COMPARISON OF BROADBAND DIRECTION OF ARRIVAL ESTIMATION ALGORITHMS

Mohamed A. Alrmah[1], Mohamed Nuri Hussin[1], Stephan Weiss[1] and Sangarapillai Lambotharan[2]

[1]Department of Electronic & Electrical Engineering, University of Strathclyde, Glasgow, Scotland, UK
[2] Advanced Signal Processing Group, Dept. of EEE, Loughborough University, Loughborough, UK
{mohamed.alrmah,mohamed.hussin,stephan.weiss}@eee.strath.ac.uk; s.lambotharan@lboro.ac.uk

ABSTRACT

This paper reviews and compares three different linear algebraic signal subspace techniques for broadband direction of arrival estimation — (i) the coherent signal subspace approach, (ii) eigenanalysis of the parameterised spatial correlation matrix, and (iii) a polynomial version of the multiple signal classification algorithm. Simulation results comparing the accuracy of these methods are presented.

1. INTRODUCTION

For broadband direction of arrival (DoA) estimation, powerful narrowband methods such as the multiple signal classification (MUSIC) algorithm [4], are not directly applicable, and approaches e.g. based on performing MUSIC in independent frequency bins are likely to result in poor performance, particularly if signal frequencies do not coincide with frequency bins [1].

Amongst dedicated broadband DoA estimation algorithms, the coherent signal subspace method (CSSM) [6] combines covariance matrices at different frequency bins coherently by means of focussing matrices whose determination has most recently be address by an auto-focussing approach in [7]. A parameterised spatial covariance (PSC) approach [2, 5] scans for possible DoAs using what will later in this paper be termed broadband steering vectors. In [1], we have exploited a polynomial matrix decomposition in [3] to generalise MUSIC to the case of spatio-temporal polynomial covariance matrices.

In this paper, we want to analyse the above three approaches and compare them for a number of example scenarios. To accomplish this, Sec. 2 introduces the data model, with narrow- and broadband approaches to DoA approaches outlined in Secs. 3 and 4. A comparison between CCSM, PSC and polynomial MUSIC (P-MUSIC) is then performed in Sec. 5.

Notation. Matrix and vector quantities are represented by upper- and lowercase bold variables, e.g. $\mathbf{A}$ and $\mathbf{a}$. The Hermitian transpose of $\mathbf{A}$ is denoted as $\mathbf{A}^{\mathrm{H}}$. Polynomial vectors and matrices are written as $\mathbf{a}(z)$ and $\mathbf{A}(z)$, with the para-hermitian $\tilde{\mathbf{A}}(z) = \mathbf{A}^{\mathrm{H}}(z^{-1})$. A transform pair $a[n]$ and $A(z) = \sum_{n=-\infty}^{\infty} a[n]z^{-n}$ is abbreviated as $a[n] \circ\!\!-\!\!\bullet A(z)$.

2. BROADBAND ARRAY DATA MODEL

This section aims to describe the model behind multichannel data collected in a vector $\mathbf{x}[n] \in \mathbb{C}^M$ by an M-element array. We assume that K far-field sources illuminate the array and contribute to $\mathbf{x}[n]$ in addition to isotropic white noise $\mathbf{v}[n]$,

$$\mathbf{x}[n] = \sum_{k=1}^{K} \mathbf{s}_k[n] + \mathbf{v}[n] = \sum_{k=1}^{K} \mathbf{a}_k[n] * s_k[n] + \mathbf{v}[n] \quad , \ (1)$$

where $s_k[n]$ is the kth source signal, $\mathbf{a}_k[n]$ the corresponding broadband steering vector, and $*$ the convolution operator, which thereby forms the contribution of the kth source to the array. The model including the steering vector in (5) only considers the angle of arrival, but neglects any attenuation in the medium.

For an arbirary array configuration, whereby $\mathbf{r}_m$ describes the coordinates of the mth array element, the broadband steering vector consists of delays

$$\mathbf{a}_k[n] = [\delta[n - \tau_{k,0}] \ \ldots \ \delta[n - \tau_{k,M-1}]]^{\mathrm{T}} , \qquad (2)$$

with the normalised delay

$$\tau_{k,m} = \frac{\mathbf{t}_k^{\mathrm{H}} \mathbf{r}_m}{cT_{\mathrm{s}}} \quad , \qquad (3)$$

whereby $\mathbf{t}_k$ is the normal vector to the planar wavefront emanating from the kth source, c the propagation speed in the medium, and T_{s} the sampling period.

For $\mathbf{s}_k[n]$ in (1), describing the contribution from the kth source to $\mathbf{x}[n]$, the first sensor signal can be taken as reference, and the relative delays of the remaining sensor signals can be characterised as

$$\mathbf{s}_k[n] = \begin{bmatrix} s_k[n] \\ s_k[n - \Delta\tau_{k,1}] \\ \vdots \\ s_k[n - \Delta\tau_{k,M-1}] \end{bmatrix} , \qquad (4)$$

with $\Delta\tau_{k,m} = \tau_{k,m} - \tau_{k,0}$. For a narrowband source with normalised angular frequency Ω, with a reference signal $s_k[n] =$

$e^{j\Omega n}$, the time delays $\Delta\tau_{k,m}$ collapse to simple phase shifts

$$\mathbf{s}_k[n] = \begin{bmatrix} 1 \\ e^{-j\Omega\Delta\tau_{k,1}} \\ \vdots \\ e^{-j\Omega\Delta\tau_{k,M-1}} \end{bmatrix} e^{j\Omega} = \mathbf{a}_{\Omega,\vartheta_k} e^{j\Omega} \quad , \quad (5)$$

where $\mathbf{a}_{\Omega,\vartheta_k}$ is termed the narrowband steering vector. For further detail, the reader is referred to [1].

As a specific case of (1), for a narrowband scenario with L narrowband sources $s_l[n]$ characterised by pairs $\{\Omega_l, \vartheta_l\}$ the array vector is given by

$$\mathbf{x}[n] = \sum_{l=1}^{L} \mathbf{a}_{\Omega_l,\vartheta_l} s_l[n] + \mathbf{v}[n] \quad , \quad (6)$$

with independent and identically distributed white noise $\mathbf{v}[n]$, such that $\mathcal{E}\{\mathbf{v}[n]\mathbf{v}^{\mathrm{H}}[n-\tau]\} = \delta[\tau]\sigma_v^2\mathbf{I}$.

3. NARROWBAND MUSIC

3.1. Narrowband Covariance Matrix

For narrowband signals with frequency Ω, only correlations for lag zero need to be considered in the covariance matrix $\mathbf{R} = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n]\} \in \mathbb{C}^{M \times M}$, where $\mathcal{E}\{\cdot\}$ is the expectation operator. This covariance matrix entirely describes the data as modelled in the narrowband scenario (6), since in the case of L independent source signals with power σ_l^2, $l \in (1, L)$,

$$\mathbf{R} = \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}_{\Omega,\vartheta_l}\mathbf{a}_{\Omega,\vartheta_l}^{\mathrm{H}} + \sigma_v^2\mathbf{I} \quad . \quad (7)$$

The maximum rank of $\mathbf{R}$, $\mathrm{rank}\{\mathbf{R}\} = M$ is achieved in the case of linear independence of all steering vectors.

For data acquired over a data window of N samples, the data matrix

$$\mathbf{X}_n = [\mathbf{x}[n-N+1]\ldots\mathbf{x}[n-1]\ \mathbf{x}[n]] \quad (8)$$

can be utilised to estimate the covariance matrix as $\hat{\mathbf{R}}_n = \frac{1}{N}\mathbf{X}_n\mathbf{X}_n^{\mathrm{H}}$. Below, we assume an appropriate estimation procedure and for convenient continue to use $\mathbf{R}$ for the analysis.

3.2. Narrowband MUSIC Algorithm

Direct eigenanalysis of the covariance matrix $\mathbf{R}$ can only lead to the correct angles of arrivals for sources, if all steering vectors in (7) are orthogonal. Otherwise, the eigenvalue decomposition (EVD)

$$\mathbf{R} = [\mathbf{Q}_s\mathbf{Q}_n]\begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix}\begin{bmatrix} \mathbf{Q}_s^{\mathrm{H}} \\ \mathbf{Q}_n^{\mathrm{H}} \end{bmatrix} \quad (9)$$

is likely to extract the steering vector of only the strongest source correctly, but otherwise contain orthonormalised basis vectors of the signal subspace in $\mathbf{Q}_s$.

The idea of the MUSIC algorithm is to scan the noise-only subspace $\mathbf{Q}_n$, which is spanned by eigenvectors corresponding to eigenvalues close to the noise floor, $\mathbf{\Lambda}_n \approx \sigma_v^2\mathbf{I}$. The steering vectors of sources that contribute to $\mathbf{R}$ will define the signal-plus-noise subspace $\mathbf{Q}_s$ and therefore lie in the nullspace of its complement $\mathbf{Q}_n$. Therefore, the vector $\mathbf{Q}_n^{\mathrm{H}}\mathbf{a}_{\Omega,\vartheta}$ has to be close to the origin for $\mathbf{a}_{\Omega,\vartheta}$ to be a steering vector of a contributing source. Therefore, in the MUSIC algorithm the inverse of the squared Euclidean norm of this vector, as proposed by [4].

$$P_{\mathrm{MU}}(\vartheta) = \frac{1}{\mathbf{a}_{\Omega,\vartheta}^{\mathrm{H}}\mathbf{Q}_n\mathbf{Q}_n^{\mathrm{H}}\mathbf{a}_{\Omega,\vartheta}} \quad , \quad (10)$$

is calculated as the MUSIC spectrum $P_{\mathrm{MU}}(\vartheta)$.

4. BROADBAND DOA ESTIMATION

4.1. Coherent Signal Subspace Method

The CSSM approach [6] calculates covariance matrices in a number of frequency bins, which are then combined such that their signal subspaces allign into one single correlation matrix to which narrowband high resolution DoA techniques such as MUSIC can be applied. The coherence across different frequency bins is created by a frequency-dependent and unitary focussing matrix $\mathbf{T}(e^{j\Omega})$, such that

$$\mathbf{R}_{\mathrm{coh}} = \sum_{i=n}^{N-1} \alpha_n\mathbf{T}(e^{j\Omega_n})\mathbf{R}(e^{j\Omega_n})\mathbf{T}^{\mathrm{H}}(e^{j\Omega}) \quad , \quad (11)$$

where α_n a weighting for maximum ratio combination of its coherently rotated contributions. In [6] and subsequent derivative works, the focussing matrix $\mathbf{T}(e^{j\Omega})$ is estimated based on a set of steering vectors. A poor estimate of the angle of arrival may also lead to poor results of this approach. However, a recent auto-focussing method in [7] allows computation based on the EVDs of $\mathrm{R}(e^{j\Omega_n})$, $n \in (0, N-1)$, in different frequency bins.

4.2. Parameterised Spatial Correlation Matrix

The idea of the broadband DoA estimation method in [2, 5] is based on testing the zero-lag coherence of a spatial correlation matrix calculated from appropriately pre-steered array data. Knowing the array configuration, a broadband steering vector similar to (2) can be defined for a specific DoA. Assuming a linear array which only resolves a single angle ϑ, the covariance matrix of the pre-steered data is given by

$$\mathbf{R}_\vartheta = \mathcal{E}\{\mathbf{y}_\vartheta[n]\mathbf{y}_\vartheta^{\mathrm{H}}[n]\} \quad (12)$$

$$\mathbf{y}_\vartheta[n] = \begin{bmatrix} x[n-\tau_0(\vartheta)] \\ \vdots \\ x[n-\tau_{M-1}(\vartheta)] \end{bmatrix} = \mathbf{\Gamma}_\vartheta[n] * \mathbf{x}[n] \quad (13)$$

with the delay $\tau_m(\vartheta)$ calculated akin to (3) and the diagonal pre-steering system

$$\mathbf{\Gamma}_\vartheta[n] = \text{diag}\{\delta[n - \tau_0(\vartheta)] \ldots \delta[n - \tau_{M-1}(\vartheta)]\} \ . \quad (14)$$

The proposed method then evaluates the maximum eigenvalue of $\mathbf{R}_\vartheta$ in (12) for a range of angles ϑ, with the best match indicated by $\vartheta_{\text{opt}} = \arg\max_\vartheta\{\max_i \lambda_i(\mathbf{R}_\vartheta)\}$, where $\lambda_i(\mathbf{R}_\vartheta)$ indicates the ith eigenvalue of $\mathbf{R}_\vartheta$.

4.3. Space-Time Covariance Matrix and Polynomial Eigenvalue Decomposition

To generalise (10) to the broadband case, we first define a polynomial space-time covariance matrix. This matrix can be decomposed by McWhirter's polynomial EVD [3], followed by an appropriate selection of a broadband steering vector to probe its noise-only subspace.

   Different from the narrowband case, in a broadband scenario time signal wavefronts travelling across the array at finite speed must be characterised by time delays rather than just phase shifts. This motivates the definition of a polynomial space-time covariance matrix $\mathbf{R}(z) \bullet\!\!-\!\!\circ \mathbf{R}[\tau]$,

$$\mathbf{R}[\tau] = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^\text{H}[n - \tau]\} \ ,$$

which includes a time delay in form of the lag value τ. This power spectral matrix can be decomposed by an iterative algorithm [3] to yield a polynomial EVD

$$\mathbf{R}(z) = \mathbf{Q}(z)\mathbf{\Lambda}(z)\tilde{\mathbf{Q}}(z) = \sum_{m=0}^{M-1} \lambda_m(z)\mathbf{q}_m(z)\tilde{\mathbf{q}}_m(z)$$

with paraunitary $\mathbf{Q}(z)$, i.e. $\mathbf{Q}(z)\tilde{\mathbf{Q}}(z) = \mathbf{I}$. The diagonal matrix $\mathbf{\Lambda}(z)$ contains the polynomial eigenvalues $\lambda_m(z)$. Thresholding the latter reveals the number of independent broadband sources contributing to $\mathbf{R}(z)$, and permits a distinction between signal-plus-noise and noise only subspaces,

$$\mathbf{R}(z) = [\mathbf{Q}_s(z)\mathbf{Q}_n(z)]\begin{bmatrix} \mathbf{\Lambda}_s(z) & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n(z) \end{bmatrix}\begin{bmatrix} \mathbf{Q}_s^\text{H}(z) \\ \mathbf{Q}_n^\text{H}(z) \end{bmatrix}$$
$$(15)$$

similar to the narrowband EVD in (9). To probe the nullspace of $\tilde{\mathbf{Q}}_n(z)$,

$$\tilde{\mathbf{Q}}_n(z) = \begin{bmatrix} \tilde{\mathbf{q}}_L(z) \\ \vdots \\ \tilde{\mathbf{q}}_{M-1}(z) \end{bmatrix} \quad (16)$$

a broadband steering vector is required instead of the narrowband one in (5).

4.3.1. Broadband Steering Vector

To accurately reflect the time delays required to describe (4), a polynomial vector containing fractional delay transfer functions is proposed here. One possibility to implement these fractional delays is by means of an appropriately sampled sinc function, such that

$$a_l[n] = \text{sinc}(nT_s - \Delta\tau_l) \quad . \quad (17)$$

With $A_l(z) \bullet\!\!-\!\!\circ a_l[n]$, a broadband steering vector can be defined as

$$\mathbf{a}_\vartheta(z) = \begin{bmatrix} A_0(z) \\ \vdots \\ A_{M-1}(z) \end{bmatrix} \quad . \quad (18)$$

The parameter ϑ on the l.h.s. of (18) indicates the dependency of $\Delta\tau_l$ on the angle of arrival. This vector is equivalent to the main diagonal of the parameterised spatial correlation matrix approach in (14).

4.3.2. Polynomial MUSIC Algorithm

Based on the concept of the narrowband MUSIC algorithm, the generalised quantity

$$\Gamma_\vartheta(z) = \tilde{\mathbf{a}}_\vartheta(z)\mathbf{Q}_n(z)\tilde{\mathbf{Q}}_n(z)\mathbf{a}_\vartheta(z)$$

is no longer a norm measuring the vicinity of $\mathbf{a}_\vartheta(z)$ to the nullspace of $\tilde{\mathbf{Q}}_n(z)$, but a power spectral density. This has motivated two versions of the a polynomial MUSIC (P-MUSIC) algorithm [1] outlined below.

Spatial P-MUSIC. The energy contained in the signal vector $\tilde{\mathbf{Q}}_n(z)\mathbf{a}_\vartheta(z)$ is related to the zero lag term $\gamma_\vartheta[0]$ of the autocorrelation-type sequence $\gamma_\vartheta[\tau] \circ\!\!-\!\!\bullet \Gamma_\vartheta(z)$. This measure is only dependent on the angle of arrival ϑ, and collects all energy across the spectrum. Instead of searching for the steering vectors providing minimum energy, the reciprocal

$$P_{SP-MU}(\vartheta) = \frac{1}{\gamma_\vartheta[0]} \quad . \quad (19)$$

is maximised by the angle of arrival ϑ of signal sources.
Spatio-Spectral P-MUSIC. With (4.3.2) describing a power spectral density, spectral clues can be exploited in addition to the spatial information extracted by (19). Therefore in addition to spatial localisation of sources,

$$P_{SSP-MU}(\vartheta, \Omega) = \left(\sum_{\tau=-\infty}^{\infty} \gamma_\vartheta[\tau]e^{-j\Omega\tau}\right)^{-1} \quad (20)$$

can determine over which frequency range sources in the direction defined by the steering vector $\mathbf{a}_\vartheta(z)$ are active. SSP-MUSIC was introduced in [1], but will be omitted from the comparison below, since the benchmark method only retrieves DoA information.

5. COMPARISON

The broadband steering vector for a linear uniform array sensors separated by distances $d = \frac{c}{f_s}$ takes on simple forms
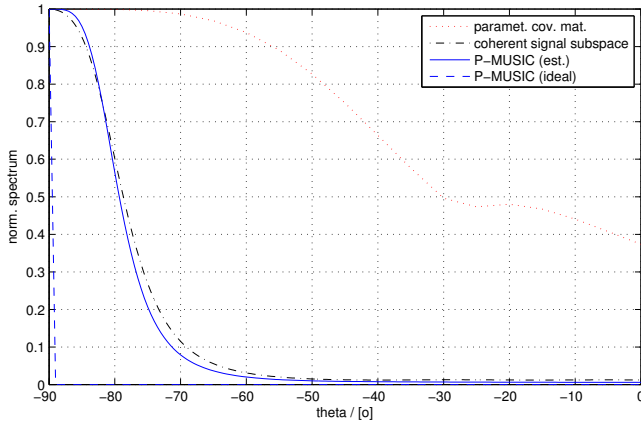
Fig. 1. Direction of arrival estimation for a single source located at end fire ($\vartheta = -90°$) using different approaches.
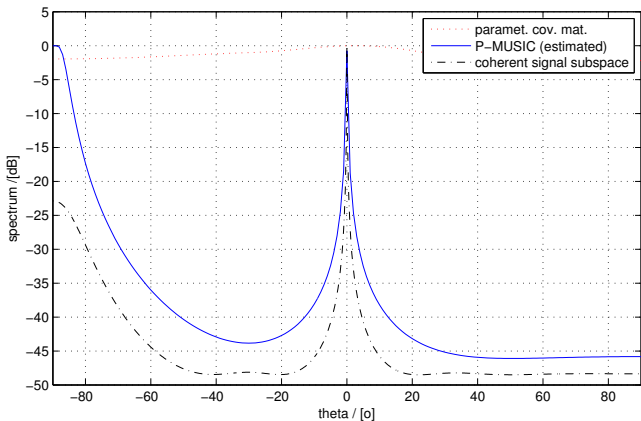

Fig. 2. Direction of arrival estimation for a scenario with two independent sources of equal strength located at broadside and end fire positions.

for the broadside and end-fire directions, where for $M = 4$ sensors

$$\mathbf{a}_{\vartheta=0°}(z) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^\mathrm{T} \quad (21)$$

$$\mathbf{a}_{\vartheta=-90°}(z) = \begin{bmatrix} 1 & z^{-1} & z^{-2} & z^{-3} \end{bmatrix}^\mathrm{T} \quad (22)$$

results, with the covariance matrix $\mathbf{R}(z) = \mathbf{a}_\vartheta(z)\tilde{\mathbf{a}}_\vartheta(z)$ for uncorrelated unit-variance sources.

Example 1. For a single source at end-fire position, the simulation results normalised to a spectrum peak value of one are shown in Fig. 1, whereby the PCV approach fares worst. Close to end-fire position, the CSS provide slightly higher resolution than P-MUSIC based on an estimate covariance matrix and iterated decomposition. Knowing the covariance matrix and its true noise subspace in the PEVD sense however can achieve a much higher selectivity, with values quickly descending towards -300dB away from end-fire.

Example 2. Assuming two independent sources at broadside and end-fire position, the space-time covariance matrix is given by $\mathbf{R}_3(z) = \mathbf{R}_1(z) + \mathbf{R}_2(z)$. The Spatial P-MUSIC algorithm identifies two large polynomial eigenvalues, and

from the noise-only subspace derives the result in Fig. 2. The DoAs and relative strengths of the sources are correctly extracted. CSS provide higher resolution towards broadside, while the end-fire source is not detected equally. The PSC approach fails to identify more than one source, and is only able to provide a very poor estimate for the broadside source. As for example 1, the spectra in Fig. 2 are normalised such that the maximum is unity.

The simulations in Example 1 suggest that the proposed P-MUSIC algorithm can outperform the other methods if an idea polynomial eigenvalue decomposition is available. In practical cases where this matrix is estimated and the decomposition iteratively approximated, the performance of P-MUSIC is often inferior to the coherent signal subspace approach [6, 7], such as in the accuracy of detecting the broadside source in Example 2. In contrast, the more recent method in [2, 5] appears weaker and is unable to resolve more than one source.

6. CONCLUSIONS

This paper has compared three linear algebraic broadband direction of arrival estimation techniques, whereby a recently proposed polynomial matrix decomposition approach to extend the MUSIC algorithm to the broadband case has been reviewed. Simulations indicate that the performance for the optimum decomposition works very well and outperforms other algorithms. The performance degrades when based on estimated values and an iterative approximate decomposition and may therefore be enhanced by the develpment of a more dedicated decomposition algorithm.

7. REFERENCES

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