility for Advanced Accelerator Experimental Tests (FACET) [8]. Both electron and positron beams are accelerated in this facility [22, 23]. Beam-driven plasma wakefields also underpin the proton beam-driven wakefield experiment, AWAKE, which will use the proton beam from CERN's Super Proton Synchrotron and a 10 m long plasma column to produce gigaelectronvolt electrons [20, 24].

Today most experiments are conducted in the bubble regime. This includes experiments at the Berkeley Laboratory Laser Accelerator Center at Lawrence Berkeley Laboratory [18, 25] and the Rutherford Appleton Laboratory's Central Laser Facility [17,26,27], as well as many other laser-plasma accelerator experiments around the globe. These experiments have demonstrated mono-energetic electron beams at the gigaelectronvolt scale and planned experiments using lasers will demonstrate acceleration

of electrons to 10 GeV. Recently, FACET experiments demonstrated high efficiency in electron beam production, where the energy transfer from the wakefield to the accelerated bunch exceeded 30% with a low energy spread [22]. Similar impressive results using positrons have also been demonstrated [23].

Despite the successes of these experiments, it is still necessary to improve beam quality, in particular, to produce low energy spread and low emittance, and improve beam focusing. Most of the experiments are guided by plasma simulations that involve high-performance computing clusters. Commonly used simulation codes include the particle-in-cell codes Osiris [28–30], VLPL [31], Vorpal [32], and Epoch [33]. These simulations have already predicted that between 10 and 50 GeV electron beams can be created in one stage of a plasma accelerator.

If plasma accelerators are to take over from conventional machines, a great deal of effort still needs to be put into efficient drivers. Suitable laser efficiencies and pulse rates seem likely with diode-pumped lasers or with fibre lasers, but effort has to be put into these schemes to meet the requirements necessary to drive a wakefield. For beam-driven systems, electron beams at 100 GeV and proton beams with teraelectronvolt energies are required. These exist at the Large Hadron Collider for protons and at FACET for electrons. For an electron–positron system, a key challenge is positron acceleration; some groups are investigating positron acceleration in wakefields. Alternatively, an electron-proton collider or a photon–photon (γ–γ) collider could be built, doing away with the need for positrons, thus saving time and effort.

In the next section, we will discuss the short-pulse laser wakefield accelerator scheme. We will present basic analytical theory that lays the groundwork for all subsequent investigations into laser-driven and beam-driven wakefield acceleration, and provide results from particle-in-cell simulations of three-dimensional laser wakefield acceleration.

2 The laser wakefield accelerator (LWFA)

In the laser wakefield accelerator (LWFA), a short laser pulse, whose frequency is much greater than the plasma frequency, excites a wake of plasma oscillations (at $\omega_{\rm p}$), owing to the ponderomotive force, much like the wake of a motorboat. Since the plasma wave is not resonantly driven, as in the beat-wave, the plasma density does not have to be of a high uniformity to produce large-amplitude waves. We start from an intense laser pulse with electric field amplitude E_0 and frequency ω_0, and define $a_0 \equiv eE_0/(m_{\rm e}\omega_0 c)$. As this pulse propagates through an underdense plasma, $\omega_0 \gg \omega_{\rm p}$, the relativistic ponderomotive force associated with the laser envelope, $F_{\rm pond} \simeq -\frac{1}{2}mc^2(\nabla a_0^2)/\sqrt{1+a_0^2}$, expels electrons from the region of the laser pulse and excites electron plasma waves. These waves are generated as a result of being displaced by the leading edge of the laser pulse. If the laser pulse length, $c\tau_{\rm L}$, is long compared with the electron plasma wavelength, the energy in the plasma wave is re-absorbed by the trailing part of the laser pulse. However, if the pulse length is approximately equal to or shorter than the plasma wavelength $c\tau_{\rm L} \simeq \lambda_{\rm p}$, the ponderomotive force excites plasma waves or wakefields, with a phase velocity equal to the laser group velocity, which are not re-absorbed. Thus, any pulse with a sharp rise or a sharp fall on a scale of $c/\omega_{\rm p}$ will excite a wake. With the development of high-brightness lasers, the laser wakefield concept first put forward by Tajima and Dawson [1] in 1979 has now become a reality. The focal intensities of such lasers are $\geq 10^{19}$ W cm^{-2}, with $a_0 \geq 1$, which is the strong non-linear relativistic regime. Any analysis must, therefore, be in the strong non-linear relativistic regime and a perturbation procedure is invalid.

The maximum wake electric field amplitude generated by a plane-polarized pulse has been given by Sprangle *et al.* [34] in the one-dimensional limit as $E_{\rm max} = 0.38a_0^2\left(1+a_0^2/2\right)^{-1/2}\sqrt{n_0}$ V cm^{-1}. For $a_0 \approx 4$ and $n_0 = 10^{18}$ cm^{-1}, then $E_{\rm max} \approx 2$ GV cm^{-1}, and the time to reach this amplitude level is of the order of the laser pulse length.

2.1 Model equations describing laser wakefield excitation

To understand the laser wakefield excitation mechanism, it is sufficient to use a model based on one-fluid, cold relativistic hydrodynamics, and Maxwell's equations, together with a 'quasi-static' approximation, a set of two coupled non-linear equations describing the self-consistent evolution in one dimension of the laser pulse vector potential envelope, and the scalar potential of the excited wakefield. Starting from the equation for electron momentum,

$$\frac{\partial \mathbf{p}}{\partial t} + v_z \frac{\partial \mathbf{p}}{\partial z} = -\left(e\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} \right) , \tag{1}$$

where

$$\mathbf{p} = m_0 \gamma \mathbf{v},\ \gamma = \left(1 + p^2/m_0^2 c^2\right)^{1/2} ,$$

m_0 and $\mathbf{v}$ being the electron rest mass and velocity.

In Eq. (1), we have assumed that all quantities only depend on z and t, z being the direction of propagation of the (external) pump and

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}_\perp}{\partial z} - \hat{z}\frac{\partial \phi}{\partial z}\ ;\ \mathbf{B} = \nabla \times \mathbf{A}_\perp;\ \ \mathbf{A}_\perp = \hat{x}A_x + \hat{y}A_y\ , \tag{2}$$

where $\mathbf{A}_\perp$ is the vector potential of the electromagnetic pulse and ϕ is the ambipolar potential due to charge separation in the plasma.

Using Eqs. (1) and (2), the perpendicular component of the electron momentum is found to be

$$\frac{p_\perp}{m_0 c} = \frac{e}{m_0 c^2}\mathbf{A}_\perp \equiv \mathbf{a}(z,t)\ , \tag{3}$$

and we can write

$$\gamma = \left[1 + \left(\frac{p_\perp}{m_0 c}\right)^2 + \left(\frac{p_z}{m_0 c}\right)^2 \right]^{1/2} \equiv \gamma_a \gamma_\parallel\ , \tag{4}$$

where

$$\gamma_a = \left(1 + \mathbf{a}^2\right)^{1/2};\ \ \gamma_\parallel = \left(1 - \beta^2\right)^{-1/2}\ , \tag{5}$$

and $\beta = v_z/c$.

The equations derived from this model are now the longitudinal component of Eq. (1), the equation of continuity, Poisson's equation, and the wave equation for $\mathbf{a}(z,t)$, which are given by

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\gamma_a \sqrt{\gamma_\parallel^2 - 1}\right) + \frac{\partial}{\partial z}\left(\gamma_a \gamma_\parallel\right) = \frac{\partial \phi}{\partial z};\ \varphi \equiv \frac{e\phi}{m_0 c^2}\ , \tag{6}$$

$$\frac{1}{c}\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}\left(n\sqrt{1 - 1/\gamma_\parallel^2}\right) = 0\ , \tag{7}$$

$$\frac{\partial^2 \varphi}{\partial z^2} = \frac{\omega_{\mathrm{p0}}^2}{c^2}\left(\frac{n}{n_0} - 1\right)\ , \tag{8}$$

$$c^2 \frac{\partial^2 \mathbf{a}}{\partial z^2} - \frac{\partial^2 \mathbf{a}}{\partial t^2} = \omega_{\mathrm{p0}}^2 \frac{n}{n_0}\frac{\mathbf{a}}{\gamma_a \gamma_\parallel}\ . \tag{9}$$

Assuming a driving pulse of the form

$$\mathbf{a}(z,t) = \frac{1}{2}\mathbf{a}_0(\xi,\tau)\mathrm{e}^{-\mathrm{i}\theta} + \text{c.c.}, \tag{10}$$

where $\theta = \omega_0 t - k_0 z$, ω_0 and k_0 being the central frequency and wavenumber, $\xi = z - v_{\mathrm{g}} t$, $v_{\mathrm{g}} = \partial\omega_0/\partial k_0$ is the group velocity and τ is a slow time-scale, such that

$$a_0^{-1}\frac{\partial^2 a_0}{\partial \tau^2} \ll \omega_0^2\ .$$

Accounting for changes in the pump due to the plasma reaction, the wave equation becomes

$$\left[2\frac{\partial}{\partial\tau}\left(\mathrm{i}\omega_0 a_0 + v_\mathrm{g}\frac{\partial a_0}{\partial\xi}\right) + c^2\left(1 - v_\mathrm{g}^2/c^2\right)\frac{\partial^2 a_0}{\partial\xi^2} + 2\mathrm{i}\omega_0\left(\frac{c^2 k_0}{\omega_0} - v_\mathrm{g}\right)\frac{\partial a_0}{\partial\xi}\right]\mathrm{e}^{-\mathrm{i}\theta} + \mathrm{c.c.}$$
$$= \left[c^2 k_0^2 - \omega_0^2 + \frac{n}{n_0}\omega_{\mathrm{p}0}^2 > \gamma_a\gamma_\parallel\right] a_0 \mathrm{e}^{-\mathrm{i}\theta} + \mathrm{c.c.}\ , \quad (11)$$

where $\omega_{\mathrm{p}0}$ is the plasma frequency of the unperturbed plasma. Equations (6), (7), (8), and (11) form the basic set for this problem in the 'envelope approximation'.

In the weak-pump, weakly relativistic regime, the solution has the structure of a wakefield growing inside the electromagnetic pulse and oscillating behind the pulse, with the maximum amplitude being reached inside the pulse. Using the quasi-static approximation, the time derivative can be neglected in the electron fluid equations, Eqs. (6) and (7), yielding the following constants:

$$\gamma_a\left(\gamma_\parallel - \beta_0\sqrt{\gamma_\parallel^2 - 1}\right) - \varphi = 1\ , \quad (12)$$

$$n\left(\beta_0\gamma_\parallel - \sqrt{\gamma_\parallel^2 - 1}\right) = n_0\beta_0\gamma_\parallel\ , \quad (13)$$

where $\beta_0 = v_\mathrm{g}/c$. The constants of integration have been chosen in such a way that

$$n = n_0,\ \gamma_\parallel = 1,\ \varphi = 0\ ,$$

for

$$\gamma_a = 1, \qquad \left(|a_0|^2 = 0\right)\ . \quad (14)$$

Using Eqs. (10) and (13), the general system, Eqs. (6)–(9), can be written as two coupled equations, describing the evolution of the laser pulse envelope $\mathbf{a}_0$ and the scalar potential φ:

$$\frac{\partial^2\varphi}{\partial\xi^2} = \frac{\omega_{\mathrm{p}0}^2}{c^2}G\ , \quad (15)$$

$$2\mathrm{i}\omega_0\frac{\partial a_0}{\partial\tau} + 2c\beta_0\frac{\partial^2 a_0}{\partial\tau\partial\xi} + \frac{c^2\omega_{\mathrm{p}0}^2}{\omega_0^2}\frac{\partial^2 a_0}{\partial\xi^2} = -\omega_{\mathrm{p}0}^2 H a_0\ , \quad (16)$$

where

$$G = \frac{\sqrt{\gamma_\parallel^2 - 1}}{\beta_0\gamma_\parallel - \sqrt{\gamma_\parallel^2 - 1}}\ ,\ H = 1 - \frac{\beta_0}{\gamma_a\left(\beta_0\gamma_\parallel - \sqrt{\gamma_\parallel^2 - 1}\right)}\ .$$

This set of non-linear equations, Eqs. (15) and (16), is obtained using a quasi-static approximation, which yields two integrals of the motion, given by Eqs. (12) and (13). The model is valid for electromagnetic pulses of arbitrary polarization and intensities $|a_0|^2 \geq 1$.

Equations (15) and (16) can be solved numerically in the stationary frame of the pulse. Equation (15), Poisson's equation for the wakefield, is solved with the initial conditions $\varphi = 0$, $\partial\varphi/\partial\xi = 0$ by a simple predictor–corrector method. The envelope equation, Eq. (16), describing the evolution of the laser pulse, is written as two coupled equations for the real and imaginary parts of a_0 and solved implicitly.

Numerical solutions of Eqs. (15) and (16) show the evolution of the excited plasma wakefield potential φ and electric field E_w, as well as the envelope of the laser pulse $|a_0|$ [35], all in one spatial dimension. Solutions for a typical laser and plasma configuration are shown in Fig. 1. There is significant distortion of the trailing edge of the laser pulse, resulting in photon spikes. The distortion occurs where the wake potential has a minimum and the density has a maximum. The spike arises as a result of the photons interacting with the plasma density inhomogeneity, with some photons being accelerated (decelerated) as they propagate down (up) the density gradient. This effect was predicted by John Dawson

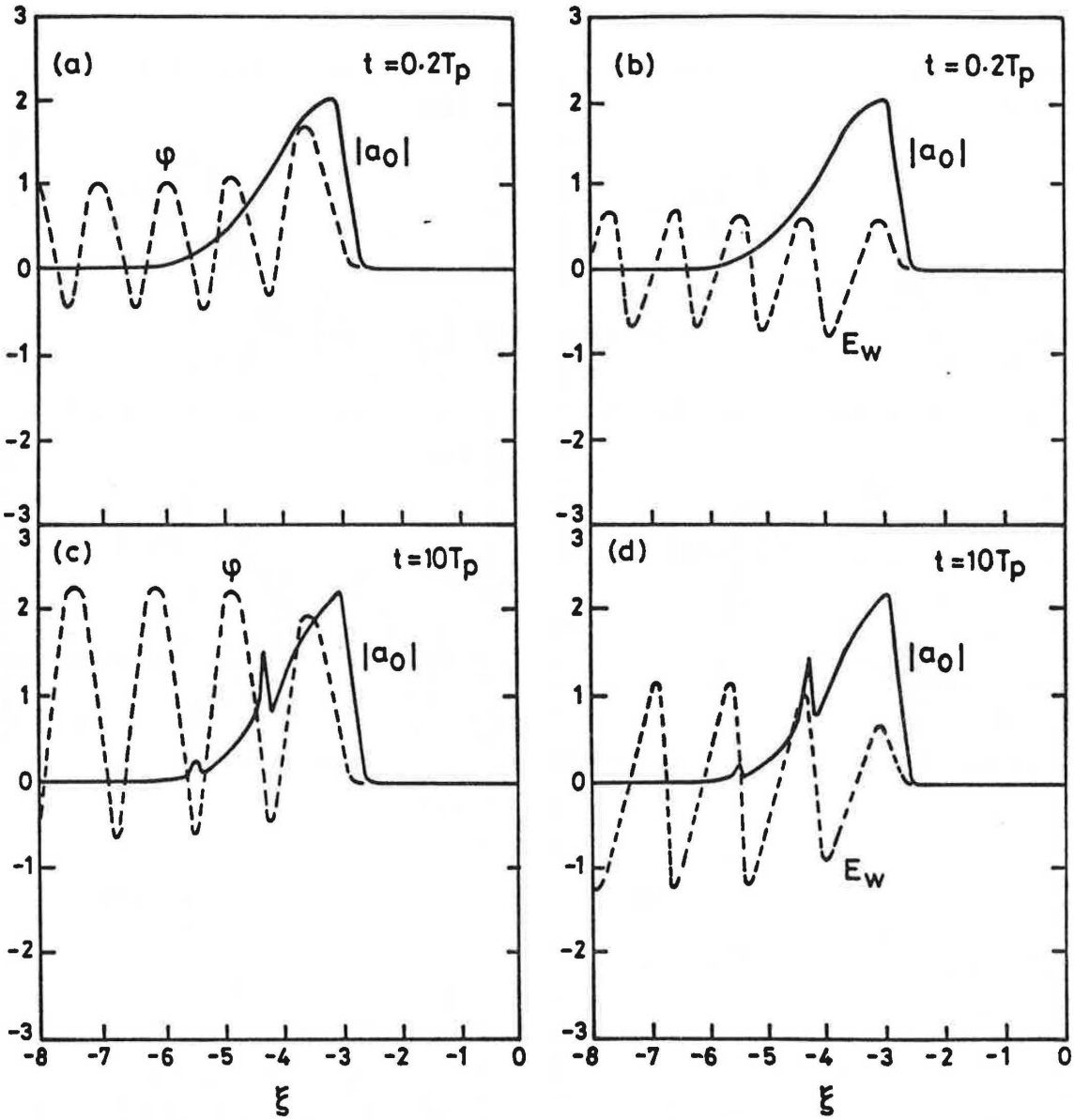

Fig. 1: The values of the magnitude of the normalized vector potential $|a_0|$ (solid curves) and scalar potential φ or wake electric field $E_{\rm w}$ (dashed curves) versus position $\xi = z - v_{\rm g}t$. Here, $|a_0^{\rm in} = 2|$, $\omega_{\rm p0}/\omega_0 = 1$, Gaussian rise $\sigma_{\rm r} = 0.25\lambda_{\rm p}$, Gaussian fall $\sigma_{\rm f} = 1.5\lambda_{\rm p}$. Curves (a) and (b) are at time $t = 0.2T_{\rm p}$; (c) and (d) are at $t = 10T_{\rm p}$.

and his group, and is called the photon accelerator [36]. The distortion of the trailing edge increases with increasing $\omega_{\rm p0}/\omega_0$. The longitudinal potential, $e\phi/(mc^2) > 1$ or $eE_z/(m_{\rm e}\omega_{\rm p0}c) > 1$, is significantly greater than for fields obtained in the plasma beat-wave accelerator. The field amplitude for the beat-wave accelerator is limited by relativistic detuning, while no such saturation exists in the laser wakefield accelerator.

When studying wakefields in more than one spatial dimension, the transverse dimensions of the driving laser pulse or particle beam become important, and the wakefield will assume a characteristic 'bubble' shape, provided the driver is sufficiently short. The bubble regime of plasma-based acceleration has been studied extensively using both analytical theory and full-scale numerical simulations [37–42]. A typical example of a three-dimensional laser-driven bubble-shaped wakefield can be seen in Fig. 2.

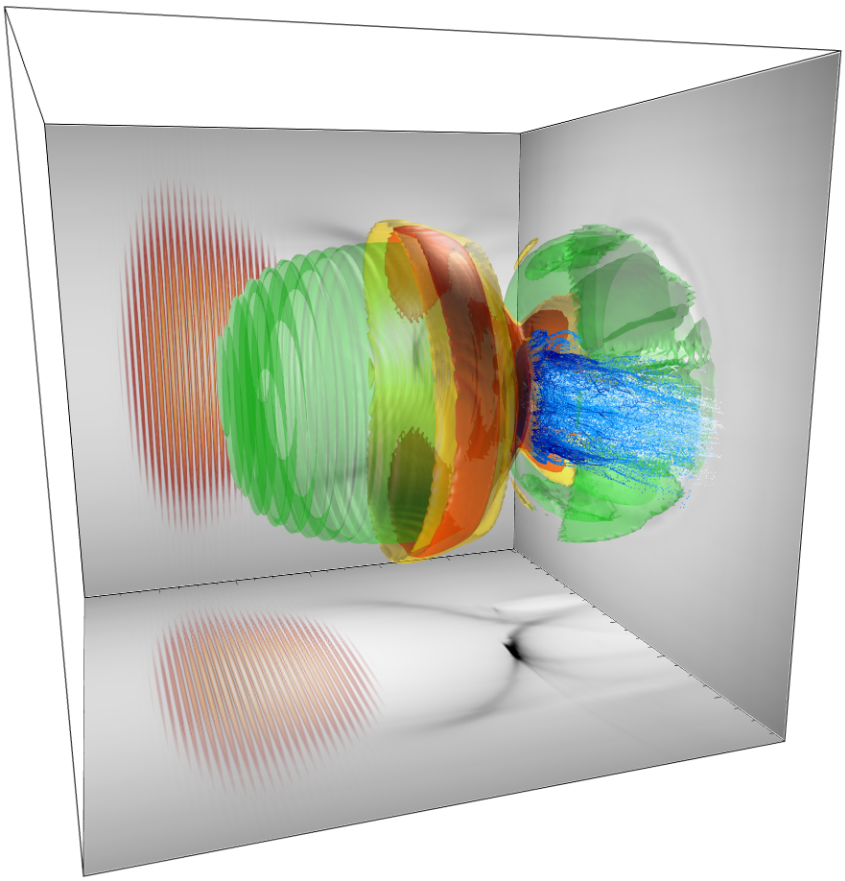

Fig. 2: An example of a three-dimensional wakefield generated by a short, intense laser pulse. Red and yellow, electromagnetic fields; green, electron density; blue, electrons trapped and accelerated by the wakefield. © J. Vieira, IST Lisbon, Portugal.

The electromagnetic fields are coloured red and yellow and the background plasma electron density is coloured green, while the population of electrons trapped and accelerated by the wakefield is coloured blue.

3 Photon acceleration

Photon acceleration is the phenomenon whereby photons interacting with a co-propagating plasma wave can increase or decrease their frequency, and thus their energy. The group speed of photons in plasma is given by $v_{\rm g} = c^2 k/\omega = c\sqrt{1-\omega_{\rm p}^2/\omega_0^2} < c$. Thus, when the photon frequency increases, its group speed in plasma also increases, hence the photon accelerates. Photon acceleration [36, 43] is intimately related to short-pulse amplification of plasma waves and is described by a similar set of equations. It has been demonstrated that a relativistic plasma wave can be generated by a series of short electromagnetic pulses [44]. These pulses have to be spaced in a precise manner to give the plasma wave the optimum 'kick'. Conversely, if the second or subsequent photon pulses are put in a different position, 1.5 plasma wavelengths behind the first pulse, the second pulse will produce a wake that is 180° out of phase with the wake produced by the first pulse. The superposition of the two wakes behind the second pulse results in a lowering of the amplitude of the plasma wave. In fact, almost complete cancellation can take place. The second laser pulse has absorbed some or all of the energy stored in the wake created by the first pulse. From conservation of photon number density, the increase in energy of the second pulse implies that its frequency has increased. The energy in the pulse is $W = N\hbar\omega$, where N is the total number of

photons in the packet. Wilks *et al.* [36] have shown that the increase in frequency of the 'accelerated' wave packet exactly accounts for the loss in the energy of the accelerating plasma wave, given by

$$\frac{\delta\omega}{\delta x} = \frac{\omega_{\rm pe}^2 k_{\rm p}}{2\omega}\frac{\delta n}{n_0} \,. \tag{17}$$

Computer simulations by Wilks *et al.* [36] show that the laser pulse increased in frequency by 10% over a distance $237c/\omega_{\rm p}$.

Photon acceleration also plays a role in laser modulational instability [45–47]. This instability occurs when the energy of a long laser pulse is bunched longitudinally by a co-propagating plasma wave. The underlying process is periodic acceleration and deceleration of the laser's photons by the density fluctuations of the plasma wave. In turn, the laser's ponderomotive force helps to enhance the plasma wave, creating a positive feedback loop (see also Ref. [48] for the relationship between the ponderomotive force and the photon dispersion in plasma). The laser amplitude modulation due to this instability can be seen in Fig. 1, frames (c) and (d).

Photon acceleration of laser light in laser wakefield interactions was first observed by Murphy *et al.* [49]. The role of photon acceleration in the laser modulational instability was studied by Trines *et al.* [50,51], while aspects of the numerical modelling of wave kinetics have been investigated by Reitsma *et al.* [52, 53].

Photon acceleration can be used as a single-shot diagnostic tool to probe laser- or beam-driven wakefields and determine their characteristics [36,49,50]. This is currently pursued within the framework of the AWAKE project [24]. Promising developments have been reported by Kasim *et al.* [54, 55].

4 Relativistic self-focusing and optical guiding

In the absence of optical guiding, the interaction length L is limited by diffraction to $L = \pi R$, where R is the Rayleigh length $R = \pi\sigma^2/\lambda_0$, and σ is the focal spot radius. This limits the overall electron energy gain in a single stage to $E_{\rm max}\pi R$. To increase the maximum electron energy gain per stage, it is necessary to increase the interaction length. Two approaches to keeping high-energy laser beams collimated over a longer region of plasma are being developed. Relativistic self-focusing can overcome diffraction and the laser pulse can be optically guided by tailoring the plasma density profile, forming a plasma channel.

Relativistic self-focusing uses the non-linear interaction of the laser pulse and plasma results in an intensity-dependent refractive index to overcome diffraction. In regions where the laser intensity is highest, the relativistic mass increase is greatest; this results in a reduction of the fundamental frequency of the laser pulse. The reduction is proportional to the laser intensity. Correspondingly, the phase velocity of the laser pulse will decrease in regions of higher intensity. This has the effect of focusing a laser beam with a radial Gaussian profile. This results in the plane wavefront bending and focusing to a smaller spot size. Relativistic self-focusing has a critical laser power threshold, $P_{\rm cr}$, which must be exceeded; this is given by [56]

$$P \geq P_{\rm cr} \sim 17\frac{\omega_0^2}{\omega_{\rm p}^2}\ \text{GW} \,. \tag{18}$$

The laser must also have a pulse length (τ) that is shorter than both a collision period and an ion plasma period, to avoid the competing effects of thermal and ponderomotive self-focusing; this condition is $\tau_{\rm L} \leq 5\times 10^9\sqrt{Z/n_0}$ ps. The shape of the self-focused pulse for intense short pulses is also interesting; at the leading edge the non-linear response is not yet established, there is a finite time for the electrons to respond, and the front of the pulse propagates unchanged. The trailing edge of the pulse compresses radially, owing to the non-linear relativistic self-focusing. It is only the trailing edge of the pulse that is channelled. Another way of forming a guided laser is to perform a density cavity with one short-pulse laser forming a plasma density channel. Alternatively, the channel could be formed by a low-current electron beam, which produces the same effect.

One drawback of the short-pulse laser–plasma interaction is that, at the extreme intensities used, a hole in electron density can be created. This would prevent a wake from being created, or it might result in a very small wake, if the residual density were many orders of magnitude smaller than the original density.

5 Discussion

Plasma acceleration processes continue to be an area of active research. Initial studies of particle acceleration have proved fruitful for current drive schemes and laser accelerators. Particle acceleration in strongly turbulent plasmas is still in its infancy and requires a great deal more research. This area of research is important in astrophysical and space plasma.

Current and future experiments, however, are very far from the parameter range of interest to high-energy physicists, who require something like 10^{11} particles per pulse accelerated to teraelectronvolt energies (for electrons), with a luminosity of 10^{-34} cm^{-2} s^{-1} for acceptable event rates to be achieved. The teraelectronvolt energy range is more than 100 times greater than a single accelerating stage could provide at present; even if the interaction length could be extended by laser channelling, there would still be the requirement of multiple staging, and the need for more energetic lasers.

Researchers, realizing that the next collider will almost certainly be a linear electron–positron collider, are proposing a novel way of building such a device, known as the 'plasma afterburner' concept [57, 58]. Several groups are also developing an entirely new type of electron lens, using focusing by a plasma, to increase the luminosity of future linear colliders [59].

This plays on the fact that relativistic electron beams can be focused by a plasma if the collisionless skin depth $c/\omega_{\rm pe}$ is larger than the beam radius. Generally, when a relativistic electron beam enters a plasma, the plasma electrons move to neutralize the charge in the beam on a $1/\omega_{\rm pe}$ time-scale. However, if the collisionless skin depth is larger than the beam radius, the axial return current flows in the plasma on the outside of the electron beam and the beam current is not fully neutralized, leading to the generation of an azimuthal magnetic field. Consequently, this self-generated magnetic field pinches or focuses the beam in the radial direction. This type of lens exceeds conventional lenses by several orders of magnitude in focusing gradient.

At present, laser-plasma accelerators are being used to accelerate electrons from hundreds of megaelectronvolts to gigaelectronvolts, and to create intense X-ray radiation via betatron motion of the accelerating electrons [26]. A future milestone to be achieved will be the 10 GeV energy level with good beam quality. Electrons in this energy range are ideal as a driver for free electron lasers; at the higher energies predicted, several gigaelectronvolts, it is possible to produce an X-ray free electron laser capable of biological investigations around the water window. Many other applications of plasma-based acceleration are also already being discussed [60].

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