Uncertainty Modelling in Reliable Preliminary Space Mission Design

Nicolas Croisard and **Matteo Ceriotti** and **Massimiliano Vasile**
University of Glasgow
Department of Aerospace Engineering
n.croisard@aero.gla.ac.uk

In the early phase of the design of a space mission, it is generally desirable to investigate as many feasible alternative solutions as possible. At this particular stage, an insufficient consideration for uncertainty would lead to a wrong decision on the feasibility of the mission. Traditionally a system margin approach is used in order to take into account the inherent uncertainties within the subsystem budgets. The reliability of the mission is then independently computed in parallel. An iteration process between the solution design and the reliability assessment should finally converge to an acceptable solution.

By combining modern statistical methods to model uncertainties and global search techniques for multidisciplinary design, the present work proposes a way to introduce uncertainties in the mission design problem formulation. By minimising the effect of these uncertainties on both constraints and objective functions, while optimising the mission goals, the aim is to increase the reliability of the produced results.

Uncertainties are usually classified in two distinct categories, aleatory and epistemic uncertainty. According to K. Sentz and S. Ferson, the definition of each type is [2002]:

Aleatory Uncertainty The type of uncertainty which results from the fact that a system can behave in random ways.

Epistemic Uncertainty The type of uncertainty which results from the lack of knowledge about a system and is a property of the analysts performing the analysis.

Aleatory uncertainties are due to the random nature of input data while epistemic ones are generally linked to incomplete modelling of the physical system, the boundary conditions, unexpected failure modes, etc.

In the particular case of preliminary space mission design, analysts face both types of uncertainty. For example, the initial velocity of the spacecraft, the gravity model or the solar radiation present aleatory uncertainties. However, most of the parameters of the spacecraft subsystems are first assessed by a group of experts, expressing their opinion on ranges of values. The uncertainty associated to those parameters is therefore epistemic.

The classical way to treat uncertainty is through probability theory. It is well suitable to mathematically model aleatory uncertainties, as far as enough data, experimental for instance, are available. Even though, the analyst still has to assume the distribution function and estimate its parameters. Moreover, the available data may be insufficient to construct an acceptable probability distribution. In this case, the uncertainty is in fact epistemic and not aleatory.

Probability fails to represent epistemic uncertainties because there is no reason to prefer one distribution function over another [Oberkampf and Helton, 2002]. When uncertainties are express by means of intervals, based on experts' opinion or rare experimental data, as it is the case in space mission design, this representation becomes even more questionable.

Here we propose to use Evidence Theory instead of probability to address this issue. The Evidence Theory, developed by G. Shafer from A.P. Dempster's original work, has be proven to model adequately both types of uncertainty.

First, the Evidence Theory does not request additional assumptions when the available information is poor or incomplete. For instance, evidence on the event $\{A \text{ or } B\}$ does not imply/require information on both events $\{A\}$ and $\{B\}$. Similarly, the knowledge of an event does not imply knowledge of its opposite (for the probability theory $P(\overline{A})=1-P(A)$).

Secondly, this theory introduces two uncertainty quantification, the Belief (Bel) and Plausibility (Pl). Comparatively to probability, Bel and Pl can be seen as defining lower and upper probabilities, Pl including the uncertainty, Bel excluding it. This approach allows for the uncertainty quantification to conform to, and only to the available information.

Similarly, two complementary cumulative functions (CCF) are defined. These functions are at the Evidence Theory what a cumulative distribution function is at probability. In the case of f, function of a vector $\mathbf{x}$ of uncertain parameters, they express the belief (CCBF) and the plausibility (CCPF) that $f(\mathbf{x}) < \nu$, ν being within the set $\mathcal{Y}$ of the values of f.

$$CCBF = \Big\{[\nu, Bel\big(f(\mathbf{x}) < \nu\big)] : \nu \in \mathcal{Y}\Big\}$$
$$CCBF = \Big\{[\nu, Pl\big(f(\mathbf{x}) < \nu\big)] : \nu \in \mathcal{Y}\Big\}$$

To illustrate how Evidence Theory can be used in the preliminary phase of a space mission design, we can investigate two cases: a low thrust escape from Earth and a low thrust gravity assist (LTGA) transfer.

For the low thrust escape, two parameters have been considered as uncertain, the specific impulse and the required electrical power. The transfer time and the final mass have be simply computed. The figure 1 presents the CCBF and CCPF of the time required to escape Earth starting from a geostationary orbit.

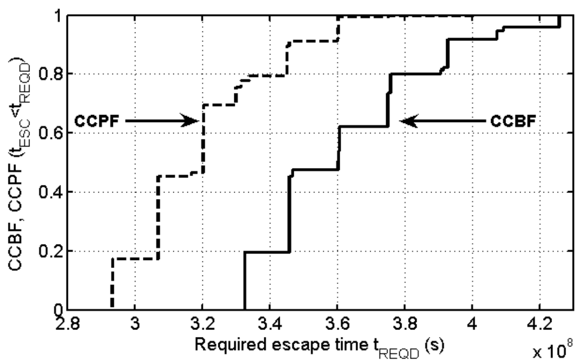

Figure 1: CCF for a low thrust Earth escape starting from GEO

In the second case example, a LTGA trajectory design, the uncertainties of 7 different parameters have been introduced in the problem formulation through Evidence Theory. Their impact were minimised along with the optimisation of both the mass of propellant and of the power system. The results on the thrust are illustrated in figure 2. For the robust solution (left) where the uncertainties have been taken into account, the needed thrust is always less than the available thrust in the worst case. However, this is not the true for the deterministic solution (right) where the needed thrust could not be supplied.

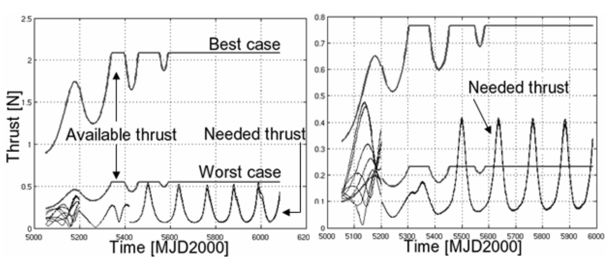

Figure 2: Available and needed thrust for two Earth-Venus-Venus-Mercury trajectories, a robust (left) and a deterministic one (right)

Evidence Theory is an interesting way to model uncertainty to increase the reliability of a space mission design. However, the application of this theory to complex engineering cases have faced significant problems, such as the choice of combination rule of information sources or the discontinuity of the CCF. Moreover, a critical issue with a high number of design parameters is the computational cost. The time required to compute each CCF is increasing exponentially with the number of parameters and the number of intervals specified for each parameter. In addition to that, the minimum and maximum values of the objective function should be calculated for each intervals.

A few solutions have previously been studied to address these issues. Solutions like those proposed by H. Bae et al. are insufficient to deal with the complexity of space mission design problems Indeed, to compute both CCF, the maximum and the minimum of the system function f has to be calculated for each different combination of parameter intervals. As an example, for a 10 parameters system, considering 10 intervals per parameter, 20 billion of vertices have to be evaluated. A solution is to use parallel computing, which could significantly improve performance. However, this mitigates but does not solve the problem's complexity.

An idea under investigation is to define a criterion on the combinations of parameter intervals to decide if it is necessary to compute the minimum and maximum of the system function. Indeed, if there is enough information to say that $f(\mathbf{x}) < \nu$ over a set of parameter intervals, then its corresponding belief (or plausibility) can be added in the CCF without calculating precisely the vertices.

In addition to this, connected intervals could be considered as a single one if, once again, the criterion assures that the constrain would be verified. This way, the number of intervals combination could drop dramatically, limiting even more the amount of required computations. By this approach we can incrementally add pieces of information to build up the belief and plausibility, though at every stage, Bel and Pl are only approximated.

Finally, to compute the vertices of expensive functions, we here propose to also use surrogate models. Kriging predictors are currently envisaged to model the system function f. As an illustration, figure 3 presents the computation time saved by using minimum finder algorithms based on Kriging predictor. This surrogate has the main advantage of providing an estimation of the error made on the predicted values, thus allowing for a guided search of the minimum.

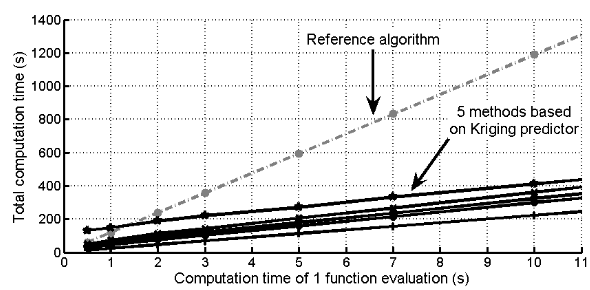

Figure 3: Time to find the minimum of Rosenbrock's function

References

[Oberkampf and Helton, 2002] W.L. Oberkampf and J. C. Helton. Investigation of evidence theory for engineering applications. In *4th Non-Deterministic Approaches Forum*, volume 1569. AIAA, April 2002.

[Sentz and Ferson, 2002] Kari Sentz and Scott Ferson. Combination of evidence in Dempster-Shafer theory. In *6th World Multi-conference on Systemics*. Cybernetics and Informatics, July 2002.