ROBUST TRANSCEIVER DESIGN FOR MIMO RELAY SYSTEMS WITH TOMLINSON HARASHIMA PRECODING

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ABSTRACT

In this paper we consider a robust transceiver design for two hop non-regenerative multiple-input multiple-output (MIMO) relay networks with imperfect channel state information (CSI). The transceiver consists of Tomlinson Harashima Precoding (THP) at the source with a linear precoder at the relay and linear equalisation at the destination. Under the assumption that each node in the network can acquire statistical knowledge of the channel in the form of a channel mean and estimation error covariance, we optimise the processors to minimise the expected arithmetic mean square error (MSE) subject to transmission power constraints at the source and relay. Simulation results demonstrate the robustness of the proposed transceiver design to channel estimation errors.

***Index Terms*—** MIMO relay, Tomlinson Harashima Precoding, robust transceiver, imperfect CSI, channel estimation error.

1. INTRODUCTION

The use of relay nodes to forward data between a source and destination pair can provide benefits over conventional point-point transmission systems such as increased network coverage as well as robustness to channel impairments. When the source, relay, and destination nodes are all equipped with multiple antennas the system is referred to as a MIMO relay network and further benefits such as increased spectral efficiency and higher data rates can be realised. Two main strategies have emerged from the study of MIMO relay systems depending on the functionality of the relaying device, with transceivers generally being classed as either decode forward or amplify forward, which are also commonly refered to as regenerative and non-regenerative relaying respectively.

Non-regenerative transceiver designs have been particularly well investigated due to their simplicity [1–5]. In [1] the optimal relay precoder and destination equaliser are found that minimise the arithmetic MSE when the source precoder is a scaled identity matrix. A unified framework that embraces most design objective functions is presented in [2] where the optimal source and relay precoders are derived for the family of Schur convex and Schur concave objective functions. In [3] a non-linear transceiver that utilises THP at the source is investigated and shown to outperform linear techniques. The works of [1–3] assume that the source, relay, and destination can acquire perfect CSI, which in practice may be an unreasonable assumption. Linear transceiver designs that are robust to channel estimation errors have been studied in [4] and [5]. In [4] the relay and destination processors are derived that minimise the arithmetic MSE where both iterative and closed form solutions are presented. In [5] robust linear transceivers are derived for any objective function that is either Schur convex or Schur concave and is an extension of the work in [2] to the scenario of channel estimation errors.

In this paper we consider the robust design of a non-linear transceiver that consists of THP at the source, linear precoding at the relay, and linear equalisation at the receiver. The rest of the paper is organised as follows. In Section 2 we review the signal model for MIMO relay systems with THP. In Section 3 we formulate the problem for finding robust processors that minimise the arithmetic MSE subject to transmission power constraints at both the source and relay terminals. The robust non-linear transceiver design is then presented in Section 4 and simulation results are shown in Section 5. Finally, conclusions are drawn in Section 6.

Notation: Matrices, vectors, and scalars are denoted by upper case bold font, lower case bold font, and lower case normal font respectively. The element in the ith row and jth column of matrix $\mathbf{A}$ is noted as $[\mathbf{A}]_{ij}$ and the ith element of vector $\mathbf{a}$ is denoted by $[\mathbf{a}]_i$. The $N \times N$ identity matrix is noted as $\mathbf{I}_N$ and $\mathbf{0}_{N\times M}$ is the $N \times M$ zero matrix. The sets $\mathbb{R}$ and $\mathbb{C}$ are the set of real and complex numbers, and in the case of matrix and vector quantities denote dimensions by a superscript. The operators $\mathbb{E}\{.\}$, $\mathrm{tr}\{.\}$, $|.|$, $\{.\}^{\mathrm{T}}$, $\{.\}^{\mathrm{H}}$, and $\{.\}^*$ denote expectation, trace, determinant, transpose, Hermitian transpose, and conjugation respectively. The notation $\mathrm{diag}[.]$ signifies a diagonal matrix and matrix rank is denoted as $\mathrm{rank}\{.\}$. The operator $\min(.)$ returns the minimum and $[x]^+ \triangleq \min(x, 0)$. The symbol $\otimes$ is the Kronecker product.

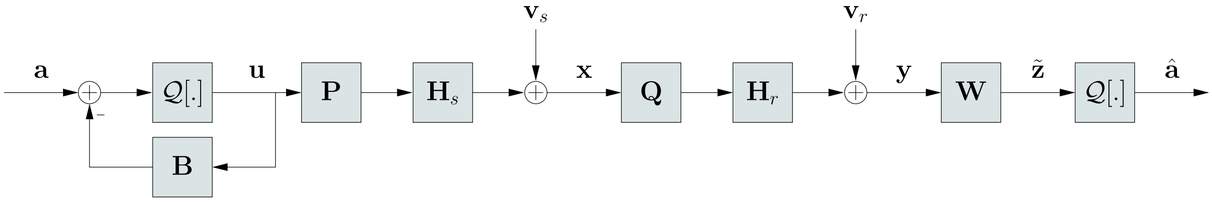

Fig. 1. MIMO relay system model with Tomlinson Harashima Precoding.

2. SIGNAL MODEL

We consider transmission of N_a data streams through a MIMO relay system equipped with N_s, N_r, and N_d antennas at the source, relay, and destination respectively. To deal with the spatial interference that occurs from the transmission of multiple data streams, we utilise THP at the source which consists of a feedback processor $\mathbf{B} \in \mathbb{C}^{N_a \times N_a}$ and linear precoder $\mathbf{P} \in \mathbb{C}^{N_s \times N_a}$. We also employ a precoder $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ at the relay and linear equaliser $\mathbf{W} \in \mathbb{C}^{N_a \times N_d}$ at the destination. This configuration is shown in Fig. 1. The symbols $\mathbf{a} \in \mathbb{C}^{N_a}$ are drawn from an M-QAM signal constellation $\mathcal{A}$ with a square Voronoi region $\mathcal{U}$ [3, 6] and are assumed to have covariance $\mathbf{R}_a = \mathbf{I}_{N_a}$. The elements of $\mathbf{u} \in \mathbb{C}^{N_a}$ are recursively computed according to

$$[\mathbf{u}]_i = \mathcal{Q}\left[[\mathbf{a}]_i - \sum_{j=1}^{i-1}[\mathbf{B}]_{ij}[\mathbf{u}]_j\right], \quad i = 1, \cdots, N_a, \quad (1)$$

where $\mathcal{Q}[.]$ denotes the modulo operation and $\mathbf{B}$ is required to be a strictly lower left triangular matrix. The operation in (1) is equivalent to $\mathbf{u} = \mathbf{U}^{-1}\mathbf{z}$, where $\mathbf{U} \triangleq \mathbf{B} + \mathbf{I}_{N_a}$ is a unit diagonal lower left triangular matrix and $\mathbf{z} \triangleq \mathbf{a} + \mathbf{d}$ is a modified data vector with $\mathbf{d}$ chosen such that $\mathbf{u}$ is bounded by the square region $\mathcal{U}$ (see e.g. [3,6–8] for details). We note that for high M-QAM symbols $\mathbf{u}$ can be assumed to have covariance $\mathbf{R}_u = \mathbf{I}_{N_a}$ [6]. The vector $\mathbf{u} = \mathbf{U}^{-1}\mathbf{z}$ is then processed by the precoder $\mathbf{P}$ and transmitted across the source-relay channel $\mathbf{H}_s \in \mathbb{C}^{N_r \times N_s}$, resulting in $\mathbf{x} \in \mathbb{C}^{N_r}$ given by

$$\mathbf{x} = \mathbf{H}_s\mathbf{P}\mathbf{U}^{-1}\mathbf{z} + \mathbf{v}_s, \quad (2)$$

where $\mathbf{v}_s \in \mathbb{C}^{N_r}$ is an additive white Gaussian noise (AWGN) vector with covariance $\mathbf{R}_{vs} = \sigma_{vs}^2\mathbf{I}_{N_r}$. The vector $\mathbf{x}$ is then linearly precoded by $\mathbf{Q}$ and the resulting symbols are transmitted over the relay-destination channel $\mathbf{H}_r \in \mathbb{C}^{N_d \times N_r}$. This results in $\mathbf{y} \in \mathbb{C}^{N_d}$ at the destination being given by

$$\mathbf{y} = \mathbf{H}_r\mathbf{Q}\mathbf{x} + \mathbf{v}_r, \quad (3)$$

with $\mathbf{v}_r \in \mathbb{C}^{N_d}$ being an AWGN vector with covariance $\mathbf{R}_{vr} = \sigma_{vr}^2\mathbf{I}_{N_d}$. Linear equalisation is then performed by $\mathbf{W}$ before the resulting symbols $\tilde{\mathbf{z}} \in \mathbb{C}^{N_a}$ are modulo reduced to the region $\mathcal{U}$ and quantised to the nearest point in constellation $\mathcal{A}$. The error covariance matrix of the system is defined as $\mathbf{R}_e \triangleq \mathbb{E}\{(\mathbf{z} - \tilde{\mathbf{z}})(\mathbf{z} - \tilde{\mathbf{z}})^{\mathrm{H}}\}$, which using (2) and (3), as well as $\mathbf{z} = \mathbf{U}\mathbf{u}$, can be written as

$$\mathbf{R}_e = (\mathbf{WHP} - \mathbf{U})(\mathbf{WHP} - \mathbf{U})^{\mathrm{H}} + \mathbf{W}\mathbf{R}_v\mathbf{W}^{\mathrm{H}}, \quad (4)$$

where for convenience we define $\mathbf{H} \triangleq \mathbf{H}_r\mathbf{Q}\mathbf{H}_s$ as the effective MIMO channel between the source and destination and $\mathbf{R}_v \triangleq \mathbf{H}_r\mathbf{Q}\mathbf{Q}^{\mathrm{H}}\mathbf{H}_r^{\mathrm{H}}\sigma_{vs}^2 + \sigma_{vr}^2\mathbf{I}_{N_d}$ is the covariance matrix of the total noise signal at the equaliser input [3].

3. PROBLEM FORMULATION

In general it is very difficult to obtain perfect CSI at all nodes in the network and channel estimation errors inevitably occur which can seriously degrade performance if unaccounted for. In this section, assuming statistical knowledge of the channel and the estimation error can be acquired, we formulate a robust optimisation problem for finding the processors $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{W}$, and $\mathbf{U}$, in the proposed system.

3.1. Channel Estimation Error

Before formulating the robust optimisation problem, we consider a statistical description of the source-relay and relay-destination channels when they are estimated incorrectly. Using the well known Kronecker model [4, 5] the channels $\mathbf{H}_s$ and $\mathbf{H}_r$ (including estimation errors) can be decomposed as

$$\mathbf{H}_s = \boldsymbol{\Upsilon}_s^{1/2}\left(\tilde{\mathbf{H}}_s + \mathbf{E}_s\right)\boldsymbol{\Xi}_s^{\mathrm{T}/2} \quad (5)$$

$$\mathbf{H}_r = \boldsymbol{\Upsilon}_r^{1/2}\left(\tilde{\mathbf{H}}_r + \mathbf{E}_r\right)\boldsymbol{\Xi}_r^{\mathrm{T}/2}, \quad (6)$$

where $\boldsymbol{\Xi}_s \in \mathbb{C}^{N_s \times N_s}$ and $\boldsymbol{\Upsilon}_s \in \mathbb{C}^{N_r \times N_r}$ are the transmit and receive side spatial correlation matrices of the source-relay channel and $\boldsymbol{\Xi}_r \in \mathbb{C}^{N_r \times N_r}$ and $\boldsymbol{\Upsilon}_r \in \mathbb{C}^{N_d \times N_d}$ are the relay-destination transmit and receive side spatial correlation matrices. The matrices $\tilde{\mathbf{H}}_s$ and $\tilde{\mathbf{H}}_r$ represent the estimated CSI of the source-relay and relay-destination channels which are assumed to contain zero mean Gaussian random variables with variances $\sigma_{h_s}^2$ and $\sigma_{h_r}^2$ respectively. In (5) and (6) the matrices $\mathbf{E}_s$ and $\mathbf{E}_r$ are the channel estimation error matrices which contain zero mean Gaussian random variables with variances $\sigma_{e_s}^2$ and $\sigma_{e_r}^2$. The channels can be represented by

$$\mathbb{E}\{\mathbf{R}_e\} = \mathbf{W}\left(\bar{\mathbf{H}}_r\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}} + \mathrm{tr}\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\boldsymbol{\Xi}_r^{\mathrm{T}}\}\bar{\boldsymbol{\Upsilon}}_r + \sigma_{vr}^2\mathbf{I}_{N_d}\right)\mathbf{W}^{\mathrm{H}} - \mathbf{W}\bar{\mathbf{H}}_r\mathbf{Q}\bar{\mathbf{H}}_s\mathbf{P}\mathbf{U}^{\mathrm{H}} - \mathbf{U}\mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}}\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}}\mathbf{W}^{\mathrm{H}} + \mathbf{U}\mathbf{U}^{\mathrm{H}} \quad (9)$$

the matrix variate complex Gaussian distributions [9]

$$\mathbf{H}_s \sim \mathcal{CN}\left(\bar{\mathbf{H}}_s\,,\,\boldsymbol{\Xi}_s \otimes \bar{\boldsymbol{\Upsilon}}_s\right) \quad (7)$$
$$\mathbf{H}_r \sim \mathcal{CN}\left(\bar{\mathbf{H}}_r\,,\,\boldsymbol{\Xi}_r \otimes \bar{\boldsymbol{\Upsilon}}_r\right). \quad (8)$$

where $\bar{\mathbf{H}}_s \triangleq \boldsymbol{\Upsilon}_s^{1/2}\tilde{\mathbf{H}}_s\boldsymbol{\Xi}_s^{\mathrm{T}/2}$ and $\bar{\mathbf{H}}_r \triangleq \boldsymbol{\Upsilon}_r^{1/2}\tilde{\mathbf{H}}_r\boldsymbol{\Xi}_r^{\mathrm{T}/2}$ are the estimates of the source-relay and relay-destination channels including spatial correlation. The Kronecker products $\boldsymbol{\Xi}_s \otimes \bar{\boldsymbol{\Upsilon}}_s$ and $\boldsymbol{\Xi}_r \otimes \bar{\boldsymbol{\Upsilon}}_r$ are the channel estimation error covariance matrices where we define $\bar{\boldsymbol{\Upsilon}}_s \triangleq \sigma_{e_s}^2\boldsymbol{\Upsilon}_s$ and $\bar{\boldsymbol{\Upsilon}}_r \triangleq \sigma_{e_r}^2\boldsymbol{\Upsilon}_r$. Before proceeding, we introduce the following lemma:

Lemma 1*:* For a random matrix $\mathbf{A} \in \mathbb{C}^{M\times N}$ with matrix variate Gaussian distribution $\mathbf{A} \sim \mathcal{CN}\left(\bar{\mathbf{A}}\,,\,\mathbf{C}\otimes\mathbf{D}\right)$, we have for any matrix $\mathbf{F} \in \mathbb{C}^{N\times N}$ that $\mathbb{E}\{\mathbf{A}\mathbf{F}\mathbf{A}^{\mathrm{H}}\} = \bar{\mathbf{A}}\mathbf{F}\bar{\mathbf{A}}^{\mathrm{H}} + \mathrm{tr}\{\mathbf{F}\mathbf{C}^{\mathrm{T}}\}\mathbf{D}$ [9].

3.2. Robust Minimum MSE Problem Formulation

We focus on transceivers that minimise the arithmetic MSE which is given by $\mathrm{tr}\{\mathbf{R}_e\}/N_a$. Since the error covariance matrix in (4) depends on the channels $\mathbf{H}_s$ and $\mathbf{H}_r$, which are unknown, a problem formulation based on the instantaneous error covariance matrix cannot be conducted. We shall thus formulate a problem based on $\mathbb{E}\{\mathbf{R}_e\}$ where the expectation is carried out with respect to $\mathbf{H}_s$ and $\mathbf{H}_r$. With the use of Lemma 1, the expectation of $\mathbf{R}_e$ can be calculated as (9) where for notational convenience we define $\mathbf{X}$ as

$$\mathbf{X} \triangleq \bar{\mathbf{H}}_s\mathbf{P}\mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}} + \mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Xi}_s^{\mathrm{T}}\}\bar{\boldsymbol{\Upsilon}}_s + \sigma_{vs}^2\mathbf{I}_{N_r}. \quad (10)$$

As well as minimising the arithmetic MSE we also wish to limit the transmission power used by the source and relay nodes which are given by $\mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\}$ and $\mathrm{tr}\{\mathbf{Q}(\mathbf{H}_s\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{H}_s^{\mathrm{H}} + \sigma_{vs}^2\mathbf{I}_{N_r})\mathbf{Q}^{\mathrm{H}}\}$ respectively. We note that the relay power consumption depends on the unknown channel $\mathbf{H}_s$ and cannot be constrained for an instantaneous channel realisation. We shall thus limit the expected transmit power of the relay node and arrive at the constrained optimisation problem

$$\min \quad \mathrm{tr}\{\mathbb{E}\{\mathbf{R}_e\}\}/N_a \quad (11)$$
$$\text{s.t.} \quad \mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\} \le P_s \quad (12)$$
$$\mathrm{tr}\left\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\right\} \le P_r, \quad (13)$$

where P_s and P_r are the maximum available power budgets at the source and relay respectively. To obtain the relay power constraint in (13) we have made use of Lemma 1 and the matrix $\mathbf{X}$ is defined in (10).

4. TRANSCEIVER DESIGN

Having formulated the robust optimisation problem for minimising the arithmetic MSE subject to transmit power constraints, we now focus on deriving the processors $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{W}$, and $\mathbf{U}$, as the solution to (11)-(13).

4.1. Optimal Equaliser and Problem Reformulation

The optimal equaliser $\mathbf{W}$ that minimises the MSE of every data stream is provided by the well known Wiener solution, which can be obtained by setting the derivative of (11) with respect to $\mathbf{W}^*$ to zero and solving for $\mathbf{W}$. This results in the optimal equaliser solution

$$\mathbf{W} = \mathbf{U}\mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}}\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}}\left(\bar{\mathbf{H}}_r\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}} + \mathrm{tr}\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\boldsymbol{\Xi}_r^{\mathrm{T}}\}\bar{\boldsymbol{\Upsilon}}_r + \sigma_{vr}^2\mathbf{I}_{N_d}\right)^{-1}. \quad (14)$$

Substituting (14) in (9), and using (10) as well as the Woodbury identity, we can write $\mathbb{E}\{\mathbf{R}_e\} = \mathbf{U}\mathbf{E}\mathbf{U}^{\mathrm{H}}$ where $\mathbf{E} \in \mathbb{C}^{N_a\times N_a}$ is defined in (15). Using the structure of $\mathbb{E}\{\mathbf{R}_e\}$ and the arithmetic-geometric mean inequality we can state that

$$|\mathbf{E}|^{1/N_a} \le \mathrm{tr}\{\mathbb{E}\{\mathbf{R}_e\}\}/N_a. \quad (16)$$

where equality is achieved when $\mathbb{E}\{\mathbf{R}_e\}$ is a diagonal matrix with equal diagonal elements. In the following we propose to minimise the lower bound of (16) and find suitable processors such that (16) holds with equality. This leads us to the optimisation problem

$$\min \quad |\mathbf{E}| \quad (17)$$
$$\text{s.t.} \quad \mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\} \le P_s \quad (18)$$
$$\mathrm{tr}\left\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\right\} \le P_r \quad (19)$$
$$\mathbb{E}\{\mathbf{R}_e\} = \varepsilon\mathbf{I}_{N_a}, \quad (20)$$

$$\mathbf{E} \triangleq \left(\mathbf{I}_{N_a} + \mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}}\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}}\left(\bar{\mathbf{H}}_r\mathbf{Q}\left(\mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Xi}_s^{\mathrm{T}}\}\boldsymbol{\Upsilon}_s + \sigma_{vs}^2\mathbf{I}_{N_r}\right)\mathbf{Q}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}} + \mathrm{tr}\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\boldsymbol{\Xi}_r^{\mathrm{T}}\}\boldsymbol{\Upsilon}_r + \sigma_{vr}^2\mathbf{I}_{N_d}\right)^{-1}\bar{\mathbf{H}}_r\mathbf{Q}\bar{\mathbf{H}}_s\mathbf{P}\right)^{-1} \quad (15)$$

where $\varepsilon \triangleq |\mathbf{E}|^{1/N_a}$ is such that (16) holds with equality. This problem can be solved by firstly finding the source and relay precoders $\mathbf{P}$ and $\mathbf{Q}$ that minimise (17) and satisfy (18) and (19), and in a second step using the remaining degrees of freedom to ensure the constraint in (20) holds with equality. We note that a similar problem formulation can be found in [3] for the case of a two-hop MIMO relay system. However the work in [3] assumed that perfect CSI could be acquired by all nodes in the network which is not the case here.

4.2. Source, Relay, and Feedback Processors

In general the optimisation problem in (17)-(19) is intractable and must be relaxed in order to obtain closed form solutions for $\mathbf{P}$ and $\mathbf{Q}$. As in [4] we propose to relax the problem by using the approximations

$$\mathrm{tr}\left\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Xi}_s^{\mathrm{T}}\right\}\bar{\boldsymbol{\Upsilon}}_s \approx \mathrm{tr}\left\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\right\}\xi_{s,1}\bar{\boldsymbol{\Upsilon}}_s \tag{21}$$

$$\mathrm{tr}\left\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\boldsymbol{\Xi}_r^{\mathrm{T}}\right\}\bar{\boldsymbol{\Upsilon}}_r \approx \mathrm{tr}\left\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\right\}\xi_{r,1}\bar{\boldsymbol{\Upsilon}}_r, \tag{22}$$

where $\xi_{s,1}$ and $\xi_{r,1}$ are the maximum eigenvalues of $\boldsymbol{\Xi}_s$ and $\boldsymbol{\Xi}_r$ respectively. Substituting the approximations of (21) and (22) into the objective function in (17) it is straightforward to show that $|\mathbf{E}|$ is a decreasing function of both $\mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\}$ and $\mathrm{tr}\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\}$, which are bounded by the constraints in (18) and (19). It then follows that the optimal $\mathbf{P}$ and $\mathbf{Q}$ should satisfy $\mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\} = P_s$ and $\mathrm{tr}\{\mathbf{Q}\mathbf{X}\mathbf{Q}^{\mathrm{H}}\} = P_r$. With these observations, and some simple deductions, we arrive at the relaxed problem given by

$$\min \quad |\mathbf{I}_{N_a} + \mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}}\tilde{\boldsymbol{\Upsilon}}_s^{-\mathrm{H}/2}\tilde{\mathbf{Q}}^{\mathrm{H}}\bar{\mathbf{H}}_r^{\mathrm{H}}\tilde{\boldsymbol{\Upsilon}}_r^{-\mathrm{H}/2}\left(\tilde{\boldsymbol{\Upsilon}}_r^{-1/2}\bar{\mathbf{H}}_r\tilde{\mathbf{Q}} \times\tilde{\mathbf{Q}}^{\mathrm{H}}\,\bar{\mathbf{H}}_r^{\mathrm{H}}\tilde{\boldsymbol{\Upsilon}}_r^{-\mathrm{H}/2} + \mathbf{I}_{N_d}\right)^{-1}\tilde{\boldsymbol{\Upsilon}}_r^{-1/2}\bar{\mathbf{H}}_r\tilde{\mathbf{Q}}\tilde{\boldsymbol{\Upsilon}}_s^{-1/2}\bar{\mathbf{H}}_s\mathbf{P}|^{-1} \tag{23}$$

$$\text{s.t.} \quad \mathrm{tr}\{\mathbf{P}\mathbf{P}^{\mathrm{H}}\} \le P_s \tag{24}$$

$$\mathrm{tr}\left\{\tilde{\mathbf{Q}}\tilde{\mathbf{X}}\tilde{\mathbf{Q}}^{\mathrm{H}}\right\} \le P_r, \tag{25}$$

where for notational convenience we introduce the variables

$$\tilde{\boldsymbol{\Upsilon}}_s \triangleq P_s\xi_{s,1}\boldsymbol{\Upsilon}_s + \sigma_{vs}^2\mathbf{I}_{N_r} \tag{26}$$

$$\tilde{\boldsymbol{\Upsilon}}_r \triangleq P_r\xi_{r,1}\boldsymbol{\Upsilon}_r + \sigma_{vr}^2\mathbf{I}_{N_d} \tag{27}$$

$$\tilde{\mathbf{Q}} \triangleq \mathbf{Q}\tilde{\boldsymbol{\Upsilon}}_s^{1/2} \tag{28}$$

$$\tilde{\mathbf{X}} \triangleq \tilde{\mathbf{Q}}(\tilde{\boldsymbol{\Upsilon}}_s^{-1/2}\bar{\mathbf{H}}_s\mathbf{P}\mathbf{P}^{\mathrm{H}}\bar{\mathbf{H}}_s^{\mathrm{H}}\tilde{\boldsymbol{\Upsilon}}_s^{-\mathrm{H}/2} + \mathbf{I}_{N_r})\tilde{\mathbf{Q}}. \tag{29}$$

To derive the matrices $\mathbf{P}$ and $\mathbf{Q}$ as the solution to (23)-(25), we firstly consider the singular value decompositions

$$\tilde{\boldsymbol{\Upsilon}}_s^{-1/2}\bar{\mathbf{H}}_s = \mathbf{U}_s\boldsymbol{\Lambda}\mathbf{V}_s^{\mathrm{H}} \tag{30}$$

$$\tilde{\boldsymbol{\Upsilon}}_r^{-1/2}\bar{\mathbf{H}}_r = \mathbf{U}_r\boldsymbol{\Delta}\mathbf{V}_r^{\mathrm{H}}, \tag{31}$$

where $\mathbf{U}_s \in \mathbb{C}^{N_r\times R_s}$, $\mathbf{V}_s \in \mathbb{C}^{N_s\times R_s}$, $\mathbf{U}_r \in \mathbb{C}^{N_d\times R_r}$, and $\mathbf{V}_r \in \mathbb{C}^{N_r\times R_r}$ are unitary, and $\boldsymbol{\Lambda} \triangleq \mathrm{diag}[\lambda_1,\cdots,\lambda_{R_s}]$ and $\boldsymbol{\Delta} \triangleq \mathrm{diag}[\delta_1,\cdots,\delta_{R_r}]$. Here $R_s \triangleq \mathrm{rank}\{\tilde{\boldsymbol{\Upsilon}}_s^{-1/2}\bar{\mathbf{H}}_s\}$ and $R_r \triangleq \mathrm{rank}\{\tilde{\boldsymbol{\Upsilon}}_r^{-1/2}\bar{\mathbf{H}}_r\}$, and for ease of exposition in the following we assume that $N_a \le \min(R_s, R_r)$. Substituting (30) and (31) into the objective function in (23) we can deduce, using the Hadamard determinant inequality and (28), that the optimal source and relay processors $\mathbf{P}$ and $\mathbf{Q}$ are given by

$$\mathbf{P} = \bar{\mathbf{V}}_s\boldsymbol{\Gamma}\boldsymbol{\Psi} \tag{32}$$

$$\mathbf{Q} = \bar{\mathbf{V}}_r\boldsymbol{\Phi}\bar{\mathbf{U}}_s^{\mathrm{H}}\tilde{\boldsymbol{\Upsilon}}_s^{-1/2}, \tag{33}$$

where $\bar{\mathbf{V}}_s$, $\bar{\mathbf{V}}_r$, and $\bar{\mathbf{U}}_s$ contain the left N_a columns of $\mathbf{V}_s$, $\mathbf{V}_r$, and $\mathbf{U}_s$, respectively, and $\boldsymbol{\Gamma} \triangleq \mathrm{diag}[\gamma_1,\cdots,\gamma_{N_a}]$ and $\boldsymbol{\Phi} \triangleq \mathrm{diag}[\phi_1,\cdots,\phi_{N_a}]$. The matrix $\boldsymbol{\Psi} \in \mathbb{C}^{N_a\times N_a}$ is a unitary matrix yet to be determined. Substituting (32) and (33) into (23)-(25), and using the decompositions in (30) and (31), the original matrix valued problem reduces to finding γ_i and ϕ_i from the problem

$$\min \quad \prod_{i=1}^{N_a}\left(1 + \frac{\gamma_i^2\lambda_i^2\phi_i^2\delta_i^2}{\phi_i^2\delta_i^2+1}\right)^{-1} \tag{34}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_a}\gamma_i^2 \le P_s \tag{35}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_a}\phi_i^2(\gamma_i^2\lambda_i^2+1) \le P_r \tag{36}$$

$$\gamma_i^2 \ge 0\;,\; \phi_i^2 \ge 0\;,\; 1 \le i \le N_a. \tag{37}$$

The solution to (34)-(37) can be obtained using an iterative power allocation algorithm similar to those in [2, 3, 5]. The remaining task now is to compute the unitary matrix $\boldsymbol{\Psi}$ in (32) and the unit diagonal lower left triangular matrix $\mathbf{U}$ such that the constraint in (20) is satisfied. In a similar fashion to that in [3] we can calculate $\mathbf{U}$ and $\boldsymbol{\Psi}$ from the matrix decomposition

$$\tilde{\mathbf{E}}^{1/2} = \sqrt{\varepsilon}\mathbf{Q}\mathbf{U}^{-\mathrm{H}}\boldsymbol{\Psi}^{\mathrm{H}}, \tag{38}$$

where $\varepsilon \triangleq |\tilde{\mathbf{E}}|^{1/N_a}$ and $\tilde{\mathbf{E}}$ is obtained from (15) by replacing $\mathbf{P}$ with $\tilde{\mathbf{P}} \triangleq \bar{\mathbf{V}}_s\boldsymbol{\Gamma}$. The decomposition in (38) is known as the equal diagonal QR decomposition and can be computed using the algorithm in [10]. With $\mathbf{U}$ and $\boldsymbol{\Psi}$ calculated in this manner the arithmetic MSE achieves the lower bound in (16).

5. SIMULATION RESULTS

We simulate a system with $N_s = N_r = N_d = 3$. The source transmits $N_a = 3$ data symbols with each symbol being drawn from a 16 QAM constellation. The SNR of the source-relay channel is $\mathrm{SNR}_s \triangleq P_s/N_s\sigma_{vs}^2$ and the SNR of the relay-destination channel is $\mathrm{SNR}_r \triangleq P_r/N_r\sigma_{vr}^2$ and is fixed at 20dB. The source-relay and relay-destination channels are modelled as in (5) and (6) with the elements of the channel estimation error matrices $\mathbf{E}_s$ and $\mathbf{E}_r$ being drawn from zero mean Gaussian distributions with variances $\sigma_{e_s}^2 =$

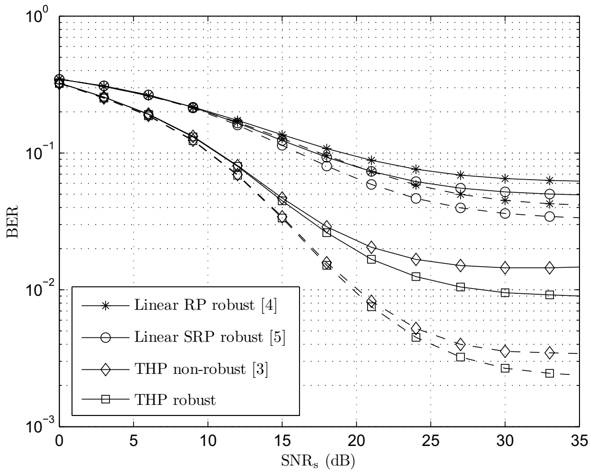

Fig. 2. BER performance for $N_a = N_s = N_r = N_d = 3$, $\mathrm{SNR_r} = 20$dB, $\varrho = \rho = 0.5$. Solid and dashed curves show performances for $\sigma_e^2 = 0.0025$ and $\sigma_e^2 = 0.001$ respectively.

$\sigma_{e_r}^2 = \sigma_e^2$. The elements of $\tilde{\mathbf{H}}_s$ and $\tilde{\mathbf{H}}_r$ are drawn from zero mean Gaussian distributions with variances $\sigma_{\hat{h}_s}^2 = \sigma_{\hat{h}_r}^2 = 1 - \sigma_e^2$. The channel spatial correlation matrices in (5) and (6) are defined by the co-efficients ϱ and ρ, and have elements given by $[\mathbf{\Xi}_s]_{ij} = [\mathbf{\Xi}_r]_{ij} = \varrho^{|i-j|}$ and $[\mathbf{\Upsilon}_s]_{ij} = [\mathbf{\Upsilon}_r]_{ij} = \rho^{|i-j|}$. We compare the performance of the proposed robust THP design to the robust linear relay precoded (RP) design in [4], the robust linear source and relay precoded (SRP) design in [5], and a non-robust THP design in [3], which only uses knowledge of the estimated channels. Fig. 2 shows the uncoded BER against $\mathrm{SNR_s}$ for $\varrho = \rho = 0.5$, with the solid curves showing performance for $\sigma_e^2 = 0.0025$ and the dashed curves showing performance for $\sigma_e^2 = 0.001$. From Fig. 2 we observe firstly that, as expected, the proposed robust THP design outperforms the non-robust THP technique in [3]. The proposed design also outperforms the robust linear transceivers in [4] and [5], and interestingly the non-robust THP transceiver also provides better performance than these designs. The additional computational complexity of the THP techniques compared to the linear transceivers is thus clearly justified by the significant improvement in performance.

6. CONCLUSIONS

In this paper we have investigated the robust design of THP tranceivers for two hop non-regenerative MIMO relay networks in the presence of channel estimation errors. The source, relay, and destination processors were designed to minimise the expected arithmetic MSE subject to transmission power constraints at the source and relay terminals. Simulation results show that this robust technique provides better BER performance compared to a non-robust THP design that does not exploit knowledge of the channel estimation error.

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