

A Simple and Accurate Approach to Solve the Power Flow for Balanced Islanded Microgrids

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Abstract—Power flow studies are very important in the planning or expansion of power system. With the integration of distributed generation (DG), micro-grids are becoming attractive. So, it is important to study the power flow of micro-grids. In grid connected mode, the power flow of the system can be solved in a conventional manner. In islanded mode, the conventional method (like Gauss Seidel) cannot be applied to solve power flow analysis. Hence some modifications are required to implement the conventional Gauss Seidel method to islanded micro-grids. This paper proposes a Modified Gauss Seidel (MGS) method, which is an extension of the conventional Gauss Seidel (GS) method. The proposed method is simple, easy to implement and accurate in solving the power flow analysis for islanded micro-grids. The MGS algorithm is implemented on a 6 bus test system. The results are compared against the simulations results obtained from PSCAD/EMTDC which proves the accuracy of the proposed MGS algorithm.

I. INTRODUCTION

Power flow studies play an important role in the planning, expansion and optimal operation of power system. Well developed power flow methods (Gauss, Gauss-Seidel, Newton-Raphson) are presented in [1]. In the recent years, scientists have focused their research in the area of grid integration of renewable energy and distributed generation (DG), which has lead to the evolution of micro-grids. These micro-grids operate either in grid connected or islanded modes. In grid connected mode, the frequency is maintained by the grid, but in islanded mode it are not constant. Several approaches have been proposed to solve power flow for islanded micro-grids [2], [3]. However, these approaches are based on the assumption to classify the droop bus (the bus at which the DG is connected) either as slack, PV or PQ bus, which is invalid in case of islanded micro-grid because size of DGs are usually small and cannot act as an infinite source of power. Also in islanded micro-grid, classification of all the DGs to be PV or PQ buses is not possible [4].

The slack bus and frequency dependency issues have been recently addressed in [5]–[7], where power flow for islanded micro-grid is proposed using droop characteristics of DGs. A novel and generalized three phase power flow solved using Newton-trust region method is proposed in [5]. In [7], particle-swarm technique is proposed for load flow of micro-grids. Literature suggests that the conventional load flow algorithms like

Gauss and Gauss Seidel (GS) are valid only for conventional power system and cannot be implemented to islanded micro-grids [5], [7].

In this paper, a simple and accurate approach is proposed that is a modification of the conventional power flow method (Gauss Seidel) to solve the power flow problem for islanded micro-grids. The method is validated by comparing the results obtained with the simulation results obtained using time domain PSCAD/EMTDC model.

II. SYSTEM MODELING

A. Load Modeling

The active and reactive load demand can be expressed as exponential functions of voltage and frequency [5].

$$P_{Lk} = P_{Lk_o} \left(\frac{V_k}{V_o} \right)^\alpha (1 + K_p(\omega - \omega_o)), \quad (1)$$

$$Q_{Lk} = Q_{Lk_o} \left(\frac{V_k}{V_o} \right)^\beta (1 + K_q(\omega - \omega_o)), \quad (2)$$

where V_o is the nominal voltage; P_{L_o} and Q_{L_o} are the active and reactive power corresponding to the nominal operating voltage, respectively; $(\omega - \omega_o)$ is the deviation in the angular frequency. K_p and K_q are the frequency sensitivity parameters [8], [9]. Exponent values (α and β) for different of loads are given in Table I.

TABLE I. LOAD TYPES AND EXPONENT VALUES [8], [10]

Load Type (L_T)	α	β
Constant Power (KP)	0.00	0.00
Constant Current (KC)	1.00	1.00
Constant Impedance (KI)	2.00	2.00
Residential (R)	0.92	4.04
Commercial (C)	1.51	3.40
Industrial (I)	0.18	6.00
Typical (T)	0.92	1.00

B. Y_{bus} Modeling

As mentioned above, the system frequency is not constant in islanded micro-grids, and thus it affects the line reactance

and as a result Y_{bus} is not constant but a function of the system frequency given by (3).

$$\vec{Y}^{bus}(\omega) = \begin{bmatrix} \vec{Y}_{11}(\omega) & \vec{Y}_{12}(\omega) & \dots & \vec{Y}_{1N}(\omega) \\ \vec{Y}_{21}(\omega) & \vec{Y}_{22}(\omega) & \dots & \vec{Y}_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{Y}_{N1}(\omega) & \vec{Y}_{N2}(\omega) & \dots & \vec{Y}_{NN}(\omega) \end{bmatrix}. \quad (3)$$

where $\vec{Y}_{kn}(\omega)$ is the admittance between bus k and n .

C. Distributed Generation (DG) Modeling

In grid connected mode, DGs are usually modeled as PV or PQ buses. In islanded mode, since there is no slack bus, it is impossible to model all the DGs as PV or PQ buses. So DGs in an islanded micro-grids are modeled as droop buses [5]. The complex power delivered to the bus can be represented by

$$S = P + jQ, \quad (4)$$

where P and Q are the active and reactive power delivered to the bus, respectively, and are given by [12]

$$\begin{cases} P = \frac{EV}{Z} \cos(\theta - \phi) - \frac{V^2}{Z} \cos(\theta) \\ Q = \frac{EV}{Z} \sin(\theta - \phi) - \frac{V^2}{Z} \sin(\theta) \end{cases} \quad (5)$$

where E is the magnitude of the inverter output voltage, ϕ is the power angle, and Z and θ are the magnitude and the phase angle of \vec{Z} , respectively. For an inverter based DG, its output impedance can be assumed to be inductive [13]. Assuming the output impedance of DG to be inductive, the above equation can be written as

$$\begin{cases} P = \frac{EV}{X} \sin(\phi) \\ Q = \frac{EV \cos(\phi) - V^2}{X} \end{cases} \quad (6)$$

where X is the output impedance of the DG. From (6), it can be observed that for a small value of ϕ , the active power is dependent upon the power angle ϕ and the reactive power is dependent upon the voltage amplitude difference. Based on the above equation the most commonly adopted droop equations for a DG can be expressed as [14], [15]

$$\omega = \omega_o - m_p P_G, \quad (7)$$

$$V = V_o - n_q Q_G, \quad (8)$$

where m_p and n_q are the frequency and voltage droop coefficients, respectively. In this paper, conventional droop equations ((7) and (8)) are considered, but the method is valid in case of resistive or complex output impedance and can be implemented by replacing conventional droop equations ((7) and (8)) with resistive or complex droop equations [12], [17].

III. PROPOSED POWER FLOW METHOD

The proposed power flow approach is based on the conventional GS method. The Modified Gauss Seidel (MGS) is developed by combining droop characteristics of DGs with the conventional GS power flow analysis.

A. Types of Buses

The well defined types of buses in the literature are slack, PV and PQ buses. The selection of bus depends upon the pre-specified quantities. In this work, DG buses are classified as VF dependent (droop) buses, in which the active and reactive powers of DGs depend upon the system frequency and bus voltage. For the reference, voltage angle of bus#1 is set to zero. In general, any bus can be a reference bus.

B. Problem Formulation

To apply GS method to an islanded micro-grid some modifications are required. Firstly, since there is no slack bus so the voltages for all the buses are variable. Secondly, the system frequency is also variable, and is required to be calculated. Also, the Y_{bus} needs to be included in the iteration procedure because Y_{bus} is a function of system frequency and it will change after every iteration. Furthermore, the losses in the system need to be distributed among the DGs. To address these issues, Modified Gauss Seidel (MGS) is proposed.

C. Modified Gauss Seidel Method

MGS is solved in two steps. The first step is same as conventional GS method but with some modification. The step starts with the assumption of voltages to be $1\angle 0^\circ$ but in this case bus#1 (conventionally treated as slack bus) is also a variable. Furthermore, frequency is also assumed to be 1 p.u. which is calculated in second step of MGS. So there are two extra variables involved in MGS calculations. The variable vector is given by

$$\mathbf{x} = [\mathbf{V}^T \ \omega]^T. \quad (9)$$

where ω is the system frequency. \mathbf{V} is the complex voltage vector (including bus#1). The steps involved in the MGS power flow are shown in Fig. 1. To solve the first step i.e. for voltages at all buses the conventional GS voltage expression is used which is given by [1]

$$\vec{V}_k^{i+1} = \frac{1}{\vec{Y}_{kk}} \left[\frac{P_k - jQ_k}{(\vec{V}_k^i)^*} - \sum_{n=1}^{k-1} \vec{Y}_{kn} \vec{V}_n^{i+1} - \sum_{n=k+1}^N \vec{Y}_{kn} \vec{V}_n^i \right], \quad (10)$$

where \vec{V}_k^{i+1} is the voltage for iteration $i+1$ at bus k . P_k and Q_k are net active and reactive powers at bus k , respectively. For all PQ buses in the system, the above equation can be solved because both P and Q are known. In case of a PV bus, the reactive power is calculated using

$$Q_k^{i+1} = -\Im \left\{ (\vec{V}_k^i)^* \left(\sum_{n=1}^{k-1} \vec{Y}_{kn} \vec{V}_n^{i+1} + \sum_{n=k}^N \vec{Y}_{kn} \vec{V}_n^i \right) \right\}. \quad (11)$$

Once the Q for the PV bus is calculated, it has to be checked for violations of its limits. Additionally, for the PV bus, once the voltage (as per (10)) is obtained, the magnitude is set back to the pre-specified value and the angle is retained. For the VF dependent, buses both active and reactive power are unknown. To calculate the active and reactive power of VF dependent buses droop equations can be used. From (7) and (8), the active and reactive power of the VF dependent bus can be calculated as

$$P_{G_k}^{i+1} = \frac{1}{m_{p_k}}(\omega_o - \omega^i), \quad (12)$$

$$Q_{G_k}^{i+1} = \frac{1}{n_{q_k}}(V_o - V_k^i). \quad (13)$$

If the active and reactive powers from (12) and (13), respectively violate the limit, they are set to their respective limit, and voltage is calculated using (10).

Once the first power flow equation is solved to obtain the voltages of all buses in the system for the $(i + 1)$ iteration, the active power losses in the system are calculated. Now the second step in MGS involves frequency calculations. Since the frequency is global, all the droop buses in the micro-grid will supply active power at the same angular frequency. In an islanded micro-grid with all the DGs operating as droop based DGs, the sum of active powers of all DGs is the total active power generation of micro-grid (P_{sys}) and can be represented as

$$P_{sys} = \sum_{k=1}^d P_{G_k} = \sum_{k=1}^d \frac{1}{m_{p_k}}(\omega_o - \omega) \quad (14)$$

P_{sys} can be replaced by the total active power demand (P_{load}) plus active power loss (P_{loss}). Thus (14), can be modified to the following

$$P_{load} + P_{loss} = \sum_{k=1}^d \frac{1}{m_{p_k}}(\omega_o - \omega) \quad (15)$$

Equation (15) can be rearranged to calculate ω as

$$\omega^{i+1} = \frac{\sum_{k=1}^d \frac{1}{m_{p_k}} \omega_o - (P_{load}^{i+1} + P_{loss}^{i+1})}{\sum_{k=1}^d \frac{1}{m_{p_k}}}. \quad (16)$$

Now the second power flow equation is solved for ω^{i+1} using (16). As the frequency changes in each iteration, Y_{bus} is calculated in each iteration and updated values are used to calculate the voltages. The error ($\Delta \mathbf{x}$) is evaluated, which is the difference of \mathbf{x}^i and \mathbf{x}^{i+1} . If the convergence criterion is met, the system line flows are evaluated.

IV. VALIDATION OF THE PROPOSED METHODS

To validate the proposed power flow method, the method is applied to a 6 bus test system (shown in Fig. 2). The parameters for the test systems are given in Appendix. Values of active and reactive power static droop gains are $9.4 \times 10^{-5} \text{ rad/s/W}$ and 0.0013 V/VAR , respectively. Three cases have been studied.

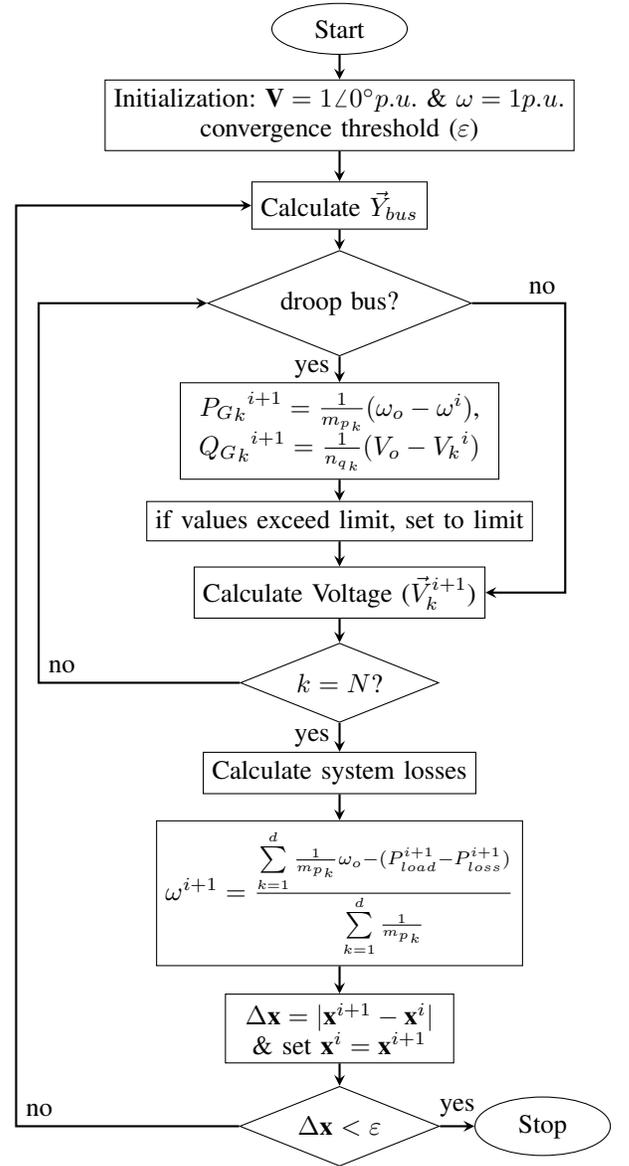


Fig. 1. MGS power flowchart

In case I, constant power load type ($L_T = KP$) is considered and the results are presented in Table II. Total active power loss in this case is 0.28 p.u., and system frequency converges to 0.99903 p.u. In case II, constant impedance load type ($L_T = KI$) is considered and the results are presented in Table III. Total active power loss in this case is 0.24 p.u., and system frequency converges to 0.99911 p.u. All the DGs supply same amount of active power because all DGs operate in droop mode and the droop coefficients are same for all DGs.

In case 3, MGS is validated for an islanded microgrid with mix of DGs operation. DG#1 operates in PV mode while DG#2 and DG#3 operate in droop mode. The PV bus (bus#3) supplies a fixed active power of 4.0 p.u. while regulating its voltage to

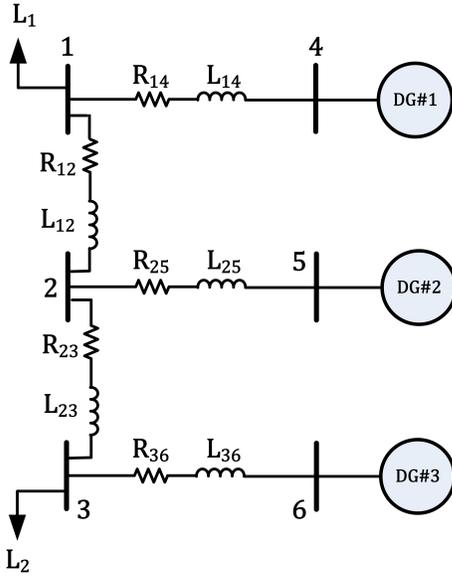


Fig. 2. Six-bus test system

TABLE II. VALIDATION RESULTS FOR CASE I

Bus	Voltage (p.u., degree)			
	Mag.		Ang.	
	MGS	PSCAD	MGS	PSCAD
1	0.9565	0.9566	0	0
2	0.9703	0.9704	-0.5603	-0.5604
3	0.9611	0.9610	-2.8710	-2.8719
4	0.9861	0.9861	-0.0875	-0.0877
5	0.9893	0.9893	-0.4775	-0.4778
6	0.9670	0.9670	-3.0696	-3.0702
Active and Reactive Power (p.u.)				
	P		Q	
	MGS	PSCAD	MGS	PSCAD
1	-4.8420	-4.8420	-3.2040	-3.2040
2	0	0	0	0
3	-6.4350	-6.4350	-4.5480	-4.5480
4	3.8529	3.8529	1.9259	1.9259
5	3.8529	3.8529	1.4781	1.4781
6	3.8529	3.8529	4.5635	4.5635
System Frequency (p.u.)				
	MGS	PSCAD		
	0.99904	0.99904		

1.002 p.u. Results for case 3 are shown in Table IV. The results from MGS power flow method are compared with the results obtained from detailed time domain PSCAD/EMTDC model to show the accuracy of the proposed method. The number of iterations in all the cases were less than 25. The maximum voltage magnitude, phase angle and system frequency error in all cases is less than 0.01%, 0.1% and 0.001%, respectively.

V. CONCLUSION

In this paper, Modified Gauss Seidel (MGS) is proposed to solve the power flow analysis for islanded micro-grids. The proposed algorithm incorporates Y_{bus} modeling, load modeling, and the DG modeling. DGs are formulated as droop buses. The proposed scheme imitates the concept of

TABLE III. VALIDATION RESULTS FOR CASE II

Bus	Voltage (p.u., degree)			
	Mag.		Ang.	
	MGS	PSCAD	MGS	PSCAD
1	0.9601	0.9600	0	0
2	0.9726	0.9725	-0.5211	-0.5213
3	0.9639	0.9639	-2.6701	-2.6706
4	0.9873	0.9872	-0.0738	-0.0739
5	0.9901	0.9901	-0.4457	-0.4458
6	0.9694	0.9694	-2.8535	-2.8538
Active and Reactive Power (p.u.)				
	P		Q	
	MGS	PSCAD	MGS	PSCAD
1	-4.4630	-4.4630	-2.9532	-2.9532
2	0	0	0	0
3	-5.9792	-5.9792	-4.2258	-4.2258
4	3.5606	3.5606	1.7617	1.7617
5	3.5606	3.5606	1.3686	1.3686
6	3.5606	3.5606	4.2340	4.2340
System Frequency (p.u.)				
	MGS	PSCAD		
	0.99911	0.99911		

TABLE IV. VALIDATION RESULTS FOR CASE III

Bus	Voltage			
	Magnitude (p.u.)		Angle (deg)	
	MGS	PSCAD	MGS	PSCAD
1	0.9703	0.9704	0	0
2	0.9780	0.9781	-0.1683	-0.1684
3	0.9656	0.9656	-2.4136	-2.4139
4	1.0020	1.0020	-0.2951	-0.2964
5	0.9938	0.9939	0.0134	0.0134
6	0.9708	0.9708	-2.5877	-2.5885
Active and Reactive Power (p.u.)				
	P		Q	
	MGS	PSCAD	MGS	PSCAD
1	-4.5585	-4.5585	-3.0164	-3.0164
2	0	0	0	0
3	-6.0000	-6.0000	-4.2405	-4.2405
4	4.0000	4.0000	2.5689	2.5689
5	3.4051	3.4051	0.8570	0.8570
6	3.4051	3.4051	4.0369	4.0369
System Frequency (p.u.)				
	MGS	PSCAD		
	0.99915	0.99915		

operation of micro-grid, where DGs share the load demands keeping the system frequency and voltages within specified limits. The method has been tested on a test systems under different load dependency conditions. A good agreement of the results indicates the accuracy of the proposed method. The convergence of GS is often criticized with the increase in size of the test system, but in case of islanded micro-grids, the proposed method can be a useful tool for planning and operation.

APPENDIX

Line data for the 6-bus test system is given in Table V.

TABLE V. PARAMETERS FOR THE 6-BUS TEST SYSTEM

Three identical DGs (10kVA 3- ϕ , 220V (L-L), 60Hz)					
Line Parameters				Load connected to F	
F	T	$R_{\phi}(\Omega)$	$X_{\phi}(mH)$	$R_{L\phi}(\Omega)$	$Q_{L\phi}(mH)$
1	2	0.43	0.318	6.95	12.2
2	3	0.15	1.843	0	0
3	6	0.05	0.050	5.014	9.4
4	1	0.30	0.350	0	0
2	5	0.20	0.250	0	0

F=From bus, T=To bus

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