Atomic interaction effects in the superradiant light scattering from a Bose-Einstein condensate

N. Piovella, L. Salasnich, R. Bonifacio

Dipartimento di Fisica, Università Degli Studi di Milano and INFM,

Via Celoria 16, Milano 20133, Italy

G.R.M. Robb

Department of Physics, University of Strathclyde Glasgow, G4 0NG, Scotland.

Abstract

We investigate the effects of the atomic interaction in the Superradiant Rayleigh scattering from a Bose-Einstein condensate driven by a far-detuned laser beam. We show that for a homogeneous atomic sample the atomic interaction has only a dispersive effect, whereas in the inhomogeneous case it may increase the decay of the matter-wave grating.

I. INTRODUCTION

The long coherence time of the Bose-Einstein condensate (BEC), now routinely produced in many laboratories, offers the possibility to study the collective motion induced by external radiation beams [1]. In particular, a single far-off resonance laser sent on an elongated BEC produces superradiant Rayleigh scattering [2, 3, 4], generating coherent backscattered radiation and splitting the condensate into fractions moving at velocities differing by multiples of $2\hbar k/m$, where $k = \omega/c$ is the wave-vector of the laser incident along the symmetry axis of the condensate, ω is the laser frequency and m is the atomic mass. Superradiant Rayleigh scattering from a BEC is the quantum analog of the collective atomic recoil laser (CARL) [5] in which the emitted radiation is not confined in a high-Q ring cavity [6]. The complete absence in a BEC of Doppler broadening due to thermal motion allows for a regime in which the atoms scatter a single laser photon and recoil with an extra momentum of $2\hbar k$ in the direction of the incident photon. The number of scattered photons and the amplitude of the density grating resulting from the interference between the two atomic wavepackets with momentum difference $2\hbar k$ are exponentially enhanced via the CARL instability [7]. In the absence of any mechanism of atomic dephasing, the process is sequential [2, 8], with a complete transfer of atoms, after the superradiant process, from the original motional state with momentum p to a state with a momentum $p + 2\hbar k$. However, in a real BEC several mechanisms contribute to the decay of the coherence between the two momentum states. Some of them are due to the decoherence induced by spontaneous emission [9] or phase diffusion [4]. The main characteristic of this kind of decoherence is irreversibility. Other mechanisms arise from inhomogeneous broadening, as those due to a finite size of the condensate wavefunction, responsible for a broadening of the atomic momentum distribution, and from mean-field broadening due to the atomic interaction [10]. It has been recently suggested that the dephasing due to inhomogeneous broadening can be reversed applying, after the superradiant scattering process, a Bragg pulse of area π inducing a superradiant echo and a further tranfer of the atoms to the final momentum state [11]. In this paper we investigate the effects of the mean-field atomic interaction on the superradiant scattering process.

II. BASIC MODEL

We consider an elongated Bose-Einstein condensate driven by a single laser incident along the positive direction of the symmetry axis z of the condensate. The laser is far-detuned from the atomic resonance, so that radiation pressure due to absorption and subsequent random incoherent, isotropic emission of a photon, can be neglected. In this regime, the atoms backscatter photons at frequency ω_s and wave vector $k_s = \omega_s/c \approx k$, recoiling with a momentum $2\hbar k$ along the same direction of the incident laser beam.

In a simplified 1D description of the process along the axis z, the evolution of the matter-wave field $\Psi(z,t)$ and of the dimensionless amplitude $a(t)$ of the scattered radiation is determined by the following self-consistent equations:

$$i\frac{\partial \Psi}{\partial t} = -\omega_r \frac{\partial^2 \Psi}{\partial \theta^2} + ig\left[a^* e^{i(\theta - \delta t)} - \text{c.c.}\right]\Psi + 2\pi\beta|\Psi|^2\Psi \tag{1}$$

$$\frac{da}{dt} = gN\int d\theta |\Psi|^2 e^{i(\theta - \delta t)} - \kappa a. \tag{2}$$

where $\theta = 2kz$, $a = (\epsilon_0 V/2\hbar\omega_s)^{1/2}E$ is the dimensionless electric field amplitude of the scattered beam with frequency ω_s, $\omega_r = 2\hbar k^2/m$ is the two-photon recoil frequency, $g = (\Omega/2\Delta_0)(\omega d^2/2\hbar\epsilon_0 V)^{1/2}$ is the coupling constant, Ω is the Rabi frequency of the laser beam of frequency $\omega = ck$, detuned from the atomic resonance frequency ω_0 by $\Delta_0 = \omega - \omega_0$, d is the electric dipole moment of the atom along the laser polarization direction, V is the volume of the condensate containing N atoms, $\delta = \omega - \omega_s$ and ϵ_0 is the permittivity of the free space. The second term on the right hand side of Eq.(1) is the self-consistent optical lattice, resulting from the interference between the laser and the backscattered radiation, whose amplitude is amplified by the matter-wave grating described by the first term on the right hand side of Eq.(2). The matter-wave field Ψ is normalized to one, i.e. $\int d\theta |\Psi(\theta,t)|^2 = 1$, and the last term on the right hand side of Eq.(1) describes the atomic interaction due to binary collisions, where $\beta = 4\hbar k a_s N/m\Sigma$, a_s is the scattering length and Σ is the condensate cross section. Eq.(2) has been written in the "mean-field" limit, which models the propagation effects of the light by replacing the nonuniform electric field by an average value and by adding to the equation a damping term with decay constant $\kappa \approx c/2L$, where L is the condensate length and c is the speed of light in vacuum.

III. HOMOGENEOUS CASE

If the condensate is much longer than the radiation wavelength and the density is uniform, then periodic boundary conditions can be assumed on θ and the wavefunction can be written as a Fourier series

$$\Psi(\theta, t) = \sum_n c_n(t) u_n(\theta) e^{-in\delta t}, \tag{3}$$

where $u_n(\theta) = (1/\sqrt{2\pi}) \exp(in\theta)$ are momentum eigenfunctions with eigenvalues $p_z = n(2\hbar k)$. Using Eq.(3), Eqs.(1) and (2) reduce to an infinite set of ordinary differential equations,

$$\dot{c}_n = -i\delta_n c_n + g(a^* c_{n-1} - a c_{n+1}) - i\beta \sum_{m,l} c_m c_l c^*_{m+l-n} \tag{4}$$

$$\dot{a} = gN \sum_n c_n c^*_{n+1} - \kappa a, \tag{5}$$

where $\delta_n = n^2 \omega_r - n\delta$ and the dot indicates the time derivative.

A. Two-level approximation

Assuming that the only two momentum levels involved in the process are the initial level n and the final level $n+1$, Eq.(4) and (5) reduce to:

$$\dot{c}_n \approx -i \left[\delta_n + \beta \left(|c_n|^2 + 2|c_{n+1}|^2 \right) \right] c_n - g a c_{n+1} \tag{6}$$

$$\dot{c}_{n+1} \approx -i \left[\delta_{n+1} + \beta \left(2|c_n|^2 + |c_{n+1}|^2 \right) \right] c_{n+1} + g a^* c_n \tag{7}$$

$$\dot{a} \approx gN c_n c^*_{n+1} - \kappa a. \tag{8}$$

Defining $S = c_n c^*_{n+1}$ and $W = |c_n|^2 - |c_{n+1}|^2$, we obtain from Eqs.(6)-(8):

$$\dot{S} = -i (\Delta - \beta W) S + g a W - \gamma S \tag{9}$$

$$\dot{W} = -2g (a S^* + \text{c.c.}) \tag{10}$$

$$\dot{a} = gNS - \kappa a, \tag{11}$$

where $\Delta = \delta_n - \delta_{n+1} = \delta - \omega_r(2n+1)$ and we have introduced a damping term in Eq.(9), to account for the decay of the coherence between the two motional states n and $n+1$. We note that when the atomic interaction is neglected ($\beta = 0$), $\Delta = 0$ is the Bragg condition of the scattering process, arising from momentum and energy conservation [12]. We observe

4

from Eq.(9) that the atomic interaction term has a dynamical dispersive effect on the Bragg resonance, proportional to the population difference W. In the linear regime, when a is still small and $W \approx 1$, the Bragg condition is $\Delta = \beta$, i.e. $\delta = \omega_r(2n+1) + n_a U/\hbar$, where $n_a U = n_a 4\pi\hbar^2 a_s/m$ is the chemical potential and $n_a = 2N/\lambda\Sigma$ is the atomic density.

In the superradiant regime the field amplitude a can be adiabatically eliminated for times much longer than κ^{-1}. In fact, let introduce the slowly varying variable $\tilde{S}(t) = S(t)e^{i\alpha(t)}$, where $\alpha(t)\Delta t - \beta \int_0^t dt' W(t')$, and let integrate Eq.(11):

$$a(t) = a(0)e^{-\kappa t} + gN \int_0^t dt' \tilde{S}(t-t')e^{-i\alpha(t-t')-\kappa t'}. \tag{12}$$

If we assume that $\tilde{S}$ and W do not change appreciably in a time κ^{-1} during the superradiant process, i.e. if $\tau_{sr} \gg \kappa^{-1}$ where τ_{sr} is a characteristic time for superradiance, then in Eq.(12) $\tilde{S}(t-t') \approx \tilde{S}(t)$ and $\alpha(t-t') \approx \alpha(t) - [\Delta - \beta W(t)]t'$. Performing the residual integration in t' and assuming $t \gg \kappa^{-1}$, we finally obtain:

$$a(t) \approx \frac{gNS(t)}{\kappa - i[\Delta - \beta W(t)]}, \tag{13}$$

so that the field a follows istantaneously the atomic evolution. Combining Eqs.(9), (10) and (13) and defining $I = |S|^2$, we obtain

$$\dot{I} = 2\left\{ \frac{GW}{1 + [(\Delta - \beta W)/\kappa]^2} - \gamma \right\} I \tag{14}$$

$$\dot{W} = -\frac{4GI}{1 + [(\Delta - \beta W)/\kappa]^2} \tag{15}$$

where $G = g^2 N/\kappa$ is the superradiant gain. Defining the characteristic time as $\tau_{sr} = 1/G$, it follows that the adiabatic approximation (13) is true for $\kappa \gg G$, i.e. for $\kappa \gg g\sqrt{N}$. Furthermore, the two-level approximation is valid if $G < \omega_r$. ¿From Eqs.(14) and (15) it is possible to derive the following analytical results:

- In the linear regime, for $W \approx 1$, the threshold condition for superradiance is $G > \gamma\{1 + [(\Delta - \beta)/\kappa]^2\}$, so that the only effect of the atomic interaction is a shift of the resonance from $\Delta = 0$ to $\Delta = \beta$.

- Neglecting decoherence ($\gamma = 0$), Eqs. (14) and (15) admit the constant of motion $4I + W^2 = 1$. Writing $2\sqrt{I} = \sin\phi$ and $W = \cos\phi$, we obtain an equation for the Bloch angle ϕ:

$$\dot{\phi} = \frac{G \sin\phi}{1 + (\Delta - \beta\cos\phi)^2/\kappa^2}. \tag{16}$$

5

Although Eqs.(16) can be solved exactly by quadrature, its solution can not be set in an explicit form. Fig.1 shows $|a|^2/N$, (a), and $P_n = |c_n|^2 = (W+1)/2$, (b), as a function of $\omega_r t$ for different values of β, obtained solving numerically Eqs.(4) and (5) for $\kappa = 20\omega_r$, $g\sqrt{N} = 2\omega_r$ and $\Delta = \beta$. The results are in excellent agreement with the numerical solution of the approximated Eq.(16), not reported in the figure. We note that the effect of the atomic interaction is only a broadening of the superradiant pulse, which still preserves the same area equal to π, transfering completely the atoms from the initial momentum state n to the final momentum state $n + 1$.

- It is easy to calculate the exact analytical solution of Eqs. (14) and (15) when $\beta = 0$. In fact, let introduce $G' = G/[1+(\Delta/\kappa)^2]$ and the new variables $x = (W-W_0)/(1-W_0)$ and $y = 2\sqrt{I}/(1 - W_0)$, where $W_0 = \gamma/G' < 1$. ¿From Eqs. (14) and (15) it follows that $x^2 + y^2 = 1$. Introducing again the Bloch angle ϕ defined such that $x = \cos\phi$ and $y = \sin\phi$, Eqs. (14) and (15) give the following equation for ϕ,

$$\dot{\phi} = G'(1 - W_0)\sin\phi, , \tag{17}$$

whose solution yields $x(t) = -\tanh[G'(1 - W_0)(t - t_D)]$, where $t_D = -\ln[|S(0)|/(1 - W_0)]/G'(1 - W_0)$ is the delay time. Coming back to the original variables we finally obtain:

$$I(t) = \left(\frac{1 - W_0}{2}\right)\operatorname{sech}^2[G'(1 - W_0)(t - t_D)] \tag{18}$$

$$W(t) = W_0 - (1 - W_0)\tanh[G'(1 - W_0)(t - t_D)] \tag{19}$$

We note that the asymptotic value of the population difference is $2W_0 - 1$, so that the fraction of atoms left in the initial state after the superradiant process is $P_n = W_0 = \gamma/G'$. Measuring experimentally G' and P_n it is possible to evaluate the decoherence rate γ [4].

IV. INHOMOGENEOUS CASE

Let now consider the case in which the condensate is described initially by a wavepacket with a finite size $\sigma_\theta = 2k\sigma_z$ and $p_z = 0$. We have solved Eqs.(1) and (2) for an initial Gaussian wavepacket of width $\sigma_\theta = 25$, $\kappa = 10\omega_r$, $g\sqrt{2N} = \omega_r$, $\Delta = 0$ and different values

of β. The numerical integration of Eqs. (1) and (2) is based on a finite-difference predictor-corrector scheme [13, 14]. Fig.2 shows the density distribution, $\rho(\theta, t) = |\Psi(\theta, t)|^2$, for $\beta = 0$ at different times, whereas fig.3 shows the corresponding momentum distribution $\rho(p_z, t) = |\tilde{\Psi}(p_z, t)|^2$, where $\tilde{\Psi}$ is the Fourier transform of the wavefunction Ψ. We observe that the superradiant process produces a condensate fraction moving with an average momentum $p_z = 2\hbar k$ and a smaller momentum spread (see fig.3). In the configuration space (see fig.2) we clearly observe the interference fringes when the two fractions overlap, whereas for longer times the recoiling atoms move away from the original condensate.

Fig.4 shows $|a|^2/N$, (a), and P_n, (b), as a function of $\omega_r t$ for different values of β, where P_n is the population of the momentum state $p_z = n(2\hbar k)$, calculated integrating the momentum distribution over an interval centered around $p_z = n(2\hbar k)$ and of length $2\hbar k$. This can be done only if the momentum distribution remains narrower than the momentum level separation, i.e. if $\sigma_{p_z} \ll 2\hbar k$.

We observe that contrary to the homogeneous case, increasing β the superradiant process becomes less efficient, decreasing the area of the superradiant pulse (see fig.4(a)) and increasing the fraction of atoms left in the initial momentum state (see fig.4(b)). This effect can be interpreted as due to a dephasing caused by a detuning from the resonance depending on the atomic density. Each atom evolves with a different detuning from the resonance, resulting in a inhomogeneous broadening of the superradiant transition and a subsequent decay of the coherence between the atoms. Similarly to the photon echo [11], it is expected that this dephasing may be partially reversed applying a suitable Bragg pulse or area π, at least for small values of β. For larger or negative values of β, it is expected that nonlinear effects are more important, and this regime will be the object of a future detailed analysis.

---

[1] See for example *Bose-Einstein Condensation in Atomic Gases*, M. Inguscio, C. E. Wieman, S. Stringari, Eds. (IOS Press Amsterdam, Oxford, Tokyo, Washington, 1999).

[2] S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, J.Stenger, D. E. Pritchard, and W. Ketterle, Science **285**, 571 (1999).

[3] M. Kozuma, Y. Suzuki, Y. Torii, T. Sugiura, T. Kuga, E. W. Hagley, and L. Deng, Science **286** 2309, (1999); S. Inouye, T. Pfau, S. Gupta, A. P. Chikkatur, A. Gorlitz, D. E. Pritchard,

and W. Ketterle, Nature **402** 641 1999.

[4] R. Bonifacio, F. S. Cataliotti, M. Cola, L. Fallani, C. Fort, N. Piovella, and M. Inguscio, E-print available at http://xxx.lanl.gov/abs/quant-ph/0306500.

[5] R. Bonifacio and L. De Salvo Souza, Nucl. Instrum. and Meth. in Phys. Res. A **341**, 360 (1994); R. Bonifacio, L. De Salvo Souza, L.M. Narducci and E.J. D'Angelo, Phys. Rev.A **50**, 1716 (1994).

[6] R. Bonifacio, G.R.M. Robb and B.W.J. McNeil, Phys. Rev.A **56**, 912 (1997).

[7] M.G. Moore and P. Meystre, Phys. Rev. A **58**, 3248 (1998); M.G. Moore, O. Zobay, and P. Meystre, Phys. Rev. A **60**, 1491 (1999); M.G. Moore and P. Meystre, Phys. Rev. Lett. **83**, 5202 (1999).

[8] N. Piovella, M. Gatelli, and R. Bonifacio, Optics Comm. **194**, 167 (2001).

[9] V.S. Letokhov and V.G. Minogin, Phys. Report **73**, 1 (1981).

[10] J. Stenger, S. Inouye, A.P Chikkatur, D.M. Stamper-Kurn, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **82**, 4569 (1999).

[11] N. Piovella, V. Beretta, G.R.M. Robb, and R. Bonifacio, Phys. Rev. A **68**, 021801(R) (2003).

[12] P.J. Martin, B.G. Oldaker, A.H. Miklich, and D.E. Pritchard, Phys. Rev. Lett. **60**, 515 (1988).

[13] E. Cerboneschi, R. Mannella, E. Arimondo, and L. Salasnich, Phys. Lett. A **249**, 495 (1998).

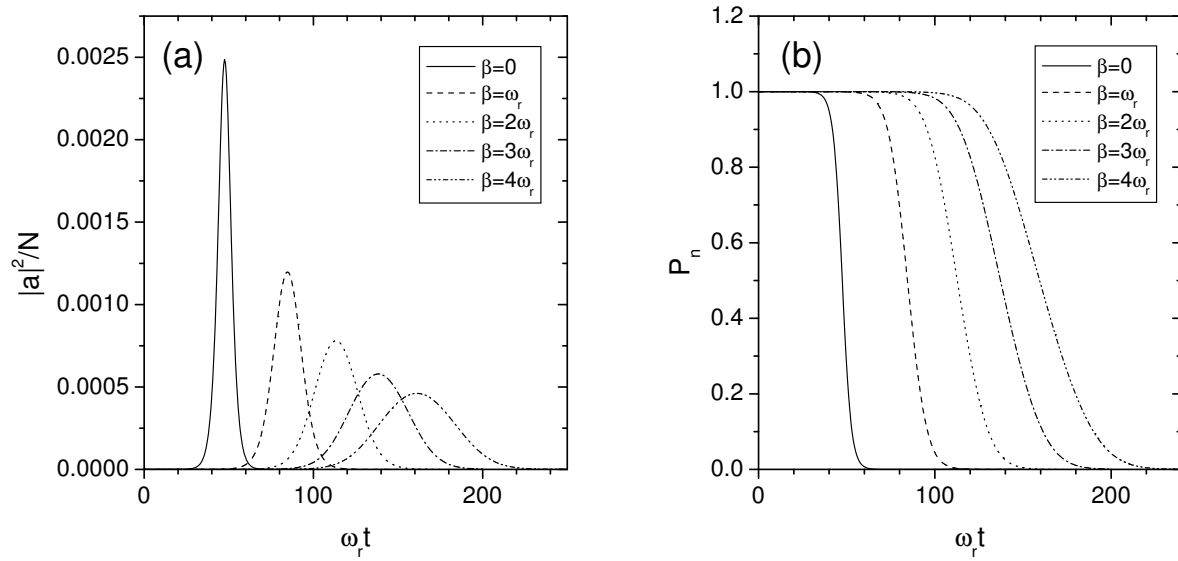[14] L. Salasnich, A. Parola and L. Reatto, Phys. Rev. A **64**, 023601 (2001).

FIG. 1: Effects of the atomic interaction on the superradiant regime in the homogeneous case: $|a|^2/N$, (a), and population fraction P_n of the initial momentum state, (b), vs. $\omega_r t$, from the numerical integration of Eqs.(4) and (5) with $\kappa = 20\omega_r$, $g\sqrt{N} = 2\omega_r$, $\Delta = \beta$ and different values of β.
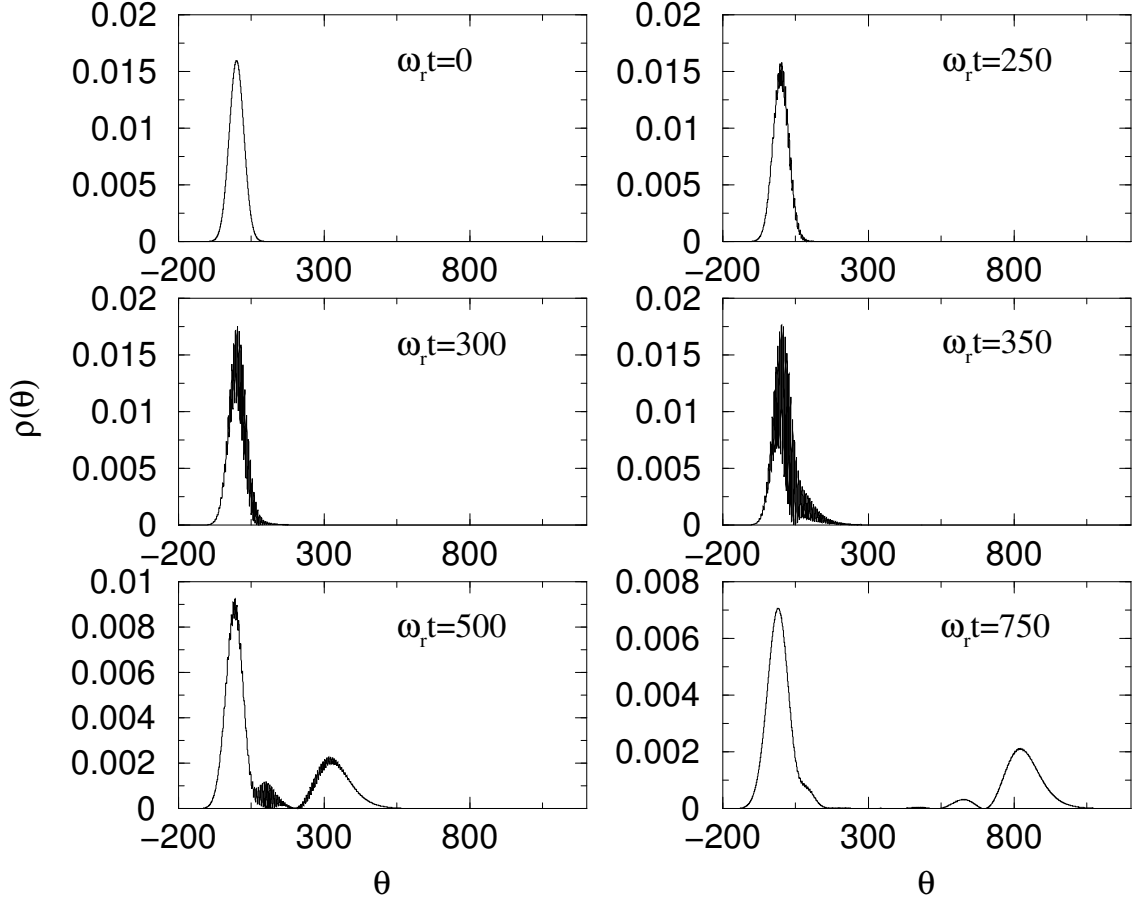
FIG. 2: Evolution of the density distribution $\rho(\theta, t) = |\Psi(\theta, t)|^2$ vs. $\theta = 2kz$ at different times, from the numerical integration of Eqs.(1) and (2) for an initial Gaussian wavepacket of width $\sigma_\theta = 25$, $\kappa = 10\omega_r$, $g\sqrt{2N} = \omega_r$, $\Delta = 0$, and $\beta = 0$.
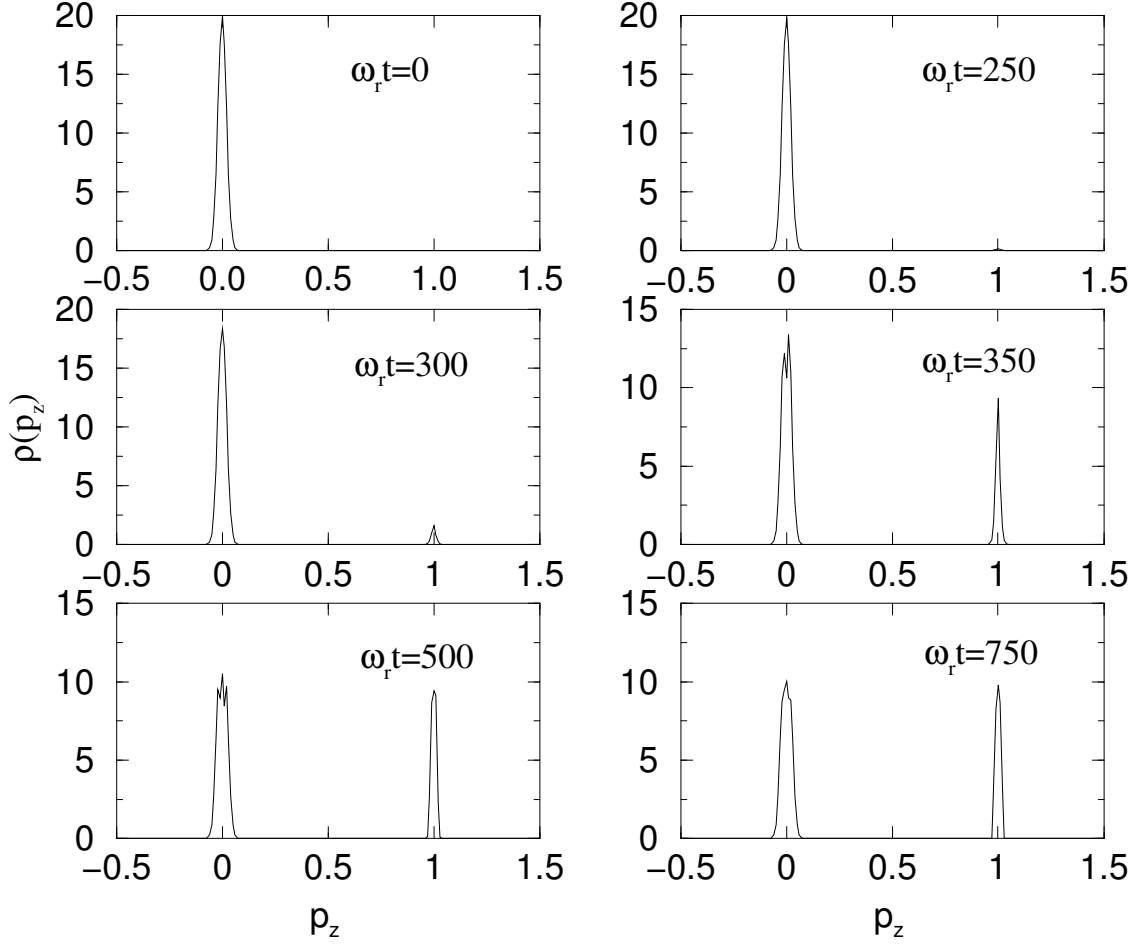
FIG. 3: Evolution of the momentum distribution $\rho(p_z, t) = |\tilde{\Psi}(p_z, t)|^2$ vs. p_z (in units of $2\hbar k$) at different times and for the same case shown in fig.2.
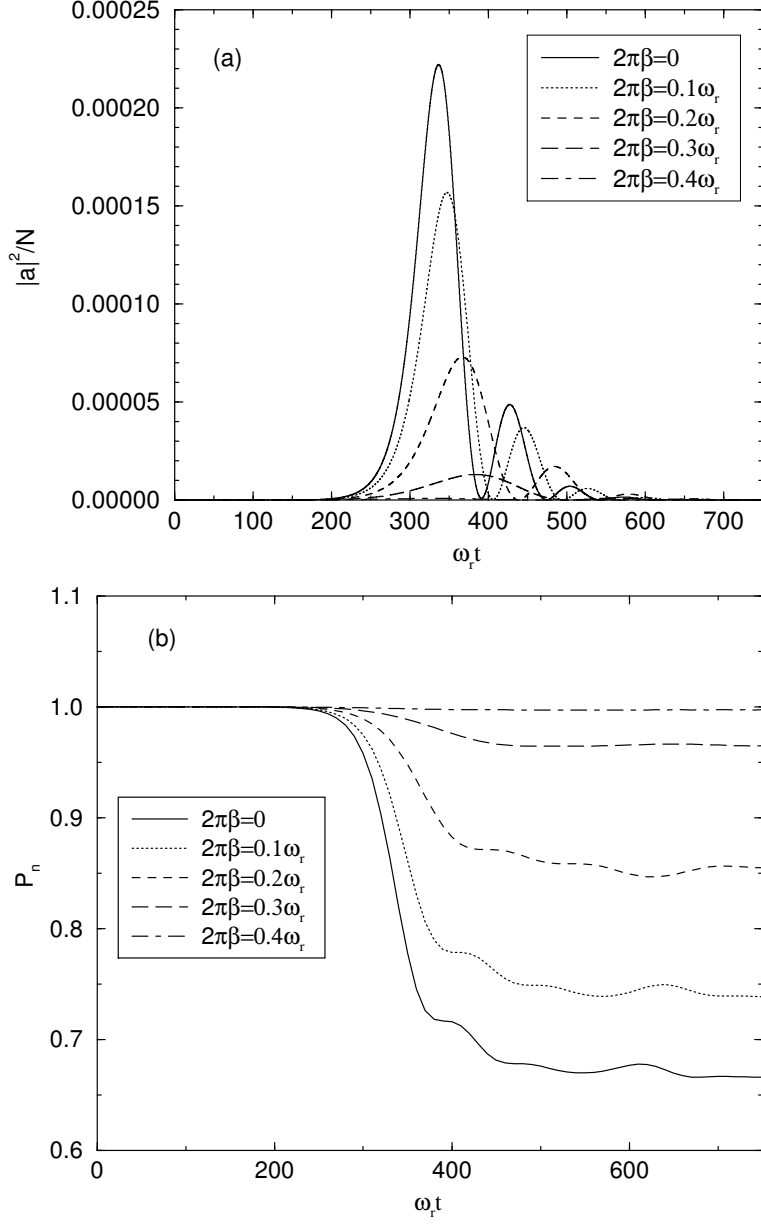
FIG. 4: Effects of the atomic interaction on the superradiant regime in the inhomogeneous case: $|a|^2/N$, (a), and population fraction P_n of the initial momentum state, (b), vs. $\omega_r t$, for the same initial conditions and parameters of the case shown in fig.2 and different values of β.