AUTHOR FINAL VERSION

An enhanced stiffness model for elastic lines and its application to the analysis of a moored floating offshore wind turbine

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Abstract: The performance of a polyester mooring line is non-linear and its elongation plays a significant role in the dynamic response of an offshore moored structure. However, unlike chain, the tension-elongation relationship and the overall behaviour of elastic polyester ropes are complex. In this paper, by applying a new stiffness model of the mooring line, the traditional elastic rod theory is extended to allow for large elongations. One beneficial feature of the present method is that the stiffness matrix is symmetric; in non-linear formulations the element stiffness matrix is often non-symmetric. The static problem was solved by Newton-Raphson iteration whereas a direct integration method was used for the dynamic problem. The mooring line tension based on the enhanced model was validated against the proprietary OrcaFlex software. Results of mooring line top tension predicated by different elongations are compared and discussed. The present method was then used for a simulation of an offshore floating wind turbine moored with taut lines. From a comparison between linear and non-linear formulations, it is seen that a linear spring model under-estimates the mean position when the turbine is operating, but over-estimates the amplitude of the platform response at low frequencies when the turbine has shut down.

Key words: Large extension; elastic rod theory; finite element method; mooring system; line tension; motion response; dynamic response

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1. Introduction

The capture of offshore wind energy plays a key role across the maritime industry (EWEA, 2013). Offshore wind turbines are becoming larger and more powerful, and are being deployed in ever-deeper water. They can be mounted on a fixed or floating base, but the former starts to lose its economic advantage for water depths larger than 60m (Goupee et al, 2014). Although the mooring system design for a floating offshore wind turbine (FOWT) has benefited from the experience of offshore oil and gas platforms, there are still several unknowns dependent on the type of floating bodies, e.g. size and environmental loading. From a report of EWEA (2013), it is recommended that more research must be done on mooring and anchoring systems for wind turbines.

Owing to the successful experience from offshore oil & gas platforms, the design and modelling of a FOWT has tended to use the same mathematical modelling and methods of solution as for offshore platforms, e.g. the hydrodynamic analysis of floating body, mooring design and the types of FOWTs (Spar, TLP and Semisubmersible, etc). The methods of analysis for the hydrodynamic aspects of a FOWT and its mooring system are the same as for offshore platforms. However, the geometry and operational water depth are different. Also, the turbine thrust force may have an effect on the motion response of the floating body and mooring line tension, and vice versa. These differences need to be examined for a FOWT.

Numerical simulations of the dynamic response of mooring lines have been studied during the past few decades, for both elastic and inelastic lines. A massless spring (e.g. Kim, et al. 2001) or the catenary equation (e.g. Agarwal and Jain, 2003) are straightforward ways to model a mooring line, but it is difficult to account for the dynamic response and the interaction between the floating body and mooring line in an accurate manner. Multi-body system dynamics (e.g. Kreuzer and Wilke, 2003) divides the mooring line into several rigid bodies, but results in a large number of degrees of freedom even for a single line. Non-linear finite element methods (FEMs), accounting for geometric and material non-linearities, have been widely used for modelling mooring line response (e.g. Kim, et al, 2013). Geometric non-linearity is needed for large displacements of the mooring line, while material non-linearity can model the time-dependent properties of a polyester rope, e.g. Young's modulus. However, a major disadvantage of FEM is the transformation

between local coordinate and global coordinate, which is often computationally-intensive. The lumped mass and spring method can be categorized as a non-linear FEM method, for which the shape function becomes a single line (Low, 2006).

Unlike traditional non-linear FEM, the elastic rod theory is a global-coordinate-based method, which is considered to be more efficient than the non-linear FEM method (Kim, et.al, 1994). The transformation between local and global coordinate is dealt within the element stiffness matrix. Following the elastic rod theory of Love (1944), Nordgren (1974) and Garrett (1982) developed this method and solved the governing non-linear equations by a finite difference method (FDM) and by FEM, respectively. Many researchers have further developed the elastic rod theory, including elongation of the line, seabed friction, non-linear material properties and the incorporation of buoys or clump weights in the mooring line model. Pauling and Webster (1986) considered the analysis of large amplitude motions of a TLP under the action of wind, wave and current, under the assumption of small line elongation. Ran (2000) proposed a finite element formulation for mooring lines and risers based on Garett's rod theory, applicable to both frequency and time domains. Based on the traditional small extensible rod theory, the incorporation of large elongation has been presented by many researchers (e.g. Chen, 2002; Tahar, 2001and Kim et al, 2011).

In the present paper, a sensible balance has been sought between efficiency and accuracy. The traditional rod theory has been extended to allow for large stretch by applying an enhanced stiffness method. By using an approximation of the non-linear tension-elongation relationship in a Taylor series expansion (Ćatipović et al., 2011), the mathematical and numerical formulation of large extensible mooring line are considered.

2. Mathematical formulation of a mooring line with large elongation

2.1 Equation of motion

For polyester mooring lines bending and torsion stiffness can be neglected, but the elongation cannot be assumed to be small. The mooring line is discretized into a number of rods and the centreline of each rod is described by a space-time curve $r(s,t)$. From Ćatipović et al. (2011), the equation of motion for a rod with large elongation can be written as:

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{dr}{ds}\right)+(1+\varepsilon)q_E=(1+\varepsilon)\rho\ddot{r} \tag{1}$$

where q_E is the load acting on the rod. Morison's equation (1950) was used to calculate the hydrodynamic loads on the mooring line (Pauling and Webster, 1986):

$$F=-C_A\ddot{r}_n+C_M\dot{V}_n+\frac{1}{2}C_D\left|V_n-\dot{r}_n\right|(V_n-\dot{r}_n) \tag{2}$$

where *n* denotes the normal component . C_A, C_M and C_D are the added mass, inertial (Morison) and drag coefficients.

The rod velocity and acceleration normal are given by

$$\dot{r}_n=\dot{r}-(\dot{r}\cdot r')\cdot r' \tag{3}$$

$$\ddot{r}_n=\ddot{r}-(\ddot{r}\cdot r')\cdot r' \tag{4}$$

$\ddot{r}$ represents the time derivation of the rod. T_E is the effective tension of the rod. ε is the elongation of the rod. ρ is the mass per length of the rod, including added mass. Following Ćatipović et al, assuming equal principal stiffness, the relationship between the effective tension and elongation can be written as

$$\varepsilon=\frac{T_E}{AE} \tag{5}$$

where *AE* is the axial stiffness

The following elongation condition then has to be satisfied

$$\frac{1}{(1+\varepsilon)^2}\frac{dr}{ds}\frac{dr}{ds}=1 \tag{6}$$

In the static problem, the weight per unit length and diameter of the mooring line are related to the elongation ε. The cross-sectional area and mass after elongation can be written as $A/(1+\varepsilon)$ and $m/(1+\varepsilon)$, respectively (Ćatipović, et,al 2011), where *A* and *m* are the cross-section area and mass of the mooring line without stretch. Applying the above relationship to the motion equation, we see that the term $(1+\varepsilon)$ cancels out when multiplied by the applied force q_E. For the hydrodynamic force calculated by Morison's equation, the mass per unit length and cross-sectional area for one element were assumed constant.

Equations (1) and (6) show the rod motion equation and elongation condition, respectively: they are non-linear. In the following section, we will describe a numerical procedure for solving this non-linear equation and the required order of approximation for the elongation condition.

2.2 Numerical Implementation

2.2.1 Static problem

For the static problem, r is independent of time. Consequently the inertial term in equation (1) is deleted. We therefore have

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{dr}{ds}\right)+q_E=0 \tag{7}$$

Using the Taylor series expansion, the elongation relationship can be written as:

$$\frac{1}{(1+\varepsilon)^2}=1-2\varepsilon+3\varepsilon^2+o(\varepsilon^3) \tag{8}$$

However, it is not clear, *a priori*, whether the third-order term should be included explicitly. In the present paper, the order of expansion and subsequent results will be discussed.

In the FEM, the variables r_i and T_E may be approximated (Garrett, 1982) as

$$r_i(s,t)=\sum_{k=1}^{4}A_k(s)U_{ik}(t) \tag{9}$$

$$T_E(s,t)=\sum_{m=1}^{3}P_m(s)\lambda_m(t) \tag{10}$$

where A_k and P_m are shape functions. The definition of the shape functions can be found in the appendix. U_{ik} and λ_m are unknown variables. The subscript *i* of U_{ik} denotes the dimension of the element. For the 3-dimensional problem, *i*=3. For *k*=1 and 3, U_{ik} represents the space position of the rod at two ends while U_{ik} denotes the space derivative at both ends for *k*=2 and 4. λ is the Lagrange multiplier. The physical meaning of λ is mooring line tension at both ends and middle of the rod.

The variable U_{ik} and λ_m are defined as:

$$U_{i1} = r_i(0,t), U_{i2} = Lr_i'(0,t)$$
$$U_{i3} = r_i(L,t), U_{i4} = Lr_i'(L,t) \tag{11}$$

$$\lambda_1 = \lambda\ (0,t), \lambda_2 = \lambda\ (\frac{L}{2},t), \lambda_3 = \lambda\ (L,t) \tag{12}$$

Using Galerkin's method (Bathe, 1996) and integrating the motion equation from 0 to L over the length of the element, the final form of motion equation for static problem in notation form can be written as

$$\hat{K}_{nijlk}\lambda_n U_{jk} - F_{il} = 0 \tag{13}$$

where

$$\hat{K}_{nijlk} = K^0_{nijlk} + \lambda_m K^1_{nmijlk} + \lambda_m \lambda_p K^2_{nmpijlk} \tag{14}$$

$$K^0_{nijlk} = \int_0^L P_n A_l' A_k' \delta_{ij} ds \tag{15}$$

$$K^1_{nmijlk} = -\int_0^L \frac{1}{EA} P_n P_m A_l' A_k' \delta_{ij} ds \tag{16}$$

$$K^2_{nmqijlk} = \int_0^L \frac{1}{(EA)^2} P_n P_m P_q A_l' A_k' \delta_{ij} ds \tag{17}$$

where δ is the Kronecker Delta function, L is the element length, and the standard double-suffix summation condition has been used.

The elongation condition, incorporating Taylor series expansion to second order, can be written as

$$\hat{B}_{mil} U_{ki} U_{kl} - C_m = 0 \tag{18}$$

where

$$\hat{B}_{mil} = B^0_{mil} + \lambda_n B^1_{nmil} + \lambda_n \lambda_p B^2_{npmil} \tag{19}$$

$$B^0_{mil} = \int_0^L P_m A_i' A_l' ds \tag{20}$$

$$B^1_{nmil} = -\int_0^L \frac{2}{EA} P_m P_n A_i' A_l' ds \tag{21}$$

$$B^2_{mnqil} = \int_0^L \frac{3}{(EA)^2} P_m P_n P_q A_i' A_l' ds \tag{22}$$

$$C_m = \int_0^L P_m ds \tag{23}$$

Recalling equation (13) and the elongation condition (18), Newton-Raphson iteration was applied to the static problem (Ran, 2000). Omitting higher order components, we have

$$R_{il}^{(n+1)} = R_{il}^{(n)} + \frac{\partial R_{il}}{\partial U_{jk}}(\Delta U_{jk}) + \frac{\partial R_{il}}{\partial \lambda_n}(\Delta \lambda_n) = 0 \tag{24}$$

$$G_m^{(n+1)} = G_m^{(n)} + \frac{\partial G_m}{\partial U_{jk}}(\Delta U_{jk}) + \frac{\partial G_m}{\partial \lambda_n}(\Delta \lambda_n) = 0 \tag{25}$$

Re-arranging the terms and writing in the matrix form, we have

$$\begin{bmatrix} K_{ijlk}^{11(n)} & K_{i\ln}^{12(n)} \\ K_{mjk}^{21(n)} & K_{mn}^{22(n)} \end{bmatrix} \begin{Bmatrix} \Delta U_{jk} \\ \Delta \lambda_n \end{Bmatrix} = -\begin{Bmatrix} R_{il}^{(n)} \\ G_m^{(n)} \end{Bmatrix} \tag{26}$$

where

$$K_{ijlk}^{11(n)} = \hat{K}_{nijlk}^{(n)} \lambda_n^{(n)} \tag{27}$$

$$K_{iln}^{12(n)} = \hat{K}_{nijlk}^{(n)} U_{jk}^{(n)} \tag{28}$$

$$K_{mjk}^{21(n)} = K_{i\ln}^{12(n)} \tag{29}$$

$$K_{mn}^{22(n)} = (B_{mnkl}^{1} + 2\lambda_p B_{pmnkl}^{2}) U_{jl}^{(n)} U_{jk}^{(n)} \tag{30}$$

$$R_{il}^{(n)} = \hat{K}_{nijlk}^{(n)} U_{jk}^{(n)} - F_{il} \tag{31}$$

$$G_m^{(n)} = \hat{B}_{mil} U_{ki} U_{kl} - C_m \tag{32}$$

The above formulation of the Newton-Raphson method can be written in matrix form

$$K^{(n)}(\Delta y) = F^{(n)} \tag{33}$$

where K and F are the same as the stiffness matrix and forcing vector in equation (26). Δy includes ΔU_{jk} and $\Delta \lambda_n$. In the static problem, n represents the step of iteration.

2.2.2 Dynamic problem

The inertial term in the equation of motion equation cannot be neglected in the dynamic problem.

$$\frac{1}{(1+\varepsilon)} = 1 - \varepsilon + \varepsilon^2 + o(\varepsilon^3) \tag{34}$$

The definition of r_i and T_E are the same as in the static case. Integrating over the element generates the discretized form of the equation of motion. Incorporating the elongation condition, we have

$$(\hat{M}_{ijlk}) U_{jk} = -\lambda_n \hat{K}_{nijlk} U_{jk} + F_{il} \tag{35}$$

$$G_m = \hat{B}_{mil} U_{ki} U_{kl} - B_m - C_{mt} \lambda_t = 0 \tag{36}$$

where

$$\hat{M}_{ijlk} = M_{ijlk} + M_{ijlk}^{a} \tag{37}$$

To solve the second-order differential equation of motion, Ran (2000) introduced a new variable V:

$$\hat{M}_{ijlk}\dot{V}_{jk} = -\lambda_n \hat{K}_{nijlk} U_{jk} + F_{il} \tag{38}$$

$$\dot{U}_{jk} = V_{jk} \tag{39}$$

To solve these two equations, we need to integrate from *t(n)* to *t(n+1)*, using the first-order Adam-Moulton method. Ran assumed a constant value $\hat{M}_{ijlk}^{(n+0.5)}$ during this time interval, leading to the equation:

$$\begin{aligned} 0 = 2G_m^{(n+1)} &\approx 2G_m^{(n)} + 2\frac{\partial G_m^{(n)}}{\partial U_{jk}}\Delta U_{jk} + 2\frac{\partial G_m^{(n)}}{\partial \lambda_n}\Delta\lambda_n \\ &= 2G_m^{(n)} + 2\hat{K}_{nijlk}U_{il}(\Delta U_{jk}) + D_{mn}^{t1(n)}(\Delta\lambda_n) \end{aligned} \tag{40}$$

Re-writing equations (38) and (40) in matrix form, we have

$$\begin{bmatrix} K_{ijlk}^{'11(n)} & K_{i\ln}^{'12(n)} \\ K_{mjk}^{'21(n)} & K_{mn}^{'22(n)} \end{bmatrix} \begin{Bmatrix} \Delta U_{jk} \\ \Delta\lambda_n \end{Bmatrix} = -\begin{Bmatrix} R_{il}^{'(n)} \\ G_m^{'(n)} \end{Bmatrix} \tag{41}$$

where

$$K_{ijlk}^{'11(n)} = \frac{2}{\Delta t^2}\hat{M}_{ijlk}^{(n+0.5)} + \lambda_n^{(n-0.5)}\hat{K}_{nijlk} \tag{42}$$

$$K_{i\ln}^{'12(n)} = 2\hat{K}_{nijlk}U_{jk} \tag{43}$$

$$K_{mjk}^{'21(n)} = 2\hat{K}_{mijlk}U_{il} \tag{44}$$

$$K_{mn}^{'22(n)} = 2(K_{mn}^{22(n)} + C_{mn}) \tag{45}$$

$$R_{il}^{'(n)} = \frac{2}{\Delta t}\hat{M}_{ijlk}^{(n+0.5)}V_{jk}^{(n)} - 2\lambda_n^{(n-0.5)}\hat{K}_{nijlk}U_{jk}^{(n)} + 3F_{il}^{(n)} - F_{il}^{(n-1)} \tag{46}$$

$$G_m^{'(n)} = 2(G_m^{(n)} - C_m) \tag{47}$$

The dynamic problem can be solved in a manner similar to the static case:

$$K^{(n)}(\Delta y) = F^{(n)} \tag{48}$$

Now, n denotes the time step (instead of the iteration step in the static analysis). The static model was first used to determine the mean position of the mooring line. The above numerical procedure was incorporated in FAST's FEM. The original FAST program, based on the assumption of small elongations was extended to allow for large elongations, and therefore suitable for polyester lines. An advantage of the present method is that the element stiffness matrix remains symmetric.

2.3 Validation of the enhanced model

The present study considered a model of a spar-type floating platform, similar to that used for a wind turbine design of the National Renewable Energy Laboratory (NREL). The platform can be moored by slack or taut mooring lines (ABS, 2014). In this paper, three equal taut mooring lines were selected for case studies. The parameters of the floating cylinder and the upper structure are the same as NREL's OC-3 Hywind Spar (Jonkman, 2010), except for the mooring system. The main properties of the wind turbine are shown in Table 1; those of the taut mooring line in Table 2. For simplicity, the *Radius to Fairleads from Platform Centreline* in Table1 was 4.7m, instead of 5.2 m.

Figures 1 and 2 show the dynamic response of line tension in sea state 6 (H=5.5m, T=11.3s). The red line shows the results of Fastlink (FAST+OrcaFlex). In Fastlink, OrcaFlex solves the dynamic response of mooring line in the time domain and passes the mooring line tension to FAST for the coupled response of the mooring system. From this comparison we can see that the Taylor expansion to second order (present) shows little difference compared with the results from third order (extended stiffness). They both show very good agreement with the lumped mass and spring method. However, when assuming small elongation (equivalent to an expansion to first order, using the governing equation and elongation condition of the rod by Pauling & Webster,1986) the blue and green lines in Figures 1 and 2 show poor results for a polyester line.

The present enhanced stiffness method is also appropriate for a slack mooring line (catenary chain, line length: 902.2m; chain mass: 77.7kg/m and elastic stiffness: 384.2E6 N). Figure 3 compares the line tension results under sea state 6 for a catenary chain. Results from the approximation to second (reduced stiffness) and third order (enhanced stiffness) generate the same results as OrcaFlex and the small elongation assumption. From a comparison of Figures 1, 2 and 3 we can see that the present method can be used for modelling both traditional materials as well as high-performance fibre. For the enhanced stiffness condition (expansion to third order), the stiffness term and elongation are:

$$\hat{K}_{nijlk} = K^{0}_{nijlk} + \lambda_m K^{1}_{nmijlk} + \lambda_m \lambda_p K^{2}_{nmpijlk} + \lambda_q \lambda_m \lambda_p K^{3}_{nmpqijlk} \tag{49}$$

$$\hat{B}_{mil} = B^{0}_{mil} + \lambda_n B^{1}_{nmil} + \lambda_n \lambda_p B^{2}_{npmil} + \lambda_n \lambda_p \lambda_q B^{3}_{npqmil} \tag{50}$$

$$K^{3}_{nmtpqijlk} = -\int_0^L \frac{1}{(EA)^3} P_n P_m P_q P_t A_l' A_k' \delta_{ij} ds \tag{51}$$

For the static problem,

$$B^{3}_{nmqtil} = -\int_0^L \frac{4}{(EA)^3} P_n P_m P_q P_t A_i' A_l' ds \tag{52}$$

while for the dynamic problem,

$$B^{3}_{npqmil} = -\int_0^L \frac{1}{(EA)^3} P_n P_m P_q P_t A_i' A_l' ds \tag{53}$$

2.4 Comparison of line tension for different approximations

In order to check further the effect of differing elongation approximations, a reduced line length (420m) was considered. Waves only were assumed, having the same height and frequency as above (sea state 6). The elongation of the mooring line is about 15%, but the difference between values of the mean and maximum line tension is around 1.3 % (see Figure 4, the mooring line layout is shown in Figure 5). So, the approximation to second order is sufficient.

3. Comparison between present method and linear spring method

3.1 Methods applied

The hydrodynamic coefficients and wave exciting forces for the OC-3 Spar-type wind turbine were pre-calculated by WAMIT and stored in FAST. Only first-order wave forces were included in the present study. The impulse response function method of Cummins (1962) was used in our time-domain study. Fourier transformations converted the frequency-dependent added mass and radiation damping terms for use in the time-domain model. The equation of motion of the floating platform is

$$[M + M^a(\infty)]\ddot{X} + KX + \int_{-\infty}^{t} R(t-\tau)\dot{X}(t)d\tau = F_e \tag{54}$$

where M and M^a are mass and added mass of the floating body, respectively; K is the hydrostatic matrix; X is the motion response of the floating body. F_e is the external force on the floating body, arising from waves, currents and wind. The HydroDyn model of FAST calculates the retardation function and motion response of the platform; the former is given

$$R(t) = \frac{2}{\pi}\int_{-\infty}^{t} b(\omega)\cos(\omega t)d\omega \tag{55}$$

The coupling between the floating body and mooring line applied a 'loose coupling' method, as introduced by Jonkman (2013).

3.2 Mooring system load-offset relationship

Figure 6 shows the surge restoring force against different initial horizontal position. From the graph we can see that the load-offset relationship is almost linear. For the linear spring method, the spring stiffness was derived from the same method- giving an initial offset and the spring stiffness was calculated with the following equation

$$K = \frac{\Delta F}{\Delta S} \tag{56}$$

where K is the spring stiffness, F and S are the force and offset of the mooring system, respectively. The off-diagonal stiffness was ignored. In other words, in the linear stiffness method, the coupling effects (e.g.

heave-pitch coupling) were not accounted for. The liner spring stiffness for surge, sway and heave are 30680.5 N/m, 29728.2 N/m and 23178 N/m, respectively.

3.3 Results of case studies under wave only and wave plus wind condition

- **Platform motion response**

The environmental conditions are shown in Tables 3 and 4. As potential theory fails to consider viscous effects, the additional linear damping was added. The additional damping for surge, sway and heave are 100000N/ (m/s), 100000N/ (m/s) and 130000N/ (m/s), respectively. Figures 7.1~7.6 show the motion RAOs for the Spar under wave only and wave plus wind condition. Under wave only condition, the amplitude of heave response is not affected by the method of analysis, but the mean heave position has seen a large difference between the two methods, as can be seen from figure 8.2 and 9.2. Surge and pitch RAOs decrease under the wave plus wind condition, compared with wave only condition.

For the wave only condition, there is little difference between the two methods for wave frequencies larger than 0.4 rad/s (e.g. Figures 10.1 and 10.3), except for the mean position of the heave motion. These results indicate that for the primary design of substructure of the FOWT under some survival conditions (e.g. seastate 7 or seastate 8), the linear spring method can be applied, as it gives results as accurate as the FEM method but with less running time. However, for the wave plus wind condition, although the amplitude of motion response shows little difference between the linear spring method and elastic rod theory, the mean position of surge and pitch were under-predicted by the linear spring method. Under the wave only condition, the floating body oscillates about its mean position, but there is a very large mean offset (e.g. about 42m in Figure 11.1 and 11.3) when considering wave plus wind condition. Under the wave plus wind condition, the turbine thrust force is much larger than the wave forces.

- **Turbine thrust force**

Figure 12 shows a comparison of mean rotor thrust force under linear spring method and present elastic rod theory. The mean thrust force is independent of wave frequency, except for wave frequency=0.24rad/s, linear spring method, but the linear spring method underestimates the mean thrust force.

- **Mooring line tension**

Figures 13 and 14 show the mooring line tension for both wave only and wave plus current condition. Under the wave only condition, the mooring line tension does not vary much; the floating body oscillates around its initial mean position. The reason for this phenomenon is because current modelling only included first-order wave forces. The second-order effects are of little importance for the Spar-type wind turbine (e.g. Roald et al, 2013). For the wave plus wind condition, the FOWT moves to a new equilibrium position and oscillates around this position, which results in one of the mooring lines becoming less taut. However, as discussed in the previous section, the proposed method is suitable for modelling both slack and taut mooring lines.

4. Conclusion

The wind turbine simulation tool FAST has been modified to examine the response of a FOWT with polyester mooring lines. A new stiffness model has been implemented to account for large elongations of the line. Its accuracy has been assessed numerically, and the results show that the proposed model is suitable for modelling both slack and taut mooring lines.

The present approach has been applied to the simulation of a taut-moored FOWT. Comparison has been made against the linear spring method. Although the mooring system's static load-offset graph is linear, the linear spring method fails to consider the dynamic response of the mooring line. It under predicts the motion of the floating body in the wave plus wind condition. This under prediction also affects the maximum mooring line tension, as its value is dependent on the instantaneous position of the floating platform.

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Appendix

SHAPE FUNCTIONS

In the FEM, the shape function A_l and P_m are defined as follows (Garrett, 1986):

$$\begin{aligned} A_1 &= 1-3\xi^2+2\xi^3 \\ A_2 &= \xi-2\xi^2+\xi^3 \\ A_3 &= 3\xi^2-2\xi^3 \\ A_4 &= \xi^2-2\xi^3 \end{aligned} \tag{A.1}$$

$$\begin{aligned} P_1 &= 1-3\xi+2\xi^2 \\ P_2 &= 4\xi\,(1-\xi) \\ P_3 &= \xi\,(2\xi-1) \end{aligned} \tag{A.2}$$

where $\xi = s/L$