A NEW CLASS OF BROADBAND ARRAYS WITH FREQUENCY INVARIANT BEAM PATTERNS

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ABSTRACT

In this paper, a new class of broadband arrays with frequency-invariant beam patterns is proposed. By suitable substitutions, the beam pattern of a continuous sensor array with continuous temporal processing can be regarded as the Fourier transform of its spatio-temporal distribution. Based on this principle, starting from the desired frequency-invariant beam pattern, and by a series of substitutions, a simple design method is derived. This method can be applied to one-dimensional (1-D) , 2-D, or 3-D broadband arrays, either with continuous arrays and signal processing or with discrete arrays and signal processing. A 2-D discrete design example is presented.

1. INTRODUCTION

In the past, broadband beamformers have been studied extensively due to its applications to sonar, radar and communications [1]. Amongst them is a class of arrays with frequency invariant beam patterns [2, 3, 4], for which a systematic method has been proposed in [4] and can be applied to one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D) arrays. The design for 1-D array is relatively simple because of the dilation property, but for higher-dimensional arrays this property is not guaranteed and can lead to complications.

In this paper, we propose a new class of frequency-invariant broadband arrays, which exploit the Fourier transform relationship between the array's spatio-temporal distribution and its beam pattern. Starting from the desired frequency-invariant beam pattern, by a series of substitutions a simple design method is found for frequency invariant beamforming design. This method can be applied to 1-D, 2-D and 3-D broadband arrays and a design example of a 2-D array is given to show its effectiveness. A previously proposed frequency invariant linear array [5] can be regarded as a special case of this new class of arrays. Comparing with [4], the main advantage of the proposed scheme lies in its extreme simplicity as result can be obtained directly by the Fourier transform.

The paper is organised as follows. Section 2 deals with the problem of continuous sensors and signal. Section 3 is focused on the case of discrete sensors and signals. A design example of a discrete planar broadband array is given in Section 4, and conclusions are drawn in Sec. 5.

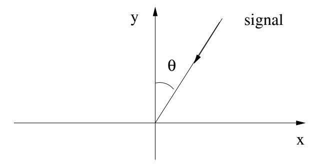

Fig. 1. A continuous sensor array aligned with the x axis.

2. FREQUENCY INVARIANT BEAMFORMING – CONTINUOUS SENSOR AND SIGNAL

Although the main purpose of this work is to derive a class of frequency-invariant broadband arrays with a finite number of sensors and discrete signals, we start with the case of continuous sensors and signals, from which we will derive the discrete case.

2.1. One-dimensional Array

Fig. 1 shows a 1-D continuous sensor array. The response of this linear array is given by

$$P(\omega,\theta) = \int_{-\infty}^{\infty} e^{-j\frac{\omega \sin\theta}{c}x} D(x,\omega)dx \ , \tag{1}$$

where c and θ are the propagation speed and angle of the impinging signal and $D(x,\omega)$ the frequency response with respect to the angular frequency ω and location x. Obviously, in general $P(\omega,\theta)$ is a function of both ω and θ, while for a frequency invariant beamformer, we require that the beam pattern $P(\omega,\theta)$ be independent of ω.

Suppose the inverse Fourier transform of $D(x,\omega)$ is $d(x,t)$, i.e.

$$D(x,\omega) = \int_{-\infty}^{\infty} d(x,t)e^{-j\omega t}dt, \tag{2}$$

then we get

$$P(\omega,\theta) = \iint_{-\infty}^{\infty} d(x,t)e^{-j\frac{\omega \sin\theta}{c}x}e^{-j\omega t}dxdt \ . \tag{3}$$

Note that $d(x,t)$ can in general be non-causal, in which case a causal approximate response can be attained by delaying and truncating $d(x,t)$.

Introducing the substitutions $\omega_1 = \frac{\omega \sin\theta}{c}$ and $\omega_2 = \omega$ into (3), we have

$$P(\omega_1,\omega_2) = \iint_{-\infty}^{\infty} d(x,t)e^{-j\omega_1 x}e^{-j\omega_2 t}dxdt \ . \tag{4}$$

From (4) we see that the beam pattern of a continuous linear array can be obtained by first applying a 2-D Fourier transform to $d(x,t)$ and then using the substitutions $\omega_1 = \frac{\omega \sin\theta}{c}$ and $\omega_2 = \omega$. The spatio-temporal spectrum of the impinging signal is located

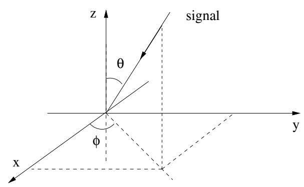

Fig. 2. A continuous 2-D sensor array in the (x,y) plane.

on the line $\omega_1 = \frac{\omega_2 \sin\theta}{c}$ of the (ω_1, ω_2) plane. If $P(\omega_1,\omega_2)$ can be expressed as a function of $\frac{\omega_1 c}{\omega_2}$, i.e. $P(\omega_1,\omega_2) = F(\frac{\omega_1 c}{\omega_2})$, where $F(\hat{\omega})$ is the frequency response of a 1-D filter ($\hat{\omega}$ is the angular frequency), then the resultant $P(\omega,\theta)$ will be independent of ω, i.e. frequency-invariant. Thus, we have a new way to design a frequency-invariant broadband beamformer. Suppose the main beam of the desired frequency-invariant beam pattern $P(\sin\theta)$ points to broadside, then the design can be divided into the following steps:

Step 1. From $P(\sin\theta)$ we obtain the frequency response of a 1-D filter $F(\hat{\omega})$ with a period of $2\omega_{1D\_1}$, which is defined as

$$F(\hat{\omega}) = P(\frac{\hat{\omega}}{\omega_{1D\_1}}) \text{ for } \hat{\omega} \in [-\omega_{1D\_1}; \omega_{1D\_1}) \ , \qquad (5)$$

where ω_{1D_1} is a constant. As the main beam is at broadside, $F(\hat{\omega}) \leq F(\hat{\omega})|_{\hat{\omega}=0}$ is a lowpass filter.

Step 2. With the substitution $\hat{\omega} = \frac{\omega_1 c}{\omega_2}\omega_{1D\_1}$, we obtain

$$\hat{P}(\omega_1,\omega_2) = \begin{cases} F(\frac{\omega_1 c}{\omega_2}\omega_{1D\_1}) & \text{for } \omega_2 \neq 0 \\ a(\omega_1) & \text{for } \omega_2 = 0 \end{cases} , \qquad (6)$$

where $a(\omega_1)$ is an arbitrary function with finite value because there is no signal existing for $\omega_2 = 0$.

Step 3. Suppose the maximum frequency of the signal of interest is ω_{1D_2}. We apply the following modification to $\hat{P}(\omega_1,\omega_2)$ to yield $P(\omega_1,\omega_2)$, the 2-D Fourier transform of the desired response $d(x,t)$,

$$P(\omega_1,\omega_2) = \begin{cases} 0 & \text{for } |\omega_2| > \omega_{1D\_2} \\ & \text{or } |\omega_1| > \omega_{1D\_2}/c \\ \hat{P}(\omega_1,\omega_2) & \text{otherwise} \end{cases} . \qquad (7)$$

This modification guarantees that the resulting $P(\omega_1,\omega_2)$ is absolutely integrable and its inverse Fourier transform exists.

Step 4. Lastly, the desired response $d(x,t)$ is obtained by the 2-D inverse Fourier transform of $P(\omega_1,\omega_2)$, which may be analytically difficult. Hence numerical methods may have to be employed.

If the main beam of the design is to point in the direction θ_0, instead of electronically steering the broadside main beam to this new direction, we can use the substitution $\hat{\omega} = (\frac{\omega_1 c}{\omega_2} - \sin\theta_0)\omega_{1D\_1}$ in the broadside main beam design. Thus for a signal from direction θ_0, $\frac{\omega_1 c}{\omega_2} = \sin\theta_0$, and

$$\hat{P}(\omega_1,\omega_2) = F((\frac{\omega_1 c}{\omega_2} - \sin\theta_0)\omega_{1D\_1}) \text{ for } \omega_2 \neq 0 \qquad (8)$$

will form the main beam pointing to the desired direction. Actually the broadside main beam design method can be applied directly to the non-broadside main beam design without any change, In this case, since the main beam of the desired beam pattern is not pointing to broadside, $F(\hat{\omega})$ will not be a lowpass filter.

2.2. Two-dimensional Array

Fig. 2 shows a 2-D continuous sensor array in the (x,y) plane. The response of this continuous array to signal from the direction (θ,ϕ) is given by

$$P(\omega,\theta,\phi) = \iint_{-\infty}^{\infty} e^{-j\frac{\omega}{c}(x\sin\theta\cos\phi + y\sin\theta\sin\phi)} \cdot D(x,y,\omega)dxdy \ , \qquad (9)$$

where $D(x,y,\omega)$ is the frequency response of the sensor at point (x,y). Similarly, using the inverse Fourier transform $d(x,y,t)$ of $D(x,y,\omega)$, we have

$$P(\omega,\theta,\phi) = \iiint_{-\infty}^{\infty} d(x,y,t)\, e^{-j\frac{\omega\sin\theta\cos\phi}{c}x} \cdot e^{-j\frac{\omega\sin\theta\sin\phi}{c}y} e^{-j\omega t} dxdydt. \qquad (10)$$

The substitution of $\omega_1 = \frac{\omega\sin\theta\cos\phi}{c}$, $\omega_2 = \frac{\omega\sin\theta\sin\phi}{c}$ and $\omega_3 = \omega$ into (10) results in

$$P(\omega_1,\omega_2,\omega_3) = \iiint_{-\infty}^{\infty} d(x,y,t)\, e^{-j\omega_1 x} \cdot e^{-j\omega_2 y} e^{-j\omega_3 t} dxdydt, \qquad (11)$$

i.e. $P(\omega_1,\omega_2,\omega_3)$ is the 3-D Fourier transform of $d(x,y,t)$. In this case, the spatio-temporal spectrum of the impinging signal lies on the lines $\frac{\omega_1 c}{\omega_3} = \sin\theta\cos\phi$ and $\frac{\omega_2 c}{\omega_3} = \sin\theta\sin\phi$. If we can express $P(\omega_1,\omega_2,\omega_3) = F(\frac{\omega_1 c}{\omega_3}, \frac{\omega_2 c}{\omega_3})$, where $F(\hat{\omega}_1,\hat{\omega}_2)$ is the frequency response of a 2-D filter, then the resulting $P(\omega,\theta,\phi)$ will be frequency-invariant. Analogously to the 1-D design in Sec. 2.1, the frequency-invariant beamformer for 2-D case can be designed as follows:

Step 1. Suppose $P(\sin\theta\cos\phi, \sin\theta\sin\phi)$ is the desired beam pattern with the main beam pointing towards broadside, then over one period $\hat{\omega}_1 \in [-\omega_{2D\_1}; \omega_{2D\_1})$ and $\hat{\omega}_2 \in [-\omega_{2D\_2}; \omega_{2D\_2})$, the 2-D periodic filter $F(\hat{\omega}_1,\hat{\omega}_2)$ is defined as

$$F(\hat{\omega}_1,\hat{\omega}_2) = P(\frac{\hat{\omega}_1}{\omega_{2D\_1}}, \frac{\hat{\omega}_2}{\omega_{2D\_2}}) \ , \qquad (12)$$

where ω_{2D_1} and ω_{2D_2} are constants and $F(\hat{\omega}_1,\hat{\omega}_2)$ is a lowpass filter.

Step 2. The substitutions $\hat{\omega}_1 = \frac{\omega_1 c}{\omega_3}\omega_{2D\_1}$ and $\hat{\omega}_2 = \frac{\omega_2 c}{\omega_3}\omega_{2D\_2}$ yield

$$\hat{P}(\omega_1,\omega_2,\omega_3) = \begin{cases} F(\frac{\omega_1 c}{\omega_3}\omega_{2D\_1}, \frac{\omega_2 c}{\omega_3}\omega_{2D\_2}) & \text{for } \omega_3 \neq 0 \\ a(\omega_1,\omega_2) & \text{for } \omega_3 = 0 \end{cases} , \qquad (13)$$

where $a(\omega_1,\omega_2)$ is an arbitrary function with finite values.

Step 3. Suppose the maximum frequency of the interested signal is ω_{2D_3}. We define

$$P(\omega_1,\omega_2,\omega_3) = \begin{cases} 0 & \text{for } |\omega_3| > \omega_{2D\_3} \\ & \text{or } |\omega_1| > \omega_{2D\_3}/c \\ & \text{or } |\omega_2| > \omega_{2D\_3}/c, \\ \hat{P}(\omega_1,\omega_2,\omega_3) & \text{otherwise.} \end{cases} \qquad (14)$$

This modification of $\hat{P}(\omega_1,\omega_2,\omega_3)$ has the same effect as in the 1-D case.

Step 4. To obtain the desired response $d(x,y,t)$, we apply a 3-D inverse Fourier transform to $P(\omega_1,\omega_2,\omega_3)$. As in the 1-D case, we may also need to delay and truncate the result to get its causal approximation.

If the main beam is desired to point towards (θ_0,ϕ_0), the substitutions $\hat{\omega}_1 = (\frac{\omega_1 c}{\omega_3} - \sin\theta_0\cos\phi_0)\omega_{2D\_1}$, $\hat{\omega}_2 = (\frac{\omega_2 c}{\omega_3} - \sin\theta_0\sin\phi_0)\omega_{2D\_2}$ can be applied to the broadside main beam design while all other design parameters remain unchanged.

2.3. Three-dimensional Array

The response of a 3-D continuous sensor array, which in addition to Fig. 2 also extends in z direction, is given by

$$P(\omega,\theta,\phi) = \iiiint_{-\infty}^{\infty} d(x,y,z,t)\, e^{-j\frac{\omega\sin\theta\cos\phi}{c}x} \cdot e^{-j\frac{\omega\sin\theta\sin\phi}{c}y} e^{-j\frac{\omega\cos\theta}{c}z} e^{-j\omega t} dxdydzdt \ . \qquad (15)$$

Here, the substitutions $\omega_1 = \frac{\omega \sin\theta \cos\phi}{c}$, $\omega_2 = \frac{\omega \sin\theta \sin\phi}{c}$, $\omega_3 = \frac{\omega \cos\theta}{c}$ and $\omega_4 = \omega$ lead to

$$P(\omega_1, \omega_2, \omega_3, \omega_4) = \iiiint_{-\infty}^{\infty} d(x, y, z, t)\, e^{-j\omega_1 x} \cdot \cdot e^{-j\omega_2 y} e^{-j\omega_3 z} e^{-j\omega_4 t} dxdydzdt . \quad (16)$$

By these substitutions, the spatio-temporal spectrum of the impinging signal falls onto the lines $\frac{\omega_1 c}{\omega_4} = \sin\theta\cos\phi$, $\frac{\omega_2 c}{\omega_4} = \sin\theta\sin\phi$ and $\frac{\omega_3 c}{\omega_4} = \cos\theta$, respectively. To design a frequency-invariant 3-D broadband array, the methods from Secs. 2.1 and 2.2 can be further extended. The procedure is analogous to the previous design steps and is omitted here.

3. FREQUENCY INVARIANT BEAMFORMING – DISCRETE SENSOR AND SIGNAL

Having established the theory for continuous sensor and signal, the aim is to extend the design to the discrete case. In general, this can be regarded as an approximation to the ideal response obtained for the continuous case, i.e. the $d(x,t)$ or $d(x,y,t)$ resulting from the proposed method need to be sampled and truncated according to the sensor pattern and the temporal processing structure. However, for a special class of sensor patterns, where all the sensors are positioned on lines parallel to the x axis (1-D array), x and y axes (2-D), or x, y and z axes (3-D) and equispaced in each of the three directions, a corresponding theory can be developed based on the Fourier transform of a discrete series rather than a continuous function.

3.1. One-dimensional Array

Suppose the sensor spacing is d_x and the signal sampling period is T. Corresponding to (3) of the continuous case, we have

$$P(\omega, \theta) = \sum_{m,n=-\infty}^{\infty} d[m,n]\, e^{-jm\frac{\omega \sin\theta}{c} d_x}\, e^{-jn\omega T} , \quad (17)$$

where $d[m,n]$ is the coefficient in the n-th position of the m-th sensor's tapped-delay line (TDL). For the same reason as in the 1-D continuous case, $d[m,n]$ is in general non-causal.

To avoid aliasing in both the spatial and temporal domains, T should be half of the period of the maximum signal frequency of interest and d_x half of its wavelength λ_{max}. Thus we have $d_x = \frac{\lambda_{max}}{2} = cT$ and $\omega T = \Omega$, where Ω is the normalised angular frequency. Without loss of generality, we always assume the same configuration for the following 2-D and 3-D cases. Therefore, (17) can be rewritten as

$$P(\omega, \theta) = \sum_{m,n=-\infty}^{\infty} d[m,n]\, e^{-jm\Omega \sin\theta}\, e^{-jn\Omega} . \quad (18)$$

By substituting $\Omega_1 = \Omega \sin\theta$ and $\Omega_2 = \Omega$, (18) yields

$$P(\Omega_1, \Omega_2) = \sum_{m,n=-\infty}^{\infty} d[m,n]\, e^{-jm\Omega_1}\, e^{-jn\Omega_2}. \quad (19)$$

Now the spatio-temporal spectrum of the impinging signal lies on the line $\Omega_1 = \Omega_2 \sin\theta$, and again a method can be developed to obtain a frequency-invariant beam pattern pointing towards broadside, comprising the steps below.

Step 1. From the desired beam pattern $P(\sin\theta)$ we derive the frequency response of a 1-D filter $F(\hat{\Omega})$, which is periodic with 2π, defined over one period as

$$F(\hat{\Omega}) = P(\hat{\Omega}/\pi) \qquad \text{for} \quad \hat{\Omega} \in [-\pi; \pi). \quad (20)$$

The obtained $F(\hat{\Omega})$ is a lowpass filter.

Step 2. With the substitution $\hat{\Omega} = \frac{\Omega_1}{\Omega_2}\pi$, for $(\Omega_1, \Omega_2) \in [-\pi; \pi)$ we have

$$P(\Omega_1, \Omega_2) = \begin{cases} F((\Omega_1/\Omega_2)\pi) & \text{for } \Omega_2 \neq 0 \\ a(\Omega_1) & \text{for } \Omega_2 = 0 \end{cases} , \quad (21)$$

where $a(\Omega_1)$ is an arbitrary function with finite values. Note that $P(\Omega_1, \Omega_2)$ is a function with period of 2π.

Step 3. Applying a 2-D inverse Fourier transform to $P(\Omega_1, \Omega_2)$ results in an infinite support of $d[m,n]$. As in the continuous case, it is difficult to obtain result analytically; therefore we can apply the 2-D inverse DFT as an approximation by sampling $P(\Omega_1, \Omega_2)$. In either case, the resulting $d[m,n]$ needs to be delayed along the n axis for reasons of causality and to be truncated according to the number of sensors and the TDL length.

For a main beam in the direction θ_0, we can simply employ the substitution $\hat{\Omega} = (\frac{\Omega_1}{\Omega_2} - \sin\theta_0)\pi$.

A similar method has been proposed in [5] to deal with the case of linear arrays with finite sensors and coefficients. Approaching from the continuous case and the discrete case with infinite number of sensors and coefficients, we can consider the problem in [5] as a special case of the proposed class of frequency-invariant broadband arrays.

3.2. Two-dimensional Array

With spatial indices l and m and the time index n, a 2-D array response is given by

$$P(\omega, \theta, \phi) = \sum_{l,m,n=-\infty}^{\infty} d[l,m,n]\, e^{-jl\frac{\omega}{c}d_x \sin\theta\cos\phi} \cdot \cdot e^{-jm\frac{\omega}{c}d_y \sin\theta\sin\phi}\, e^{-jn\omega T} . \quad (22)$$

Assuming $d_x = d_y = \frac{\lambda_{max}}{2} = cT$, we have

$$P(\omega, \theta, \phi) = \sum_{l,m,n=-\infty}^{\infty} d[l,m,n]\, e^{-jl\Omega \sin\theta\cos\phi} \cdot \cdot e^{-jm\Omega \sin\theta\sin\phi} e^{-jn\Omega} . \quad (23)$$

Substituting $\Omega_1 = \Omega \sin\theta\cos\phi$, $\Omega_2 = \Omega\sin\theta\sin\phi$ and $\Omega_3 = \Omega$ into (23) gives

$$P(\Omega_1, \Omega_2, \Omega_3) = \sum_{l,m,n=-\infty}^{\infty} d[l,m,n] e^{-jl\Omega_1} e^{-jm\Omega_2} e^{-jn\Omega_3}. \quad (24)$$

The spatio-temporal spectrum of the impinging signal in the 2-D case lies on the lines $\frac{\Omega_1}{\Omega_3} = \sin\theta\cos\phi$ and $\frac{\Omega_2}{\Omega_3} = \sin\theta\sin\phi$, respectively. Based on the design of the 2-D continuous array, we perform the 2-D discrete design case as follows:

Step 1. Suppose $P(\sin\theta\cos\phi, \sin\theta\sin\phi)$ is the desired beam pattern pointing towards broadside. Then the 2-D filter $F(\hat{\Omega}_1, \hat{\Omega}_2)$ with a period of 2π is defined as

$$F(\hat{\Omega}_1, \hat{\Omega}_2) = P(\hat{\Omega}_1/\pi, \hat{\Omega}_2/\pi) \text{ for } (\hat{\Omega}_1, \hat{\Omega}_2) \in [-\pi; \pi) , \quad (25)$$

which has a 2-D lowpass characteristic.

Step 2. With the substitutions $\hat{\Omega}_1 = \frac{\Omega_1}{\Omega_3}\pi$ and $\hat{\Omega}_2 = \frac{\Omega_2}{\Omega_3}\pi$, we obtain $P(\Omega_1, \Omega_2, \Omega_3)$ defined over an interval of one period $\Omega_1, \Omega_2, \Omega_3 \in [-\pi; \pi)$ as

$$P(\Omega_1, \Omega_2, \Omega_3) = \begin{cases} F((\Omega_1/\Omega_3)\pi, (\Omega_2/\Omega_3)\pi) & \text{for } \Omega_3 \neq 0 \\ a(\Omega_1, \Omega_2) & \text{for } \Omega_3 = 0 \end{cases} , \quad (26)$$

where $a(\Omega_1, \Omega_2)$ is an arbitrary function with finite values.

Step 3. Applying a 3-D inverse Fourier transform (or DFT as an approximation) to $P(\Omega_1, \Omega_2, \Omega_3)$ returns the desired response

$d[l,m,n]$. For a causal and practical result, a truncation in the spatial l and m domains and the temporal n domain is necessary with a possible shift in time n prior to truncation in order to ensure causality.

For a main beam in the direction (θ_0, ϕ_0), we can substitute $\hat{\Omega}_1$ and $\hat{\Omega}_2$ in step 2 of the design by $(\frac{\Omega_1}{\Omega_3} - \sin\theta_0\cos\phi_0)\pi$ and $(\frac{\Omega_2}{\Omega_3} - \sin\theta_0\sin\phi_0)\pi$, respectively.

3.3. Three-dimensional Array

With spatial indices k, l, m, and the TDL index n, the response of a 3-D array is given by

$$P(\omega,\theta,\phi) = \sum_{k,l,m,n=-\infty}^{\infty} d[k,l,m,n]\; e^{-jk\frac{\omega\sin\theta\cos\phi}{c}d_x} \cdot \\ \cdot\, e^{-jl\frac{\omega\sin\theta\sin\phi}{c}d_y} e^{-ml\frac{\omega\cos\theta}{c}d_z} e^{-jn\omega T}. \quad (27)$$

With $d_x = d_y = d_z = cT$, $\Omega = \omega T$, the substitutions $\Omega_1 = \Omega\sin\theta\cos\phi$, $\Omega_2 = \Omega\sin\theta\sin\phi$, $\Omega_3 = \Omega\cos\theta$ and $\Omega_4 = \Omega$ into (27) yield

$$P(\Omega_1,\Omega_2,\Omega_3,\Omega_4) = \sum_{k,l,m,n=-\infty}^{\infty} d[k,l,m,n]\; e^{-jk\Omega_1} \cdot \\ \cdot\, e^{-jl\Omega_2} e^{-jm\Omega_3} e^{-jn\Omega_4}. \quad (28)$$

Base on the above substitutions, a 3-D array with frequency-invariant beam pattern can be developed analogously to the previous designs, for which we omit the details here.

4. DESIGN EXAMPLE

To show the effectiveness of the proposed method, we give a simple design example for an equispaced planar array with 24×24 sensors and a TDL length of 24. Its main beam is to point towards broadside. The desired frequency-invariant response is given by

$$P(\theta,\phi) = \frac{1}{49}\sum_{l,m=-3}^{3} e^{-jl\pi\sin\theta\cos\phi} e^{-jm\pi\sin\theta\sin\phi}, \quad (29)$$

which is the response of a planar array with uniform weighting to a signal with frequency $\Omega = \pi$. Following the method outlined in Sec. 3.2, utilising a 3-D inverse DFT we obtain a 4-D beam pattern, of which some aspects are characterised in Figs. 3 and 4.

The response to a single frequency $\Omega = 0.85\pi$ is given in Fig. 3, while the frequency-invariant property can be appreciated by a representative slice of the beam pattern at $\phi = 210^\circ$ shown in Fig. 4. Note that for $\Omega \geq 0.35\pi$ the beam pattern is almost frequency-invariant. The reason for the degradation for $\Omega < 0.35\pi$ is that the sample density of the spatio-temporal spectrum in this region is lower than that at higher frequencies, and hence cannot be represented sufficiently when applying inverse DFTs.

5. CONCLUSIONS

A new class of broadband arrays with frequency-invariant beam patterns has been proposed. It can be applied to arrays with continuous or discrete sensors and signals. The main advantage of this method is its simplicity by relying mostly on substitutions and Fourier transforms. A design example for an equispaced broadband planar array has been given, which shows a satisfactory frequency-invariant characteristic over a large range of frequencies.

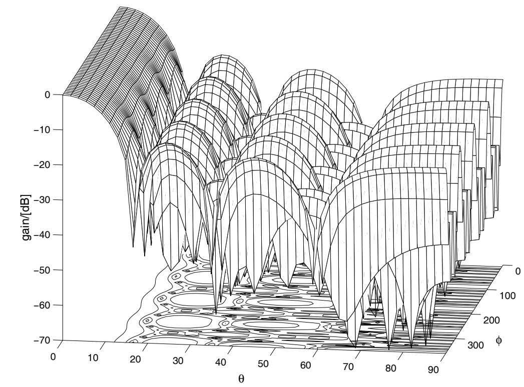

Fig. 3. The resulting beam pattern at $\Omega = 0.85\pi$.

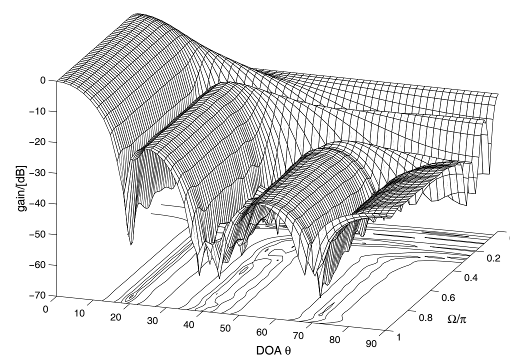

Fig. 4. A slice of the beam pattern at $\phi = 210^\circ$.

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