The Effect of Water Stage on The Infiltration Rate For Initially Dry Cchannels

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Abstract

Several hydrological models that are used for simulating water flow in rivers and channels are based on the shallow water equations as in Copeland & El-Hanafy, (2006) and Sanders & Katopodes (2000) or Saint Venant equations (El-Hanafy & Copeland, 2007a) and (Ding & Wang 2004). Both the shallow water equations and the Saint Venant equations form a system of partial differential equations which represents mass and momentum conservation along the channel and include source terms for the bed slope and bed friction. The quantity of infiltrated water into the channel bed should be added as a source term in both the mass the momentum equations. Akanbi & Katopodes, (1998) suggested an approximation of seepage outflow in the momentum equation that is based upon Green and Ampt infiltration rate equation.

The well known formula for the infiltration rate by Green and Ampt neglects the effect of the water stage (water head) above the soil surface; this assumption is physically acceptable when the water depth is small compared to the other terms in Green-Ampt infiltration rate equation. But when simulating water flow over an initially dry bed or studying flood wave propagation over a thin initial of water depth then the water depth should be taken into consideration when calculating the infiltration rate despite its requiring more mathematical computations.

This paper presents a staggered finite difference scheme for the channel routing based upon Saint Venant equations and uses the method of characteristics to interpolate the downstream boundary conditions after modifications to suit the case of a shallow water initial depth followed by a flood event (El-Hanafy & Copeland, 2007b). The modified method of characteristics is implemented to achieve a transparent down stream boundary. The relation between the water depth and the infiltration rate has been derived for Saint Venant equations and it is concluded that the effect of water stage has a positive effect on the infiltration rate as was expected.

Keywords
Green Ampt equation; Method of Characteristics; flood prediction; shallow water equations; St. Venant equations.

1 - Introduction

Simulating water flow in a river channel either by the shallow water equations or Saint Venant equations is not a recent topic in the field of computational fluid dynamics, one of the most important challenges is to take the effect of infiltrated water into the channel bed into consideration. Akanbi & Katopodes (1998) modelled the flood wave propagation on an initially dry bed using the finite element technique; Fiedler & Ramirez (2000) applied the MacCormack finite difference scheme to simulate the discontinuous shallow flow over an infiltrating surface. Although both these two works take the effect of infiltration rate into consideration they neglect the effect of the water stage above the soil surface in order to simplify the computations.

Infiltration is the process of water penetration into the soil and it is an important process in the hydrological cycle by which surface runoff and groundwater recharge can be linked. The Richard's equation which is a partial differential equation is used to evaluate the infiltration rate that penetrates the soil surface vertically (Smith & Woolhiser, 1971) as cited by Shaohua at al, (2002). Based on this, three infiltration-capacity formulas by Horton, Philip, and Green-Ampt have been derived. These three formula were assessed by Shaohua at al, (2002) and from their recommendations, it was found that the exact solution of Richard's equation is best fitted by the Green-Ampt infiltration rate equation.

The initial Green-Ampt model (1911) was the first physically-based model/equation describing the infiltration of water into soil. It has been the subject of considerable developments in soil physics and hydrology owing to its simplicity and satisfactory performance for a great variety of water infiltration problems. This model yields the infiltration rate as an implicit function of time (i.e., given a value of time (t), value of the infiltration rate (f) can be obtained only by direct calculation). But one of the important features of the well known Green-Ampt formula is the neglecting of the surface ponding on the infiltration rate.

In this article, efforts were devoted to solve the well known Green-Ampt Formula using Newton Raphson without neglecting the ponding effect on the infiltration rate. The purpose, however, was not to construct a new infiltration formula but to assess the effect of water stage above the soil surface on the infiltration rate while simulating a water flow in initially dry river streams.

2 - Governing equations for the open channel flow

The Saint Venant equations (SVEs) that take the effect of infiltration rate into consideration form a system of partial differential equations which represents mass and momentum conservation along the channel and include source terms for the bed slope and bed friction. These equations may be written as:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - f \cdot b = 0 \tag{2.1}$$

$$\frac{\partial Q}{\partial t} + gA\left(\frac{\partial H}{\partial x} + \frac{\partial z}{\partial x}\right) + \frac{\partial (Qu)}{\partial x} + k\frac{Q|Q|}{A\ R} + \left(\frac{u.f}{2}\right). b = 0 \tag{2.2}$$

where t is time; x is the horizontal distance along the channel; Q is the discharge; A is the flow cross section area; H is the total water stage; g is the gravitational acceleration; z is the vertical distance between the horizontal datum and the channel bed as function (x,t); S_0 is the bed slope = $-\frac{\partial z}{\partial x}$; k is a friction factor = g/C^2 according to Chezy or = $gn^2/R^{(1/3)}$ according to Manning; and $\frac{\partial(Qu)}{\partial x}$ is the momentum flux term, or convective acceleration; b is the channel bottom width and f is the infiltration rate. The effect of infiltration rate is added to the (SVEs) using the Green-Ampt model as shown in Figure 1,as follows:

$$f = \frac{dF}{dt} = -K\left(\psi_f + \frac{F}{\theta_s - \theta_i} - H\right)\frac{\theta_s - \theta_i}{F} \tag{2.3}$$

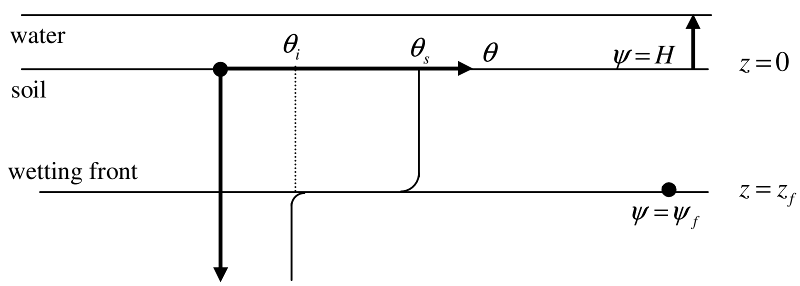

FIGURE 1. Green- Ampt formula

Where F is the cumulative depth of infiltration (-ve); K is saturated hydraulic conductivity; ψ_f is suction at the wetting front (negative pressure head); θ_i is initial moisture content; θ_s is saturated moisture content; and H is the depth of ponding.

The estimation of the momentum loss due to seepage (u.f/2) used in the momentum equation (2.2) follows work by Abiola & Nikaloaos (1998).

Assume H is small relative to the other terms and the previous equation simplifies to the Green-Ampt infiltration rate equation.

$$f = \frac{dF}{dt} = -K\left(\psi_f \frac{\theta_s - \theta_i}{F} + 1\right) \tag{2.4}$$

But if the case under consideration is to simulate flood wave propagation over an infiltrating surface and the initial condition is a dry channel bed as shown in Figure 2, the previous assumption is not acceptable.

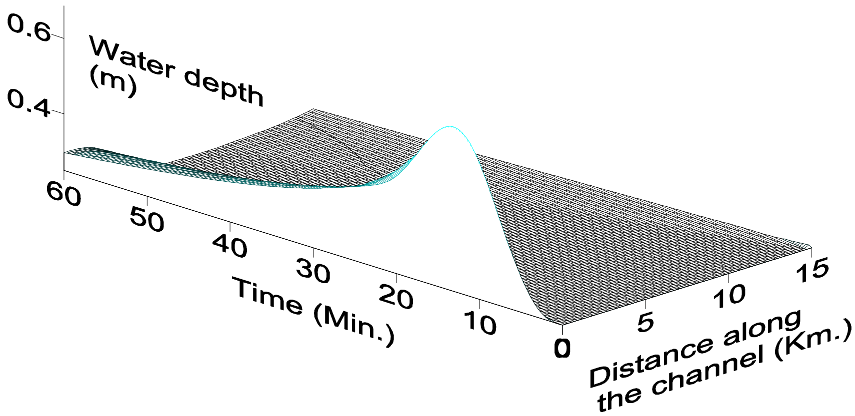

FIGURE 2. Flood wave propagation in a an initially dry bed channel

From Figure 2 it is clear that the initial condition is zero flow, so the effect of the water depth could be definitely neglected. But by focusing on the flood wave along the time axis at the upstream boundary there is a significant change in the water stage, which in turn should affect the infiltration rate.

3 – Solution of Implicit Green-Ampt equation:

The main problem that arises when evaluating the infiltration rate using the exact formula, equation (2.3) is that the cumulative depth of infiltration, F is an implicit function. The well known Newton Raphson could by used to find a solution for this implicit function as will be described later.

Rearranging equation (2.4) gives the cumulative infiltration, F as a function of infiltration rate, f but still neglects the water depth, H.

$$\frac{f}{K}+1=-\psi_f\frac{\theta_s-\theta_i}{F} \quad \rightarrow \quad F=-\psi_f\frac{\theta_s-\theta_i}{1+(f/K)} \qquad (3.1)$$

Now, to take the effect of ponding and to introduce time dependence in equation (2.3) which is a relation between f and F, let $\Delta\theta=\theta_s-\theta_i$ and separate variables in equation (2.3);

$$\int\frac{F}{\Delta\theta\left(\psi_f+\frac{F}{\Delta\theta}-H\right)}dF=\int -K_s dt \qquad (3.2)$$

$$\int\frac{F}{(\Delta\theta\,\psi_f+F-\Delta\theta\,H)}dF=\int -K_s dt \qquad (3.3)$$

Use the following u substitution to integrate equation (3.3)

$u=\Delta\theta\,\psi_f+F-\Delta\theta\,H$

$F=u-\Delta\theta\,\psi_f+\Delta\theta\,H$

$du=dF$

Substitute the above into the differential equation.

$$\int\frac{u-\Delta\theta\,\psi_f+\Delta\theta\,H}{u}du=\int -K_s dt \qquad (3.4)$$

Integrate the previous equation.

$$(u-\Delta\theta\,\psi_f\ln u+\Delta\theta\,H\ln u)\,\Big|=-K_s t\,\Big| \qquad (3.5)$$

Substitute the original variables back into integrated equation.

$$\left[\begin{array}{l}\Delta\theta\,\psi_f+F-\Delta\theta\,H-\Delta\theta\,\psi_f\ln(\Delta\theta\,\psi_f+F-\Delta\theta\,H)\\+\Delta\theta\,H\ln(\Delta\theta\,\psi_f+F-\Delta\theta\,H)\end{array}\right]\Big|_{F_1}^{F_2}=-K_s t\,\Big|_{t_1}^{t_2} \qquad (3.6)$$

Although t can be expressed explicitly as a function of F but F is an implicit function of t. so one of the most common methods to find the roots is Newton Raphson. The Newton Raphson formula consists geometrically of extending the tangent line at a current point xi until it crosses zero (Press et al, 1992) as shown in Figure 3.

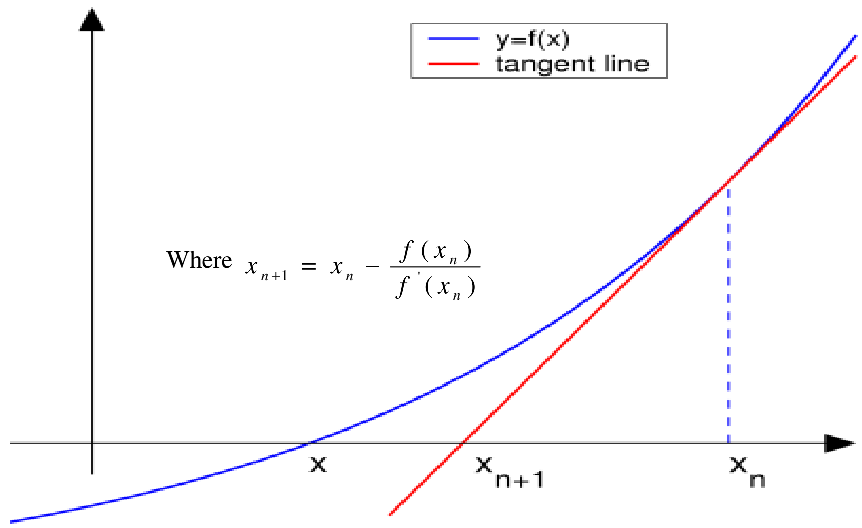

FIGURE 3. The geometrical principle of Newton Raphson Formula

Once the values of the cumulative depth of infiltration are obtianed at each node within the discrizing scheme, the infiltration rate, f is calculated as $f = \frac{\Delta F}{\Delta t}$

4 – Numerical approach:

4.1 – Model discretization

A simple space and time staggered finite difference mesh is used to discretise the domain as depicted in Figure 4. The approximation of the derivatives $\frac{\partial h}{\partial x}$ is $\frac{\left[h_i^j - h_{i-1}^j\right]}{\Delta x}$. A first order upwind scheme is used for the convective term $\frac{\partial (Qu)}{\partial x}$ which is discretised with two point upwind difference expression or a weighted average of centered and upwind difference expressions:

$$\frac{\partial (Qu)}{\partial x} = \frac{Qu(i+1) - Qu(i-1)}{2\Delta x} + \frac{0.5\left(Qu(i-2) - 3Qu(i-1) + 3Qu(i) - Qu(i+1)\right)}{3\Delta x} \quad (4.1)$$

See (Fletcher, 1991), (Leonard, 1983) and (Falconer and Liu, 1988) for more details. The discharge Q is marched forward in time using the momentum equation, equation (2.2) as follow:

$$Q_i^{j+1} = Q_i^j - \left(\frac{gA_i^j}{Tw_i^j}\right)\left(\frac{\Delta t}{\Delta x}\right)\left[A_i^j - A_{i-1}^j\right] - gA_i^j.\left(\frac{\Delta t}{\Delta x}\right)(z_i - z_{i-1}) - \Delta t.\,g\,n^2.\frac{Q_i^j\left|Q_i^j\right|}{AR^{(4/3)}} - \Delta t.\frac{\partial(Qu)}{\partial x} - \frac{\Delta t}{2}.b_{i-1}\, f_i^j\, u_i^j \quad (4.2)$$

The flow cross section, A, is marched forward in time using the continuity equation, equation (2.1)

$$A_i^{j+1} = A_i^j - \left(\frac{\Delta t}{\Delta x}\right)\left[Q_{i+1}^{j+1} - Q_i^{j+1}\right] + \Delta t.b_i\, f_i^j \quad (4.3)$$

where; Tw is the channel top width, and b is the channel bottom width. The initial conditions are A_i^1 and Q_i^1, while the boundary conditions are Q_1^{j+1} at the upstream boundary and A_{nx}^{j+1} at the downstream boundary, the upstream condition is the inflow hydrograph; the downstream condition must be interpolated using the method of characteristics (MOC) as described in Abbott (1977) and French (1986) see below.

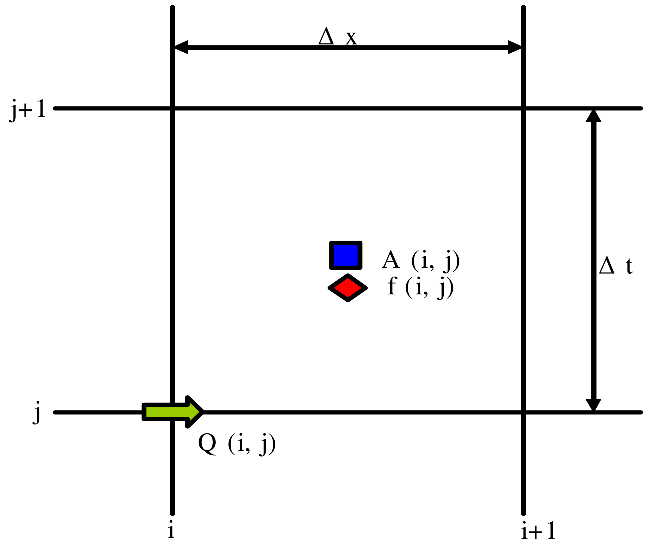

FIGURE 4. The discretization scheme for Saint Venant equations

4.2 -Method of Characteristics (MOC):

Following a standard text such as (Abbott, 1977) and (French, 1986), the characteristics of the Saint Venant equations can be derived. The final form is:

$$\Delta Q + (-u \pm c)\Delta A + g\ n^2 \frac{Q|Q|}{AR^{(4/3)}} + gA\frac{\partial z}{\partial x} - (-u \pm c) f\ b + \frac{f\ u}{2} b = 0 \qquad (4.4)$$

Where; Δ indicates a total change in the variable along the characteristic path. equation (4.4) is conceptually correct from the mathematical point of view, but when dealing with a wave propagating over an initially dry bed we cannot interpolate the unknown value of the cross section area, A at the downstream boundary (point B) in Figure 5 from a unknown value backward in time (point A) in Figure 5 directly since due to nonlinearity, the characteristic is not a straight line but it is a curve as shown by the dashed line in Figure 5. So the Modified Method Of Characteristics (MMOC) see (El-Hanafy & Copeland, 2007b) for more details should be implemented rather than the common Method Of Characteristics (MOC). These formulas are used to interpolate boundary values of B at the downstream boundary (x = nt) from a known value of A at x = nt-1 as multi segments.

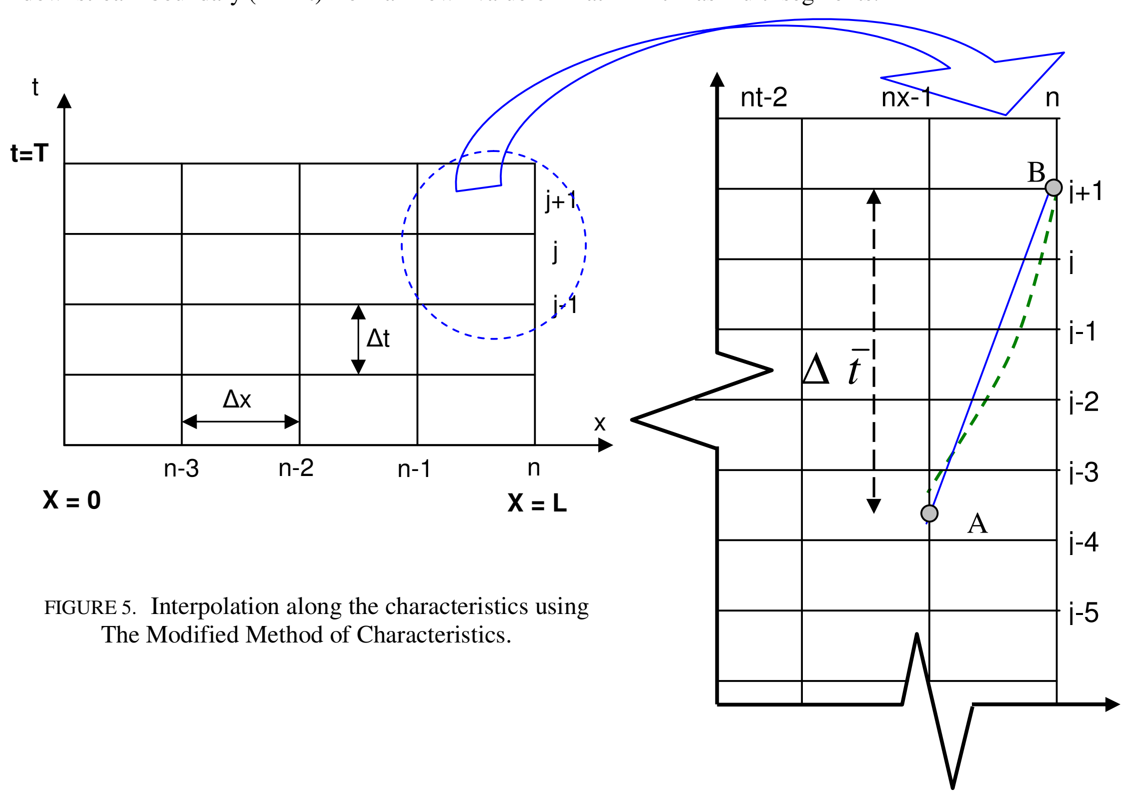

FIGURE 5. Interpolation along the characteristics using The Modified Method of Characteristics.

5 – Test case 1:

This case is a bench mark, idealized case since the studied channel has a trapezoidal cross section as shown in Figure 6 and the channel is frictionless and horizontal. The main purpose of this case is to investigate the effect of the water depth on the infiltration rate. If the driving condition upstream is a steady flow and identical with a uniform initial condition and the infiltration rate is absolutely neglected, then the values of both the discharge and the water depth along the channel should remain as it is without any variation as shown in Figure 7.

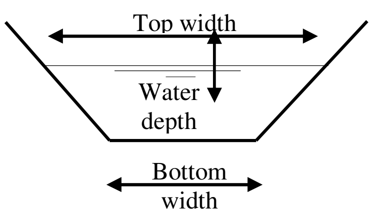

FIGURE 6. Channel cross section

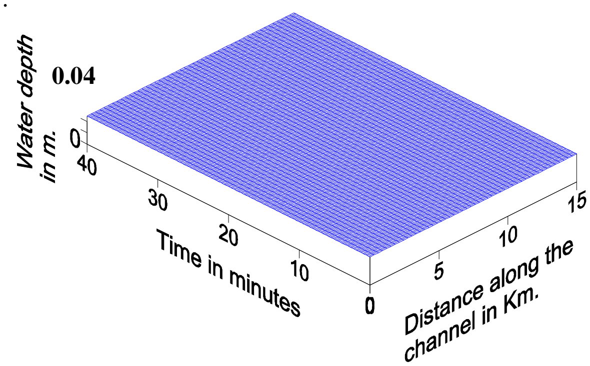

FIGURE 7. Steady uniform flow

If the infiltration rate is involved as a source term in Saint Venant equations (SVEs), equations (2.1 and 2.2) without taking the effect of the water depth, i.e. implement equation (2.4) for the calculation of the infiltration rate, then the water depth is expected to decrease along the time axis since the infiltration rate is function of time as shown in Figure 8.

If the exact solution of Green-Ampt equation is implemented to calculate the infiltration rate, and taking the effect of ponding into consideration, equation (2.3) then the water depth is expected to be less than the previous calculation shown in Figure 8. The result of this case is shown in Figure 9 and by comparing Figure 8 and Figure 9 it is apparent that when the effect of the water depth on the infiltration rate is ignored the infiltrated water into the soil is less than if the water depth is taken into consideration. This is why in Figure 9 the water depth decreases more quickly than in Figure 8 so it drops down from 0.05 m to nearly zero within 33 min. only, while in Figure 8 it takes about 40 minutes to reach the zero stage

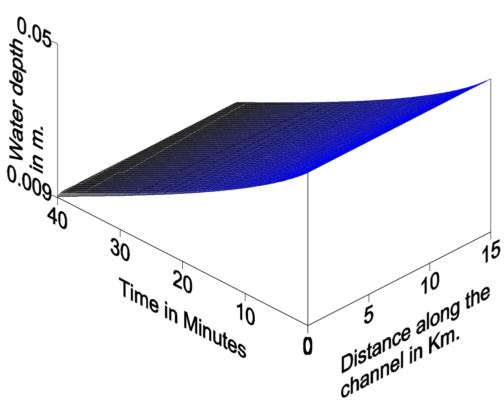

FIGURE 8. Water depth within the channel & the effect of water depth on the infiltration rate is neglected

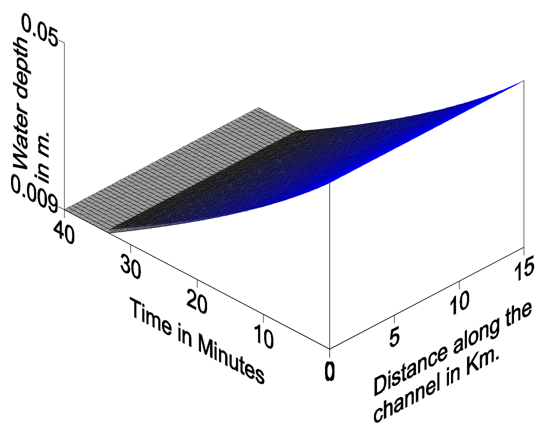

FIGURE 9. Water depth within the channel & taking the effect of water depth on the infiltration rate into consideration

Furthermore, Figure 10 shows the rates of infiltration evaluated from formulas, equation (2.3) and equation (2.4), which emphasis that the rate of infiltration increase when taking the effect of ponding into consideration, in other word neglecting the ponding effect results in under estimates of the infiltration rate.

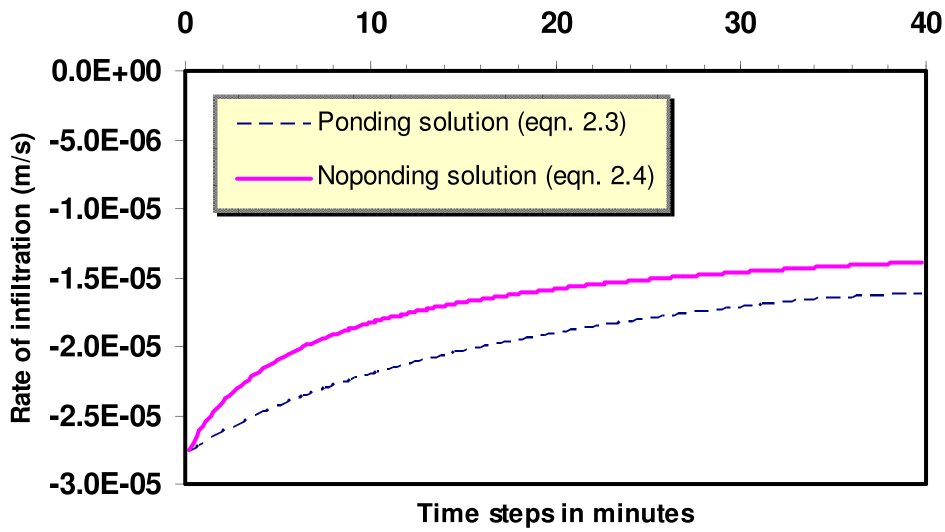

FIGURE 10. Water depth within the channel

6 – Test case 2:

At this case the stream is 15 km long and its cross section is rectangular section with 3.0 m bottom width. Flow in the channel is simulated for a period of 2.5 hour during which time a sinusoidal hydrograph shape of duration 50 minute as shown in Figure 11 is introduced at the upstream boundary and passes along the channel to represent a flash flood event. The peak discharge is $0.16\ m^3.s^{-1}$.

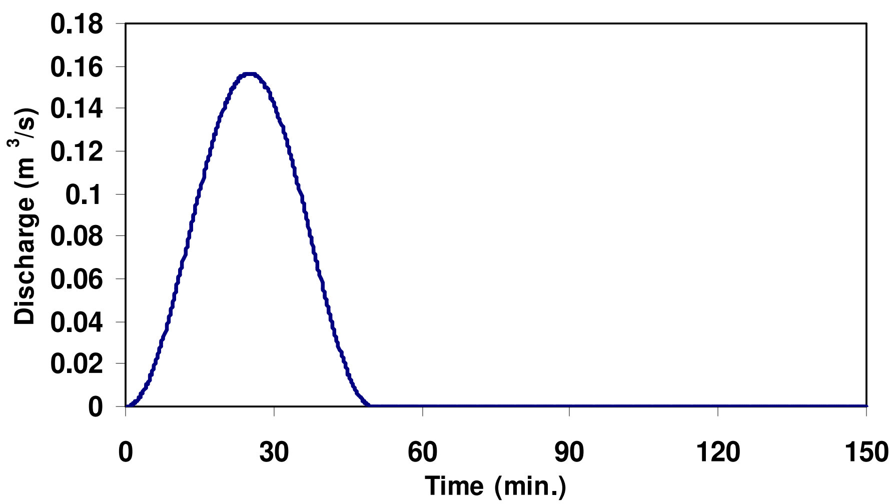

FIGURE 11. a flash flood hydrograph at the upstream boundary.

The effect of the infiltration rate on the flow is significant as illustrated in Figure 12 which represents the flood passage along the channel. The water depth decreased from 15 cm to zero within 2 hr. and 20 min. at the down stream boundary, while the flood wave reached a zero flow within 2.5 hr.

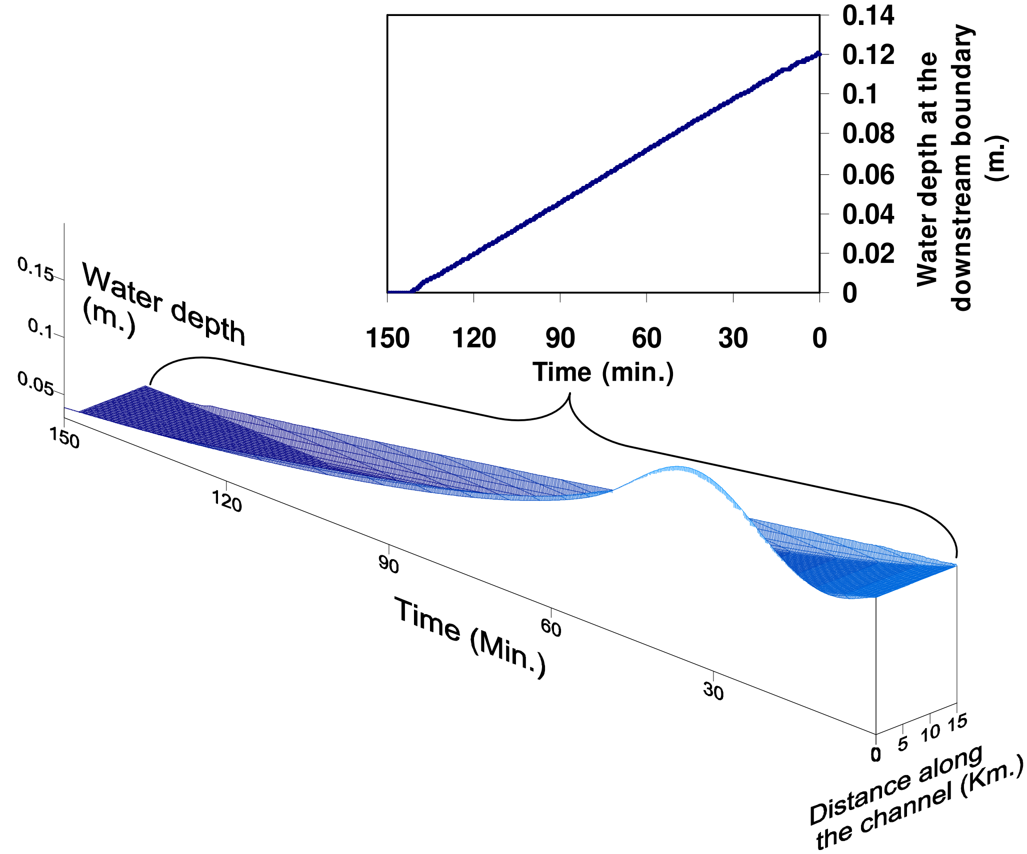

FIGURE 12. Flash flood wave propagation

Figure 13 show the distribution of the infiltration rate along the channel for 2.5 hr. and it is clear how the values of the infiltration are responding to the water depth along the trajectory of the flood wave.

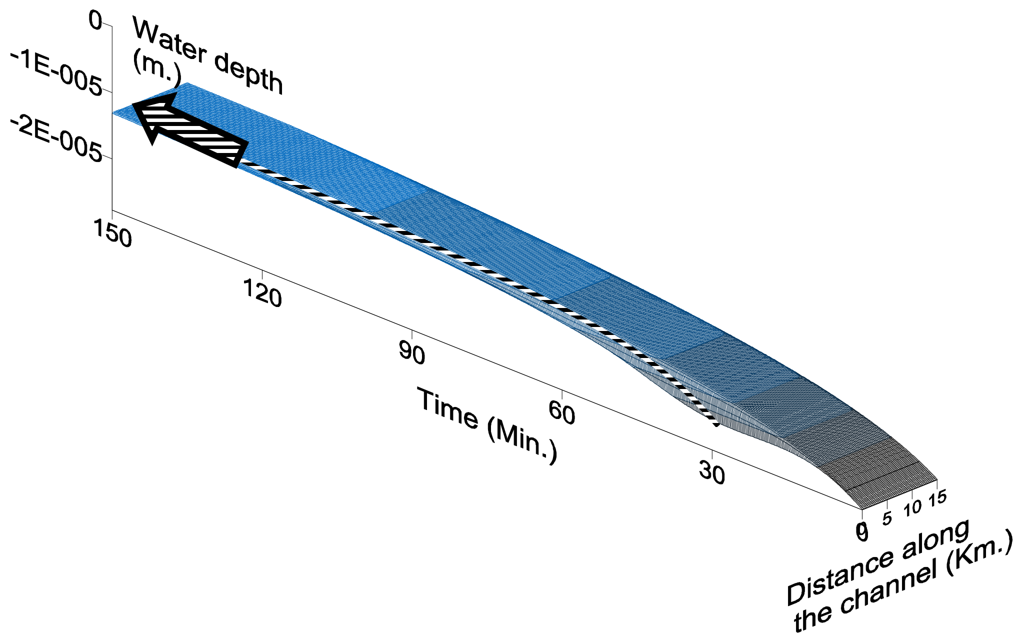

FIGURE 13. distribution of the infiltration rate

7 – Conclusions:

This paper shows that, although Green-Ampt Formula is considered one of the most appropriate formula for the calculation of the infiltration rate (Shaohua at al, 2002), some precautions should be considered while implementing it. As shown, one of the most important factors that greatly affects the calculations of the infiltration rate is the effect of ponding above the soil surface on the infiltration rate which in turn should reduce the water stage of the water flow. So in a case where the studied channel is an irrigation channel for example, it is clear now that the total infiltrated water to the ground will be greater than if it is calculated without taking the effect of ponding. Also, the water stage, which is an important factor in the design of irrigation structures, will be less than if it is calculated without taking the effect of ponding.

If the purpose of a study is not only to simulate the flow but also to evaluate the sensitivity of the flow to some controls such as the bed friction, bed slope and infiltration rate as in the work by El-Hanafy & Copeland (2007c), in this case the estimation of the sensitivity of the flow to the infiltration rate will be less if the effect of ponding is ignored see Figure 10

The present more complete solution for Green-Ampt equation allows a realistic computation of the infiltration rate, water flow conditions, and sensitivity analysis. Results of the sensitivity of the flow to the infiltration rate are shown in separate publications El-Hanafy et al, (2007d).

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