Photon energy upshift by gravitational waves from a compact source

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We consider the propagation of light from an isolated source that also emits gravitational waves. The eikonal approach is employed to determine the transfer of energy from the gravitational to electromagnetic radiation. A mechanism is found in which a photon "surfs" on the gravitational wave. For black hole events, a significant upshift of photon energy can occur according to a power-law buildup over the radial distance. This surprising effect may be responsible for some of the unexplained high energy phenomena in the cosmos involving gamma rays or other astro-particles.

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The ray description of light has long offered a powerful theoretical tool in probing spacetime structure and its effect on matter kinetics. It is the very approach adopted by Einstein, together with the constancy of the speed of light, in arriving at the celebrated special relativity a century ago. The subsequent advent of general relativity requires far more sophisticated analysis of null geodesics and light cones to unravel local and global spacetime properties that are often counterintuitive. These include the existence of black hole and cosmological horizons as well as gravitational lensing, focusing and red/blue shifts.

The anticipated direct detection of gravitational waves (GWs) [1] and ongoing intense research into GW interaction with matter [2, 3, 4] and astro-particle physics [5] have stimulated a great surge of interest in novel astronomical phenomena that involve both high energy jets and gravitational radiation [1, 6]. For events as violent as the core-collapse of stars and merger of black holes, one of the largest portions of energy released is through gravitational radiation. This leads to a number of important issues. For instance, does there exist any mechanism by which some of the GW energy may be transferred to photons and other particles? If such a mechanism exists, how relevant is it for energetic events like gamma ray bursts (GRBs) [1, 7, 8]? Furthermore, to what extent can such a mechanism be responsible for the unexplained sources of energy for the observed ultra-high energy cosmic rays [5]?

In this letter, we address these issues by exploring a hitherto undiscovered process in which light particles and GWs from an unstable compact source may interact in a resonant manner, causing significant energy gain by the particles. In particular, we shall focus on the GW interaction with photons with a view to further dedicated treatments for other particles such as neutrinos. The ray description of light propagation will be adopted, as appropriate for photons (at 1 eV or larger) having a wavelength much shorter than that of the GWs (of a few km or larger) under consideration.

In some sense, the effect of photon energy upshift by GWs is in close analogy with that by plasma waves [9, 10]. In the latter case, as a medium is involved, photon energy upshift is accompanied by the acceleration of the photon group velocity, the effect has been coined as "photon acceleration". Some pioneering theoretical works have been done to replace plasma waves by idealized plane GWs [2, 3], where it has been found that the photon energy may increase for a period before it falls. Recent development of the "collapsar" model of GRBs strongly suggests that gamma ray bursters may emit significant GWs [1, 7, 8]. However, to date no attempts have been reported to couple the predicted GWs with the gamma rays. In view of this, the present work will consider the gravitational radiation and explore the consequent interaction with co-moving photons.

Below, the geometric units of $c = G = 1$ are adopted unless otherwise indicated. The notation for differential geometry follows mainly that of [11]. In particular, the Greek and Latin coordinate indices range over 0–3 and 1–3 respectively. The essential picture to envisage consists of an isolated source of GWs and photons. The former perturb the asymptotically flat spacetime such that, in coordinates $(x^\mu) = (x^0, x^a) = (t, x, y, z)$, the spacetime metric assumes the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and $h_{\mu\nu} \to 0$ at infinity. This paradigm is appropriate for astrophysical conditions where the expansion of the Universe is negligible, as will be justified towards the end of this letter. The total energy of the photons are taken as insufficient to back-react on the GW metric. The radiation from the source will be detected by a remote observer. Both the source and observer are assumed to be stationary with respect to the Minkowski metric $\eta_{\mu\nu}$ and follow the asymptotic "cosmological time" flow given by $\partial/\partial t$.

Note that although there exist a large class of Robinson-Trautman solutions for spherical GWs, the associated singularity structures unfortunately render these solutions unsuitable to described gravitational radiation from an isolated source [12]. In astrophysics, GWs from such a source have very small amplitude in the far field, where the GW interaction with photons under consideration take place. Therefore, the lowest order post-Newtonian approximation to general relativity in terms of linearized gravity provides an adequate description.

In treating the photons, we shall take into account not only their trajectories but also their energy in reference to the "cosmological time" $t = x^0$. To this end, consider the eikonal function $\psi = \psi(x^\mu)$ along the light rays. It gives rise to the wave 4-vector as $k_\mu = \partial\psi/\partial x^\mu$. For any spacetime metric $g^{\mu\nu}$, the eikonal function ψ satisfies the dispersion relation for photons given by $g^{\mu\nu}k_\mu k_\nu = 0$. The angular frequency of the photon with respect to t is given by $\omega := -k_0 = -\partial\psi/\partial t$. (Here ":=" means "is defined as".) This may be compared with the proper frequency ω_τ given by $\omega_\tau := -\partial_\tau\psi = \omega/\sqrt{-g_{00}}$ with respect to the proper time τ along ∂_t such that $\mathrm{d}\tau^2 = -g_{00}\mathrm{d}t^2$. It follows from the dispersion relation that the positive photon frequency ω takes the form

$$\omega = \sqrt{\omega_1^2 + \omega_2^2} - \omega_2 \tag{2}$$

where

$$\omega_1 := \sqrt{\frac{g^{ab}k_a k_b}{-g^{00}}}, \quad \omega_2 := \frac{g^{0a}k_a}{-g^{00}}. \tag{3}$$

The photon dynamics is generated from a variational principle using the action integral

$$S = \int k_\mu \, \mathrm{d}x^\mu = \int (k_a \dot{x}^a - \omega)\, \mathrm{d}t \tag{4}$$

where the over dot denotes a t-derivative. This action takes a canonical form in terms of the phase variables (x^a, k_a) and Hamiltonian $\omega = \omega(x^a, k_a, t)$ specified in (2), yielding the following canonical equations of motion:

$$\dot{x}^c = \frac{\partial\omega}{\partial k_c}, \quad \dot{k}_c = -\frac{\partial\omega}{\partial x^c}. \tag{5}$$

It is important to note that the Hamiltonian ω is necessarily time dependent if the spacetime metric is non-stationary.

Return to the metric of the form (1) and treat $h_{\mu\nu}$ perturbatively to its linear order henceforth. For our purpose it suffices to restrict to $h_{00} = 0$, hence $g_{00} = g^{00} = -1$ (to linear order in $h_{\mu\nu}$), which makes t proper and simplifies subsequent analysis. Substituting this condition and (1) into (2) we see that

$$\omega = k - \frac{1}{2k}\, h^{ab}k_a k_b + h^{0a}k_a \tag{6}$$

where $k^2 := k_x^2 + k_y^2 + k_z^2$. Therefore the canonical equations of motion (5) reduce to

$$\dot{x}^c = \frac{k^c}{k} - \frac{1}{k}\, h^{ca}k_a + \frac{1}{2k^3}\, h^{ab}k_a k_b k^c + h^{0c} \tag{7}$$

$$\dot{k}_c = \frac{1}{2k}\, \partial_c h^{ab}\, k_a k_b - \partial_c h^{0a} k_a. \tag{8}$$

As in standard linearized gravity, consider the trace-reversed metric perturbation $\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}h$, with the trace term $h = \eta^{\mu\nu}h_{\mu\nu}$, satisfying the Lorentz gauge condition $\bar{h}^\nu{}_{\mu,\nu} = 0$. The gravitating source has the full quadrupole moment

$$Q_{ab}(t) = \int \rho\, x^a x^b\, \mathrm{d}^3x \tag{9}$$

where ρ denotes the mass density. We shall evaluate the metric perturbation at a large distance L compared with the size of the source. Within the level of accuracy adopted, $\bar{h}_{\mu\nu}$ is given by the quadrupole formula

$$\bar{h}_{ab} = \frac{2}{L}\, \ddot{Q}_{ab}(t - L) \tag{10}$$

along with $\bar{h}_{00} = \bar{h}_{0a} = 0$ [11]. To express the metric perturbation itself, it is more convenient to introduce the trace-reversed quadrupole moment given by

$$\bar{Q}_{ab}(t) = \int \rho\, (x^a x^b - \frac{1}{2}\,\delta_{ab}\, r^2)\, \mathrm{d}^3x \tag{11}$$

where $r^2 := x^2 + y^2 + z^2$. For simplicity the radial motion inside the source is assumed to be negligible. Hence,

$$h_{ab} = \frac{2}{L}\, \ddot{\bar{Q}}_{ab}(t - L) = \frac{2}{L}\, \ddot{Q}_{ab}(t - L) \tag{12}$$

along with $h_{00} = h_{0a} = 0$. Based on the above discussion we may now investigate the dynamics of photons on the GW background. Of particular interest in our analysis is the asymptotic behaviour of photons originated from the source. It is clear that in the far field the photon trajectories are essentially radial. Let us take a photon travelling, say, in the x-direction. Thus we have $L(t) = x(t)$. For simplicity, introduce $K(t) := k_x(t)$. By retaining only the leading contributions in (7) and (8), we seek to find the asymptotic behaviours of $L(t)$ and $K(t)$ starting from the initial position $L = L_0$ and momentum $K = K_0$ at $t = 0$. This leads to the following equations

$$\dot{L} = 1, \; \dot{K} = \frac{K}{2}\, \partial_x h_{xx}. \tag{13}$$

The first equation above yields immediately $L = t + L_0$. To solve the second equation, we calculate that

$$\partial_x h_{xx} = -\frac{2}{x}\, \dddot{Q}_{xx}(t - x) - \frac{2}{x^2}\, \ddot{Q}_{xx}(t - x) \tag{14}$$

using (12). Neglecting the second term in (14) and substituting the remaining part into (13) we obtain

$$\frac{\mathrm{d}K}{\mathrm{d}L} = Q\, \frac{K}{L} \tag{15}$$

where the relation $L = t + L_0$ has been used to turn K into a function of L and the dimensionless quantity

$$Q := -\dddot{Q}_{xx}(-L_0) \tag{16}$$

has been introduced. It is worth stressing that Q is a constant of motion that depends on the phase of GW at the initial position of the photon. The argument $-L_0$ in (16) counts for the retarded time required for GW to reach $x = L_0$ at $t = 0$ from the centre of the source where $x = 0$. The asymptotic momentum for the photon is therefore obtained by solving (15) to be

$$K = K_0 \left(\frac{L}{L_0} \right)^Q . \tag{17}$$

Remarkably, this expression states that, depending on the sign of Q, the asymptotic momentum of the photon can either increase or decrease monotonically, as the result of the photon "surfing" the GW at different phases.

This statement has to be augmented with certain physical considerations. If the photon's momentum drops to the extent that its wavelength is comparable to that of the GW, then it can no longer be described as a particle and must be treated as a wave. For $Q > 0$, as the photon gains more momentum, it behaves even more particle-like. However, several effects may come into play to cap the photon energy upshift. First, the back-reaction from the photons will become significant if their total energy is comparable to that of the GW. Secondly, the GW amplitude drops as $1/L$ and will cease to dominate over surrounding spacetime geometry when it becomes close enough to the level of stochastic GW background. Above all, upon reaching the energy levels of 10^{12}–10^{15}eV, individual photons travelling along the GW will be scattered by ambient photons including those from starlight and the cosmic microwave background (CMB), by exchanging e^-e^+ pairs, and hence lose the status of being free from non-gravitational interactions.

It would appear from the dispersion relation (6) that the speed of the photon differs from the speed of light, which has been normalized to unity. This is simply because the speed of the photon is evaluated here with respect to the auxiliary Minkowski metric $\eta_{\mu\nu}$. The speed of the photon in vacuo is, of course, the speed of light, when evaluated with respect to the physical curved metric $g_{\mu\nu}$. However, this could cause a concern that photons might de-phase from the GWs that travel at the speed of light with respect to $\eta_{\mu\nu}$. Nonetheless, a closer inspection of this dispersion relation reveals the following important properties. Any photon moving in the x-direction at a GW phase where $h_{xx} = 0$ has speed equal to unity. At this phase if $\partial_x h_{xx} > 0$ then the energy of this photon will continue to increase while co-moving with the GW at the same speed. Photons following not far behind experiencing $h_{xx} < 0$ will catch up while those travelling slightly ahead and experiencing $h_{xx} > 0$ will slow down.

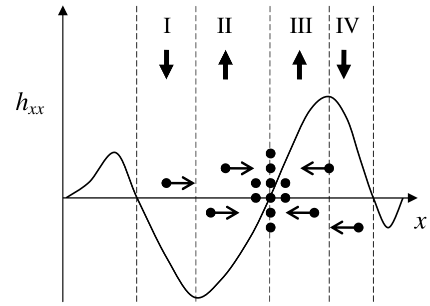

FIG. 1: This diagram illustrates how the energy upshift of photons by an expanding GW is enhanced by "phase lock-in". As shown, consider a portion of the h_{xx} component of the GW far off the radiating source along the x-axis. It is divided into four phases, separated by dashed vertical lines, according to the signs of h_{xx} and $\partial_x h_{xx}$. Owing to the positive $\partial_x h_{xx}$ of phases II and III, the co-moving photons within these phases gain energy, as indicated by upwards arrows. In contrast, photons within phases I and IV, where $\partial_x h_{xx} < 0$, suffer from a loss of energy, as indicated by downwards arrows. With respect to the auxiliary Minkowski metric, photons (dots with right-arrows) within phases I and II, where $h_{xx} < 0$, travel faster than the GW whereas photons (dots with left-arrows) within phases III and IV, where $h_{xx} > 0$, travel slower than the GW. As such, photons tend to be bunched between phases II and III, where $h_{xx} = 0$ and $\partial_x h_{xx}$ may well be maximum. The bunched photons are indicated by the un-arrowed dots.

In the long run, these photons tend to be bunched around the GW phases where $h_{xx} = 0$ and $\partial_x h_{xx} > 0$ thereby enjoying continuous energy rise. This is illustrated in Fig. 1. The same argument applies for photons radiating in any other direction. The result is a modulation of the photon distribution on the GW wavelength scale.

Let us remark on the role of gauge at this point. Within the framework of linearized general relativity using the Lorentz gauge condition (without loss of generality), it is well known that under any infinitesimal coordinate transformation with the generating functions $\xi^\mu = \xi^\mu(x^\nu)$ satisfying $\xi_{\mu,\nu}{}^{\nu} = 0$, the metric perturbation transforms according to

$$h_{\mu\nu} \to h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \tag{18}$$

whereas other metric independent tensor fields stay invariant. It follows that the calculation of the photon wave 4-vector k^μ described above is gauge invariant. To be reassured, let us consider a case study which exemplifies that gauge transformations cannot induce photon energy shift. Let the "metric perturbation" be generated by $\xi_x = f(t - x)$ along with other $\xi_\mu = 0$. Though merely gauge, this form resembles a travelling GW in the x-direction. Yet, does it influence the energy of co-moving photons? To check this, consider the resulting nonzero metric perturbation components given by

$$h_{xx} = 2\dot{f}, \quad h_{tx} = -\dot{f}. \tag{19}$$

It follows at once from (6), (8) and noting that $h^{0a} = -h_{0a}$ that $\omega = k_x$ and $\dot{k}_x = 0$, i.e. no photon energy shift, as expected.

We shall estimate the size of the effect of the photon energy upshift by GWs from a compact astronomical source. To keep our discussion generic it is not necessary to analyze any specific theoretical template for gravitational radiation of high post-Newtonian order. Partly inspired by the recent suggestion that the core-collapse model for GRBs with gravitational radiation may involve fragmentation into two in-spiral compact objects [6], we proceed with the present simplistic lowest order approach to GWs from a binary system. We expect the study of such a system to represent similar processes and to provide good order-of-magnitude guidance on photon energy upshift by gravitational radiation from violent astronomical events.

Consider a binary of black holes of masses m_1 and m_2 with separation R. Denote the total mass by $M := m_1 + m_2$ and reduced mass by $\mu = m_1 m_2/M$. Choosing the origin of (x^a) as the centre of mass, the orbit of the reduced mass on the x-y plane is given by [13]:

$$x = R\cos\Omega t,\; y = R\sin\Omega t,\; z = 0 \tag{20}$$

where $\Omega = \sqrt{M/R^3}$. This yields the nonzero components of the quadrupole moment: $Q_{xx} = \mu R^2\cos^2\Omega t$, $Q_{yy} = \mu R^2\sin^2\Omega t$, and $Q_{xy} = \mu R^2\sin\Omega t\cos\Omega t$. It then follows from (16) that

$$Q = 4\mu R^2\Omega^3\sin 2\Omega L_0. \tag{21}$$

Photons starting from the initial position L_0 such that $Q > 0$ above will be subject to an energy upshift in accord with the power law (17). The maximum energy upshift occurs when $Q = Q_{\max} = 4\mu R^2\Omega^3$. It is instructive to introduce $\lambda := m_2/m_1$ as the ratio of the masses and $\sigma := R/M$ as the separation in the unit of the total mass. In terms of these dimensionless parameters the maximum energy upshift exponent takes the form

$$Q_{\max} = \frac{4\lambda\,\sigma^{-5/2}}{(\lambda+1)^2}. \tag{22}$$

It is remarkable that $Q_{\max}$ is independent of the total mass of the system. Here the greatest effect is achieved with two equal masses, i.e. $\lambda = 1$. In this case we have simply $Q_{\max} = \sigma^{-5/2}$. Its magnitude falls sharply for $\sigma > 2$ corresponding to a separation larger than the sum of the black holes' radii. Although our calculation above is based on the lowest order approach, it has an indicative value. In particular, this expression suggests that a linear photon energy upshift with distance may be attainable for binary black hole mergers with $\sigma = 1$.

The above discussion suggests that the GWs from a black hole forming event may interact strongly with photons from (the vicinity of) the same source, regardless of its total mass. For the formation of a solar-sized black hole, if we take $Q = 1$, then, using (17), a photon with initial energy 1 eV at $L_0 = 150$ km from the source will acquire an energy of up to 1 MeV, i.e. in the gamma ray domain, when travelling a distance of only 1 AU. This photon may travel a further 15,000 light years, corresponding to a moderate 10% of a galactic diameter, to reach the energy of 10^{15}eV before being scattered by CMB. The scattering could initiate a cascade of cosmic rays.

In conclusion, by analyzing the "surfing" of photons in GWs emitted by a compact source, we have arrived at a universal mechanism whereby photon energy may be significantly upshifted. We have demonstrated that photons, under some conditions, are energized linearly with distance. They are also bunched into structures on the GW wavelength scale. This may count for the source of energy carried off by photons observed as gamma ray events. On applying to other astro-particles such as neutrinos, the universality of this mechanism may bear important implications on further problems in ultra-high energy cosmic rays. The progress on research into these directions will be reported elsewhere.

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