MAXIMUM ENTROPY FLOWS FOR SINGLE-SOURCE NETWORKS

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ABSTRACT

This paper was prompted by growing evidence that Shannon's measure of uncertainty can be used as a surrogate reliability measure for water distribution networks. This applies to both reliability assessment and reliability-governed design. Shannon's measure, however, is a non-linear function of the network flows. Therefore, the calculation of maximum entropy flows requires non-linear programming. Hence, a simpler, more accessible method would be most useful. This paper presents an alternative and rigorous method for calculating maximum entropy flows for *single-source* networks. The proposed method does not involve linear or non-linear programming. Also, it is not iterative. Consequently, the method is very efficient. In this paper, the methodology is described, several examples are presented and an algorithm is suggested.

KEYWORDS: Networks, water supply, entropy, reliability

NOTATION

E_n, E_{kn} = event identified by subscript

I = set of all source nodes

K = arbitrary positive constant

N = number of nodes

ND_n = set of all links or nodes immediately downstream of node n

NP_n = number of paths from the source to node n

NU_n = set of all links or nodes immediately upstream of node n

$p_{0i} = q_{0i}/T_0$

$p_{nj} = q_{nj}/T_n$

$p(E_{kn})$ = probability of E_{kn}

$P_n \equiv p(E_n)$ = probability of E_n

q_{i0} = external outflow at node i

q_{nj} = flow from node n to node j

q_{0i} = external inflow at node i

S = Shannon's entropy

S_n = entropy of outflows at node n

S_0 = entropy of external inflows

T_n = total outflow from node n

T_0 = total supply or demand

x^* = optimum value of x

1. INTRODUCTION

Very recently, there has been growing interest in the potential applications of the maximum entropy formalism in water distribution networks. The areas of interest, so far, have been assessment of reliability, layout optimization and/or optimum design with reliability considerations. So far, the results have been encouraging.

Awumah, Goulter and Bhatt[3,4] used the Shannon entropy function[5] as the basis of some measures for network redundancy. The development of these measures involved some intuitive input. They presented evidence[4] that the measures could be used for the design of reliable water distribution networks. The minimum cost gradient formulation of Quindry, Brill and Liebman[7] was modified slightly by replacing all the minimum diameter constraints by minimum nodal entropy limits, one for each node. Awumah, Goulter and

Bhatt[4] observed that the modified, non-linear model, could be used for optimizing both the layout and diameters of the network.

Also, Awumah and Goulter[2] obtained trade-off curves for a sample network. These included a curve for cost versus reliability. The reliability measure used was node pair reliability. A second curve related cost to network entropy. The shape of these curves showed a remarkable degree of similarity. If this holds for water distribution networks generally, it could be interpreted as evidence of a close relationship between entropy and (mechanical) reliability.

Tanyimboh and Templeman[9] have rigorously established the appropriate entropy function for the flows of a looped transportation network. The approach relies on a multiple probability space model and the conditional entropy formula of Khinchin[8]. It applies to any network with known nodal inflows and outflows. Also, it requires a specified flow direction for each arc. Maximum entropy flows were calculated for a sample network and it was observed that, for the sample network, there was uniformity in the probability that certain key nodes receive their flow from each source. The importance of this uniformity in the context of reliability was highlighted.

Subsequently, Tanyimboh and Templeman[10] collated evidence of the need for uniformity of the flows and/or diameters of a distribution network. The multiple probability space conditional entropy measure was then incorporated as a constraint in a non-linear cost minimization model. They observed that as the specified lower bound upon the value of entropy was increased, so too did the resilience of the resulting design. Resilience represents the ability to handle load patterns which are different from those specifically designed for; in the example of Ref. [10] these different cases were pipe failures and/or large fire-fighting demands. Also, it was noted that, on average, the diameters increased. This was taken to be evidence of a correlation between entropy and (mechanical) reliability. Another related observation, and perhaps the

most important, was that implicit tree-type branchedness (Templeman[11]) decreased as the entropy increased.

The above research provides ample justification for more extensive research into the properties and other potential applications of maximum entropy flows. However, the entropy function is non-linear. Also, maximum entropy flows must satisfy flow equilibrium at each node. Consequently, the determination of maximum entropy flows needs constrained non-linear programming. This is rather restrictive. A simpler way of calculating maximum entropy flows would therefore be highly desirable. This paper presents a simple method for calculating maximum entropy flows for *single-source* networks. The proposed method is path based. However, explicit path enumeration is not used. This has been made possible through a simple, but efficient, algorithm for determining the number of paths from the source to each node. Also, neither linear nor non-linear programming is involved. Further, unlike non-linear programming, the procedure is not iterative. Thus the method has a very high computational efficiency.

In this paper, first, the multiple probability space entropy function is presented. Then, some of the results obtained by Awumah, Goulter and Bhatt[4] are interpreted on the basis of this function. This is followed by a description of the proposed method of calculating maximum entropy flows for single-source networks. Examples are solved and it is shown that the results are the same as those given by maximizing the entropy subject to continuity at each node. Also, an algorithm for determining the number of paths from the source to each node is presented. Finally, another algorithm, for calculating maximum entropy flows for a single-source network is presented.

2. ENTROPY FUNCTION FOR LOOPED FLOW NETWORKS

For a distribution network with loops and all the flow directions specified, Shannon's entropy may be written as in Eq. (1).

$$S/K = S_0 + \sum_{n=1}^{N} P_n S_n \tag{1}$$

where S is the entropy (Shannon[9]), K an arbitrary positive constant, and P_n, $n = 1,...,N$, the probability of flow arriving at node n, $n = 1,...,N$, where N is the total number of nodes in the network. The value of P_n may be obtained from Eq. (6), which will be derived shortly. The other terms are defined below. The entropy of the external inflows is S_0 where

$$S_0 = -\sum_{i \in I} p_{0i} \ln p_{0i} \tag{2}$$

In the above equation, I is the set of all source nodes and p_{0i} is the proportion of the total supply to the network that is provided by source i. Its value is given by

$$p_{0i} = \frac{q_{0i}}{\sum_{i \in I} q_{0i}} = \frac{q_{0i}}{T_0}, \quad \forall i \in I \tag{3}$$

where q_{0i} is the external inflow at node i and T_0 is the total supply or demand. In Eq. (1), S_n, $n = 1,...,N$, is the entropy of the outflows, including any demand, at node n, $n = 1,...,N$. It is defined by Eqs. (4) and (5) in which p_{nj} is the fraction of T_n carried by link nj, where T_n, $n = 1,...,N$, is the total outflow, including any demand, from node n, $n = 1,...,N$. Also, p_{nj}, $\forall n$, $j = 0$, represents the fraction of T_n that satisfies consumption at node n.

$$S_n = -\sum_{nj \in ND_n} p_{nj} \ln p_{nj}, \quad \forall n \tag{4}$$

The set ND_n, $n = 1,...N$, consists of all the outflows, including any demand, from node n, $n = 1,...,N$.

$$p_{nj} = \frac{q_{nj}}{\sum_{nj \in ND_n} q_{nj}} = \frac{q_{nj}}{T_n}, \quad \forall n, \forall nj \in ND_n \tag{5}$$

The symbol q is used for both internal and external inflows and outflows. For an external inflow, the first subscript will be zero and the second, the source node number. Also, the second subscript for a demand will be zero whereas the first will be the number of the corresponding node. Otherwise, q_{ij}, $i, j = 1,...,N$, is the pipe flow from node i to node j. Tanyimboh and Templeman[10] stated that Eq. (6) is a convenient formula for the node probabilities, P_n, $n = 1,...,N$.

$$P_n = T_n / T_0, \forall n \tag{6}$$

There follows a simple derivation of this equation. Define E_n, $n = 1,...N$, as the event that a particle in the network reaches node n, $n = 1,...,N$. Then, $P_n \equiv p(E_n)$, $n = 1,...N$, is the probability that the event E_n, $n = 1,...,N$, occurs. In general, each event is conditional upon the events upstream of it. These events are consequently not independent. The probability rule for conditional events may therefore be written, as in Eq. (7), for these events.

$$p(E_k \cap E_n) = p(E_n | E_k) p(E_k), \quad \forall n, \forall k \in NU_n \tag{7}$$

where NU_n, $n = 1,...N$, is the set of all link and external inflows at, and all nodes immediately upstream of, node n, $n = 1,...,N$. By virtue of Eq. (7), the entropy of a network cannot be a simple sum of the entropy at each node (Khinchin[8]).

The ratio q_{ij}/T_0, $i, j = 1,...,N$, is the flow carried in link ij, expressed as a proportion of the total supply. In other words, q_{ij}/T_0 is the probability that a particle entering the network will flow through link ij. For the set of all links converging on a node, these probabilities are mutually exclusive. A particular particle can arrive at the node through any, but only one, of the links. Also, as flow cannot arrive at the node other than by the links converging on that node, the set is exhaustive. For a node $n, n=1,...,N$, E_n happens whenever there is flow in a link kn, $kn \in NU_n$. Let E_{kn}, $\forall n$, $kn \in NU_n$, be the event that there is flow in link kn. Thus

$$p(E_n | E_{kn}) = 1, \ \forall n, \ \ \forall kn \in NU_n \tag{8}$$

The probability of flow arriving at node n is the joint probability that flow reaches the node by all links supplying it, i.e.,

$$p(E_n) = p(\bigcup_{kn \in NU_n} \{ E_{kn} \cap E_n \}) \tag{9}$$

Therefore, applying Eq. (7) for $p(E_{kn} \cap E_n)$,

$$p(E_n) = p(\bigcup_{kn \in NU_n} \{ E_{kn} \cap E_n \}) = \sum_{kn \in NU_n} p(E_{kn} \cap E_n) = \sum_{kn \in NU_n} p(E_{kn}), \quad n=1,...,N. \tag{10}$$

The second equality of Eq. (10) holds because the E_{kn}, $\forall n$, $\forall kn \in NU_n$, have been shown to be mutually exclusive. As explained, $p(E_{kn})$ is given by q_{kn}/T_0. Substituting for $p(E_{kn})$ in Eq. (10) gives the desired result, Eq. (6).

The Eqs. (1) to (6) apply to a network with loops, in which the flow direction in all pipes is defined. Therefore, to use Eq. (1), any non-looped portions of a network under consideration should be omitted when evaluating the network entropy.

The equations suggested by Awumah, Goulter and Bhatt[4] will be examined next. These equations appear superficially to resemble the current Eqs. (1) to (6) but are different in detail. Reasons for preferring the current equations will be presented. Eqs. (6) and (8) of Awumah, Goulter and Bhatt[4] are reproduced

here as Eqs. (A) and (B). In Eqs. (A) to (C), the original notation has been preserved, where possible.

$$\hat{S} = -\sum_{j=1}^{N}\sum_{i=1}^{n(j)} (q_{ij}/Q_0) \ln(q_{ij}/Q_0) \tag{A}$$

where $\hat{S}$ is the network entropy and Q_0 is the sum of all *link flows*, as opposed to T_0 which is the total supply or demand. Also, $n(j)$ is the number of *internal inflows* at node j.

$$\hat{S} = \sum_{j=1}^{N} (Q_j/Q_0)\bar{S}_j - \sum_{j=1}^{N} (Q_j/Q_0) \ln(Q_j/Q_0) \tag{B}$$

in which $\bar{S}_j$ is the entropy of the *internal inflows* at node j. It is the same as S_j in the original publication. Therefore,

$$\bar{S}_j = -\sum_{i=1}^{n(j)} (q_{ij}/Q_j) \ln(q_{ij}/Q_j), \quad j=1,...,N \tag{C}$$

Also, Q_j is the sum of the *internal inflows* at node j, as opposed to T_j which is the sum of all inflows, including external inflows, at the node.

Comparing (A) to (C) with the current (1) to (6), the flows or elementary events, q_{ij}, overlap. This is readily seen from the consideration that $q_{ij} \cap q_{jk} \neq 0$, $\forall i, j, k = 1,...,N$. It follows that the probability-like terms q_{ij}/Q_0 in Eq. (A) are not independent. As such, Eq. (A) (or the equivalent Eq. (B)) is not appropriate for those terms (Shannon[9]).

Also, perhaps the most obvious difference between Eqs. (1) and (A) is that the latter does not (directly) account for the spatial distribution of the external inflows and outflows. In Eq. (1), the relative magnitudes of the sources is accounted for by S_0. Also, the abstraction at each node is accounted for. This is achieved by defining the nodal entropies S_n as the entropy of all the outflows, including consumption, at each node.

3. CALCULATING MAXIMUM ENTROPY FLOWS FOR SINGLE-SOURCE NETWORKS

In a single-source network, all paths start at the source. Consider any demand node served by more than one path. Given no further information about the paths, there is no reason for any path to be preferred over any other path to the demand node. This accords with Laplace's principle of insufficient reason. More appropriately, it is a direct consequence of the maximum entropy formalism[6]. Therefore, all the paths supplying a node should have the same probability of doing so. This means that flow to the node should be distributed equally amongst all the paths supplying the node.

Therefore, to obtain the maximum entropy flows, each node should be taken in turn and its demand divided equally amongst all paths supplying the node. The final network flows are then obtained by superposition of these path flows. That is, the flow for all paths through a link should be added to obtain the flow in that link. These are the link maximum entropy flows. The maximum value of the flow entropy for the network may then be calculated, using Eq. (1).

The network of Figure 1 will be used to demonstrate the above points. The demand at each node is treated separately, as shown in Figure 2. In Figure 3, the flow in each arc is obtained by adding the flow in all paths using that arc. These are the maximum entropy flows. Substituting these flows in Eq. (1) gives $(S/K)^* = 2.159$

To check that the above values are correct, Eq. (1) was maximized, subject to flow equilibrium at each node and non-negativity of all the link flows. First,

all the flows were expressed in terms of three selected flows using four of the nodal continuity equations. Equilibrium at the fifth node holds automatically because the inflows balance the outflows. The resulting objective function, with only three variables, was then maximized subject to lower bounds on these variables. The lower bounds were calculated from the non-negativity conditions. The NAG library routine E04JAF was used for the maximization. The results are shown below. They are identical to those obtained above.

$$(S/K, q_{14}, q_{13}, q_{23}, q_{35}, q_{12}, q_{25}, q_{34})^*$$

$$= (2.159, 5.000, 18.000, 18.000, 16.000, 26.000, 8.000, 10.000)$$

Maximum Entropy Flows Algorithm and Example

The method of superposition used in Figures 2 and 3 is not very practical. First, nodal flow routing, as in Figure 2, requires the tracing of all paths to each node. Also, a knowledge of the interdependencies between the paths serving each node is needed. For example, consider Figure 2d. Link 1-2 carries a flow of 16 units because it is known that the link is shared by two paths serving node 5. On the other hand, the flow in link 1-3 is 8 units because it is known that only one path serving node 5 uses that link. Furthermore, in a looped network, each link will be processed many times, as in Figure 2, for example. It would be more efficient if the flow in each link could be calculated in a single operation. In consequence, the approach of Figure 2 and 3 is quite laborious. Also, the effort needed increases very significantly as the number of nodes or links increases. The method to be described next is derived from the method of superposition of path flows which has been presented. However, it addresses all the above weaknesses, except path enumeration, which will be dealt with shortly. The description of the method is fairly general but is based on the network of Figure 1 for clarity. Following the description, algorithms are presented for the proposed method.

Consider Figure 4. The number of paths to each node is enclosed in a box next to the node. Nodes 4 and 5 are terminal nodes, which do not have any link outflows. The procedure starts with any terminal node; say, node 4. The total outflow at that node is divided by 3, this being the number of paths to it. The quotient is then multiplied by 1 and 2 respectively, these being the respective number of paths to nodes 1 and 3, the immediate upstream supply nodes to node 4. The products, respectively, are the flow in links 1-4 and 3-4.

The next step is to choose any node immediately upstream, whose link outflows have all been calculated. The procedure explained for node 4 is repeated with *all* the appropriate upstream nodes taking the place of nodes 1 and 3. Returning to Figure 4, both nodes 1 and 3 have unknown link outflows. In consequence, they cannot be treated yet.

At this point, the procedure stops and re-starts at any terminal node that has not yet been dealt with. That is, node 5 in the present example. If node 5 is processed as explained for node 4, the flow in link 2-5 is 8 units and for link 3-5, 16 units.

At this stage, the only unprocessed node with all its outflows known is node 3. Its total outflow is 36 units. This flow is partitioned according to the aforementioned procedure in the ratio 1:1 between the two incoming links. This is equivalent to dividing the total outflow from node 3 by 2 and multiplying the result by 1 in each case.

The flow in link 1-2 can now be found. It is the sum of the outflows from node 2, including the demand at node 2. The process ends here.

The flows obtained by the procedure just described are identical to those found by superposing equal path flows for each node.

A further refinement to the method concerns path enumeration. Path enumeration is not a realistic proposition, even for networks of only modest size. However, there is a way round this difficulty. It can be observed, for example in Figure 4, that each boxed number is the sum of all the boxed numbers immediately upstream. In other words, the number of paths to each node is the sum of the number of paths to all nodes upstream of, and directly supplying, the node being considered. This is a fact which can be exploited to weight the nodes and thus avoid explicit path enumeration. The steps involved are as follows.

1. Assign [1] to the source.

2. Select any node whose upstream nodes have all been processed. Add the numbers assigned to all nodes immediately upstream of the chosen node. Assign the total to the present node.

3. Repeat step 2 until all nodes have been processed.

Throughout, the assumption that the direction of flow in each link is known continues to apply. Also, it must be noted that this method of calculating the number of paths to each node applies to single-source networks only.

A final detail of the proposed method for calculating maximum entropy flow for single-source networks is concerned with the order in which nodes can be processed when weighting the nodes or calculating link maximum entropy flows. For both node weighting and flow calculation, it is desirable to know, at each stage, which nodes can be processed. When calculating flows, a node can be processed only if all its link outflows are known. Consider Figure 4 again. In describing how link flows are calculated, the nodes were selected in the order: 4,5,3,2. Another possible order is: 5,4,3,2. These two sequences are, respectively, the reverse of the two possible sequences for calculating the number of paths supplying each node. Consequently, a possible nodal sequence for flow distribution will be available if the nodes are numbered such that the order matches the node weighting sequence, as in Figure 4, for example. Thus the right sequence is obtained if each node is numbered only after all nodes upstream of it have been numbered. The algorithms presented subsequently herein, for node weighting and for calculating maximum entropy flows, assume that the nodes of the network have been numbered according to this convention. The nodes may therefore be numbered with the following algorithm.

Node Numbering Algorithm

1. Number the source with 1. Set n to 1.
2. Increase n by 1.
3. Select any node whose immediate upstream nodes have all been numbered. Number it with n.
4. If $n = N$, exit. Otherwise, continue.
5. Go to step 2.

Simple algorithms are now presented for node weighting and flow distribution, respectively. Before applying these algorithms, the nodes must first be numbered with the node numbering algorithm. Define NP_n, $n = 1,...N$, to be the number of paths from the source to node $n, n = 1,...,N$.

Node Weighting Algorithm

1. Set n to the source number, 1. Set NP_n to 1.
2. Increase n by 1.
3. Calculate NP_n:

$$NP_n = \sum_{k \in NU_n} NP_k$$

4. If $n = N$, exit. Otherwise, continue.
5. Go to step 2.

Flow Distribution Algorithm

1. Set n to the number of nodes, N.
2. Calculate T_n:

$$T_n = \sum_{nj \in ND_n} q_{nj}$$

3. Calculate q_{kn}, $\forall kn \in NU_n$:

$$q_{kn} = T_n \times \frac{NP_k}{NP_n}$$

4. If $n = 1$, go to step 7. Otherwise, continue.
5. Reduce n by 1.
6. Go to step 2.
7. Calculate S^*, if necessary. Exit.

4. GENERAL NETWORKS

The algorithms in this paper are rigorous for single source networks. However, they are in general inapplicable to multiple-source networks, for the following reasons. The proposed method is a direct application of the following result: maximization of Shannon's entropy function, subject only to normality of the probabilities, leads to the uniform probability distribution in which all the probabilities are equal. The corresponding result for a general network with multiple sources requires all the sources to contribute the same quantity of flow to the total supply; S_0 in Eq. (2) attains its highest possible value if all the p_{0i} are equal. In general, this condition will not be met as the flow from different sources will usually be unequal. Furthermore, the flow directions in a multiple-source network will be such that, for each node, the path flows will be unequal in general.

However, in any network, if the flow directions and the distribution of the source flows are such that all paths serving a node can carry the same amount of flow, the proposed method will give the right result. This also applies to any multiple-source network that is effectively operating as a single-source network. Two examples are next provided to illustrate some of the above points. However, it must be stressed that the proposed method is intended as an alternative to numerical optimization, for single-source networks only.

A sample two-source network in which all the conditions for uniformity of the path flows are satisfied is shown in Figure 5a. Link 1-2 is a direct connection between the sources. The node weighting algorithm may be applied to multiple-source networks with source-source connection. However, each source is given a weight of unity in step 1. Thus, suppose the sources were replaced by a supersource numbered 0 with 55 units. Suppose, further, that link 1-2 were replaced by a direct link from the supersource to nodes 1 and 2 respectively as shown in Figure 5b. If the node weighting algorithm is carried out on this transformed, but equivalent network, both nodes 1 and 2 would be assigned a weight of 1. This provides confirmation that each source in a multiple-source network, with *all* sources interconnected, should have a weight of unity. It may be noted that there need not be a direct link for every source-source combination. It is sufficient that each source be directly connected to at least one other source.

To obtain the maximum entropy flows for multiple-source networks with source-source connections, the flow algorithm may be applied as described for single-source networks, but with a slight modification. In step 2, $(T_n - q_{0n})$ is found, and used in step 3, instead of T_n . The external inflow q_{0n} will be zero for all nodes other than source nodes. Also, this modified version of the algorithm may be used for networks having a single source. Finally, using Eq. (1), the value of S^* may be calculated, once the maximum entropy flows are available. The problem of Figure 5a was solved by both numerical optimization using the NAG library routine E04JAF and the present method. Both methods gave the same result, of which the flows are shown in Figure 5a.

However, in a general network, if at least one of the requirements for equality of path flows is not satisfied, the single-source method cannot be used. For example, in Figure 6, the direction of flow in the source-connecting link is the reverse of the direction in Figure 5a. The problem of determining the maximum entropy flows for the network of Figure 6 cannot be solved by the present single-source method. This problem was solved using the NAG library routine E04JAF. The vector describing the optimum point is

$$(S/K, q_{23}, q_{13}, q_{34}, q_{35}, q_{21}, q_{14}, q_{25})^*$$

$$= (1.947, 12.917, 28.871, 8.871, 22.917, 0.000, 6.129, 7.083)$$

That the optimum point contains a zero element is an indication that the single-source method will not solve this problem. This example shows that the present method is in general inapplicable to multiple-source networks.

Comments on Figures 5a and 6

Several interesting comments may be made on the maximum entropy flows of Figure 5a and 6. Figure 6 has a lower value of S^* = 1.947 than Figure 5a, whose value is 2.020. This bodes well for the possible use of flow entropy in layout and reliability optimization for several reasons. First, q^*_{21}, but not q^*_{12}, being zero is desirable. It would be reasonable to augment a smaller source for even greater flexibility. On the other hand, because of the position of node 3, there is no need for flow to be transferred from source node 2 to source node 1. At the optimum, the network of Figure 5a, with three loops, has more redundancy than the network of Figure 6, with two loops. It is therefore fitting that Figure 5a, with a better layout and higher level of flexibility/redundancy, should have a higher value of entropy. Also, by correctly setting certain link flows to zero, entropy maximization has the capability of identifying superfluous links. This is highly desirable in the context of layout optimization. Furthermore, in both Figures 5a and 6, the flow from node 3 to node 4 is greater than the direct supply from source node 1 to node 4.

Similarly, the flow from node 3 to node 5 is greater than the direct supply from source node 2 to node 5, in both figures. This is desirable, from a resilience/flexibility standpoint, if there is variation in the source supplies. *Node 3 has a direct connection to both sources and the flow in links 3-4 and* 3-5 may vary considerably if the source supplies vary. Therefore, designing these links to have a larger capacity would enhance the network's flexibility. Similarly, a larger capacity for these links is desirable if the demands at nodes 4 and 5 vary. The same arguments as for varying source supplies apply.

5. CONCLUSIONS AND SUMMARY

A rigorous, simple, non-iterative algorithm for calculating maximum entropy flows for single source networks has been presented. Although the method is path-based, a simple node-weighting technique is used to avoid path enumeration. The above properties give the procedure a high computational efficiency. This can be very useful in a design or reliability framework, where very many function evaluations may be necessary. The routine lends itself to both manual computations, for small networks, and implementation on a computer, for large systems.

A suggested possible application of the proposed approach is in the design of flexible single-source water distribution networks by linear programming. In the Alperovits and Shamir[1] method, for example, the proposed routine would give the flows that the pipes should be designed to carry. The gradient step of the linear programming gradient method would not be needed. Also it would be useful to compare the reliability of such a design to other designs. Furthermore, there is some evidence (Tanyimboh and Templeman[10]) that the cost of a water distribution network designed to carry maximum entropy flows may not be much higher than if the network is designed with minimum-sized loop-completing links.

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FIGURE CAPTIONS

Figure 1	Single-source network.
Figure 2	Equal path flows from the source (node 1) to each of the demand nodes 2,3,4 and 5.
Figure 3	Maximum entropy flows for the network of Figure 1 found by superposing the path flows of Figure 2.
Figure 4	Number of paths to each node for the network of Figure 1.
Figure 5a	Maximum entropy flows for a two-source network with equal path flows to each demand node.
Figure 5b	Supersource representation of the network and flows of Figure 5a.
Figure 6	Maximum entropy flows for a two-source network with unequal path flows to each demand node.

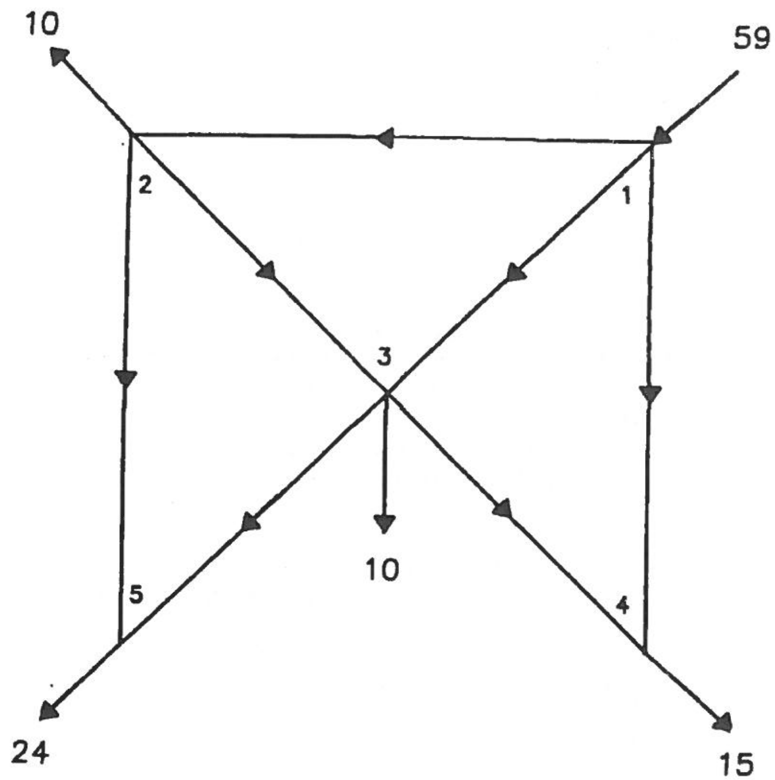

Figure 1 Single source network

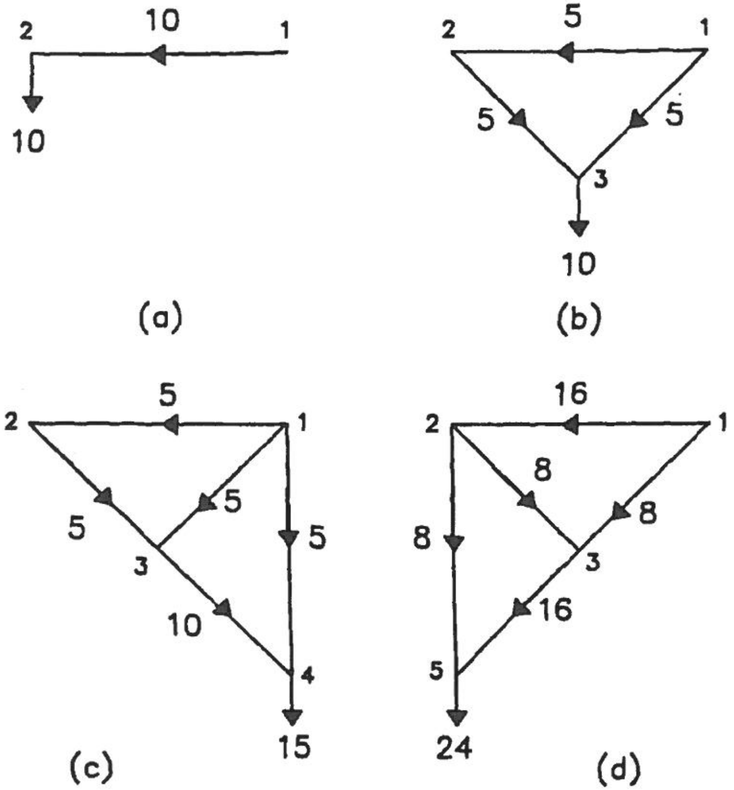

Figure 2 Equal path flows from the source (node 1) to each of the demand nodes 2,3,4 and 5

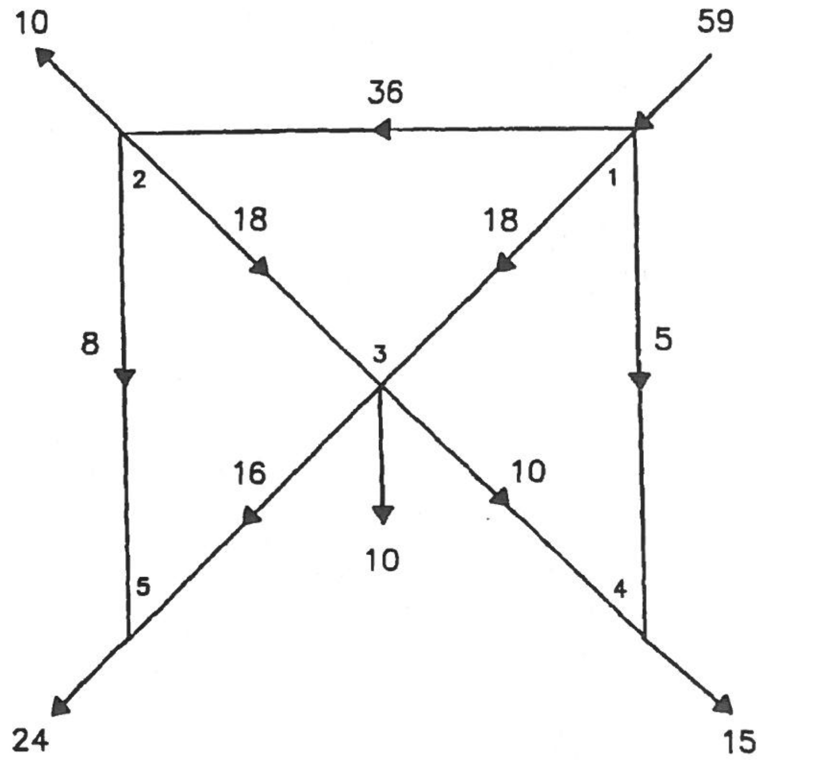

Figure 3. Maximum entropy flows for the network of Figure 1 found by superposing the path flows of Figure 2

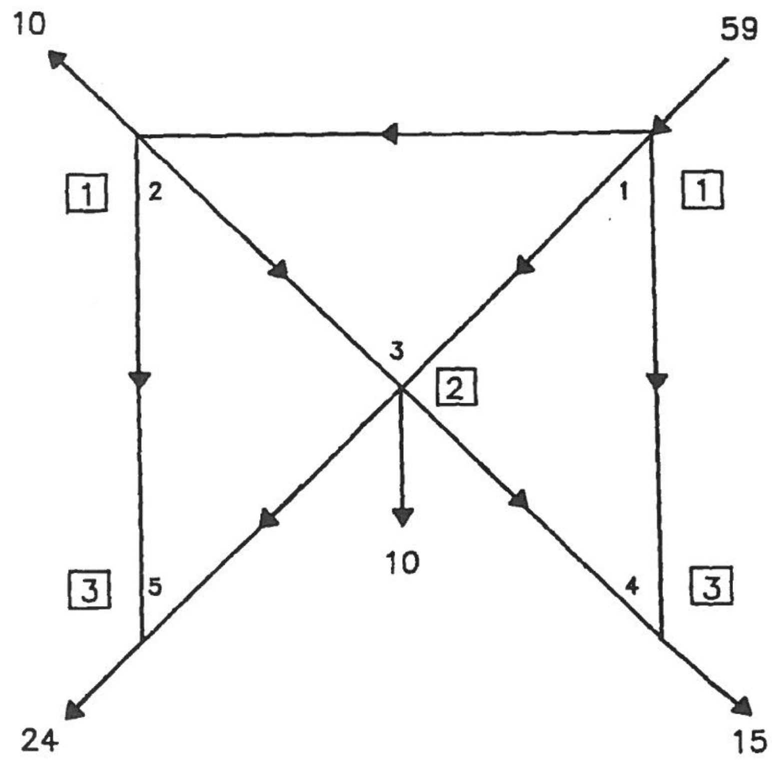

Figure 4 Number of paths to each node for the network of Figure 1

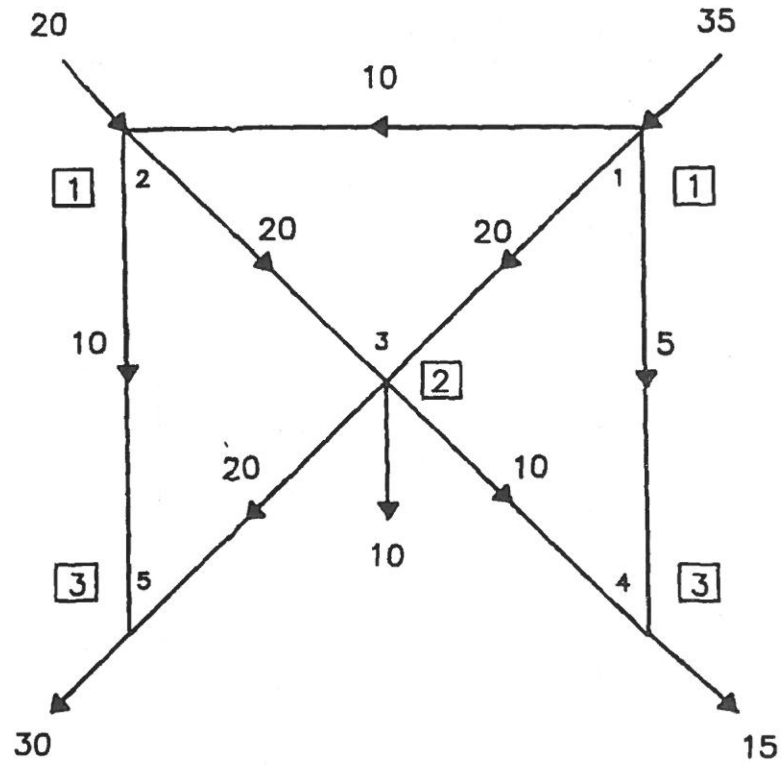

Figure 5a Maximum entropy flows for a two-source network with equal path flows to each demand node.

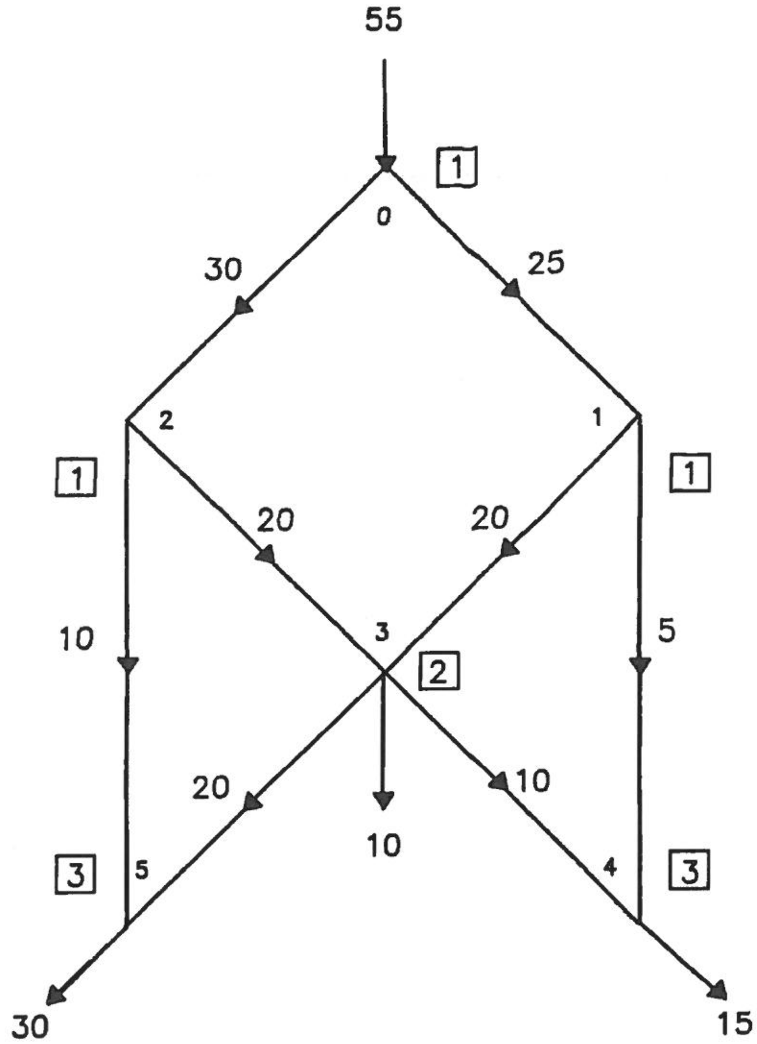

Figure 5b Supersource representation of the network and flows of Figure 5a.

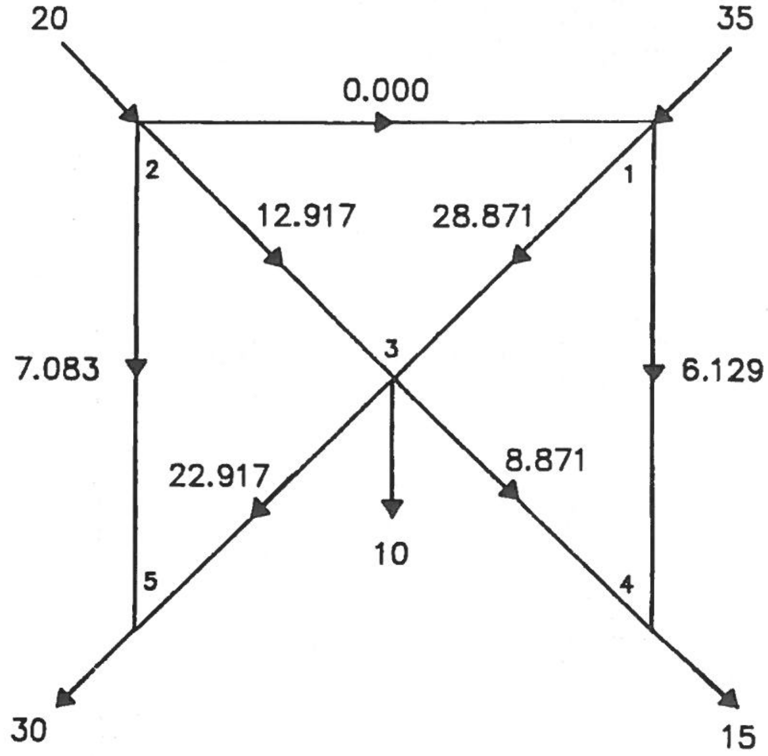

Figure 6 Maximum entropy flows for a two-source network with unequal path flows to each demand node