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ANALYTICAL CONTROL LAWS FOR INTERPLANETARY SOLAR SAIL TRAJECTORIES WITH CONSTRAINTS

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ABSTRACT

An indirect method is used to obtain an analytical control law for a spacecraft with a low-thrust propulsion system which is constituted by a solar sail coupled with a solar electric thruster. Constraints on the control inputs for such as the system need to be taken into account for the design of a control law to avoid reducing control performance, even though the solar electric thruster is employed as an auxiliary system capable of increasing the thrust magnitude of the sailcraft. The aim of this paper is to derive an analytical control law for a system with input constraints. A barrier function is used to analytically obtain a control law without a computationally expensive iterative algorithm. Therefore, using the analytic method presented, a transfer orbit can be readily calculated with an onboard computer. Pontryagin's maximum principle is also used to obtain an optimal control law to compare with the proposed control law. The proposed control law is demonstrated as suitable for an example transfer problem between circular and coplanar orbits.

NOMENCLATURE

m = sailcraft mass
a = thrust acceleration
r = San-sail distance
$\mathbf{v}$ = velocity vector of sailcraft
υ = radial velocity
θ = polar angle
ω = angular velocity of azimuth angle
T = thrust
P = electric thruster input power
c = speed of light
$\mathbf{u}$ = control variable vector
λ = adjoint variable
ν = Lagrange multiplier
A = sail area
α = sail pitch angle
β = lightness number
σ = sail loading
μ = gravity constant of Sun
L = solar luminosity

Subscript
0 = initial value
f = terminal value
p = propellant
r = radial direction
θ = circumferential direction
E = Electrical thruster
S = Solar sail
⊕ = Earth
♂ = Mars

Superscript
^ = unit vector
* = optimal value

1. INTRODUCTION

The concept of using solar radiation pressure as a means of propulsion is attractive despite the thrust magnitude being small. Indeed, solar ailing is increasingly being considered by ESA, NASA and JAXA for future science missions. A solar sail has a significant advantage in propulsive efficiency. In particular, its effective specific impulse is superior to electric propulsion for long duration missions [1]. There is an ability to exploit an increase in mission duration since a solar sail does not require propellant and is completely reusable, i.e. it can continue to produce thrust for an indefinite time. In spite of its advantage, very little effort has been expended to date on the development of the solar sail concept into a practical propulsion system.

Trajectory optimization for a solar sail has been a focus of development activities as well as other forms of low-thrust propulsion [2] – [9]. Shrivastava et al. [2] introduced a control law for a panel leading to maximum changes in various orbital elements. The control law was obtained by using parametric optimization. Kim et al. [3] obtained highly accurate solutions for the trajectory by applying computational schemes such as adaptive simulated annealing and a quasi-Newton method. As a smart method for global trajectory optimization, Dachwald [4] used artificial neural networks which can be combined with evolutionary algorithms. Otten et al. [5] and Mengali et al. [6] used a direct method to obtain near-optimal solar sail trajectories. An indirect optimization procedure was used to assess the performance of solar sails for interplanetary missions [7], [8]. However, using control laws which depend on a computational scheme or need recursive calculation to derive an optimal solution, an optimal trajectory cannot be obtained in real-time. It is preferred that an analytical control law [9] is available which can recalculate an optimal trajectory. Furthermore, such a control law provides significant operational benefits, particularly for future highly autonomous missions.

Propulsion by means of a solar sail does not require propellant. However, the thrust provided by a solar sail is limited, and large sails are required to keep the trip time reasonable. The idea of combining a solar electric propulsion (SEP) systems with a solar sail as a hybrid propulsion system has been discussed under simplified assumptions [8]. Input constraints due to mechanical and electrical power constraints exist when the hybrid propulsion system is treated in the sailcraft. If the constraints are not taken into account for analysis, they result in serious performance deterioration, i.e. it might not be possible to realize a desired optimal orbit transfer. The optimal trajectory for such as the constrained system has been derived by using mathematical programming which is not necessarily appropriate so as to update an optimal trajectory. The Pontryagin's maximum principle is used to obtain a control law by a solving two-point boundary value problem. However, such control laws are not suitable for the objective of this study because nonlinear equations need to be solved in an iterative algorithm on-board.

In this study, an analytical control law is derived for the problem by using the barrier function method. The method, whose advantage is to transform a constrained optimal problem into an unconstrained optimal problem, has been used to solve a constrained optimal problems. We apply this method for the constrained system considered here. A performance index which contains a barrier function is defined to derive an analytical control law by solving a two-point boundary value problem. Lagrange multipliers used as adjoint variables can be calculated by solving algebraic equations which are expressed by the boundary conditions. As a result, efficient computation is possible in real-time. Moreover, the sail pitch angle and the electric power are controlled in a feasible region defined by constraints. A control law which is derived by using the maximum principle is also described. The proposed method is demonstrated for an Earth-Mars trajectory optimization problem.

2. SYSTEM MODEL

2.1 Equations of motion

For the sake of simplicity, sailcraft motion in the ecliptic plane is assumed. Coplanar circular orbits are also assumed for the planets. The environmental forces acting on the sailcraft system are due to the gravitational field and the radiation pressure of the Sun. The heliocentric equations of motion for the sailcraft are as follows:

$$\dot{\mathbf{r}} = \mathbf{v} \tag{1a}$$

$$\dot{\mathbf{v}} = -\frac{\mu}{r^2}\hat{\mathbf{r}} + \beta\frac{\mu}{r^2}(\hat{\mathbf{r}}\cdot\hat{\mathbf{n}})^2\hat{\mathbf{n}} + \frac{T}{m}\hat{\mathbf{n}} \tag{1b}$$

$$\dot{m} = -\dot{m}_p \tag{1c}$$

where $\hat{\mathbf{r}}$ and $\hat{\mathbf{n}}$ denote the unit vector directed along the Sun-sail line and normal to the sail surface, respectively. SEP is used to assist the solar sail. Thrust gimballing of the SEP system is not considered to avoid complexity, so that the SEP thrust is along the sail normal.

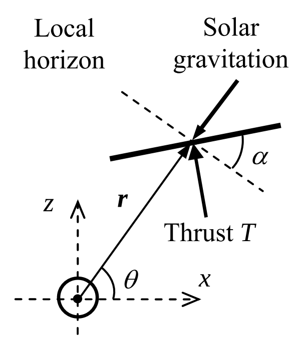

Fig.1 Schematic of parameters for spacecraft motion.

The complete system dynamics are quite complex. The sale pitch angle α, which is the angle between the sail and the local horizon in Fig.1, is an input constraint with admissible values for its range from $-\pi/2$ to $\pi/2$. As mentioned before, the angle of the thrust which is generated by the SEP system is fixed to avoid complexity, i.e. the thrust is generated on the normal line to the sail surface.

The SEP system is modeled through a polynomial approximation for the thrust and the propellant mass flow rate. The time derivative of propellant mass, $\dot{m}_p$, is assumed to be a function of the total electric thruster input power P of the SEP system. The relationships $\dot{m}_P = \dot{m}_P(P)$ and $T = T(P)$ are expressed using a second-order polynomial approximation [8].

$$\dot{m}_P = \sum_{i=0}^{2} a_i P^i \tag{2}$$

$$T = \sum_{i=0}^{2} b_i P^i \tag{3}$$

where a_i and b_i are constant coefficients which are identified through experimental data. These values are shown later.

2.2 Solar sail model

A solar sail consists of a large surface with reflective material such as a metallized plastic film and a supporting structure. Any coupling between the SEP system and the solar sail propulsion system is neglected.

The dimensionless sail loading parameter, β, which is used in Eq.(1b) will be defined as the ratio of the solar radiation pressure acceleration to the solar gravitational acceleration. This parameter is referred to as the lightness number of the solar sail. The lightness number can also be written as the following equation.

$$\beta = \frac{\sigma'}{\sigma} \tag{4}$$

$$\sigma' = \frac{L_s}{2\pi \mu c} \tag{5}$$

where $\sigma \equiv m/A$,which is a generalized sail loading, varies with time because the total sailcraft mass decreases according to Eq.(1c). The critical solar sail loading parameter σ' is a unique constant which is a function of the solar mass and the solar luminosity. This is found to be 1.539 g/m^2 [10].

3. CONTROL LAW WITH INPUT CONSTRAINTS

Control laws are designed for an inequality constrained problem (ICP). Firstly, a control method by using a barrier function is explained to derive an analytical control law. A control law using Pontryagin's maximum principle is also described to clarify the advantage of the proposed control method for the problem treated.

3.1 Barrier function

When solving an optimal control problem the performance index of a system with state variable vector $\mathbf{x}(t)$ is defined as

$$J = \int_{t_0}^{t_f} L(\mathbf{x}(t),\ u(t))dt \tag{6}$$

where t_f denotes the terminal time of a mission. Then the Hamiltonian H is defined using the function $L(\mathbf{x}(t), u(t))$ in this optimization problem as follows:

$$H = L(\mathbf{x}(t), u(t)) + \boldsymbol{\lambda}^T \dot{\mathbf{x}} \tag{7}$$

where $\boldsymbol{\lambda}$ is the adjoint variable vector. The state equation is expressed as $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u$. Euler-Lagrange equations based on the calculus of variations are derived as follows:

$$\dot{\boldsymbol{\lambda}}^T = -\frac{\partial H}{\partial \mathbf{x}} = -\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{A} \tag{8}$$

$$\frac{\partial H}{\partial u} = \frac{\partial L(\mathbf{x}(t),\ u(t))}{\partial u} + \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{b} = 0 \tag{9}$$

The variation in J must be zero for arbitrary variations in u, i.e., it is necessary to satisfy the optimality condition with respect to u in Eq. (9).

The function $L(\mathbf{x}(t), u(t))$ is usually defined in the barrier function method as

$$L(\mathbf{x}(t), u(t)) = f(\mathbf{x}(t), u(t)) + \rho B(\mathbf{x}(t), u(t)) \tag{10}$$

where ρ and $B(\mathbf{x}(t), u(t))$ denote the barrier parameter and barrier function, respectively. As previously mentioned, a feature of the method is that it transforms a constrained problem into a non-constrained problem. Furthermore, the method is extremely robust and it provides a considerable amount of structural information about the ICP. In particular, the method does not require the solution of the ICP to be a Karush-Kuhn-Tucker point. The barrier parameter, ρ, is calculated in the barrier function method so as to maximize the Hamiltonian. In this study, the parameter is treated as a constant so that recursive calculation is avoided as much as possible.

It is difficult to apply the proposed guidance law to the problem because of the nonlinearity of the vehicle's dynamics in Eq.(1). Thus, its equations of motion should be linearized by using feedback linearization. The new control inputs to the system which are expressed in the polar coordinate system are defined as follows:

$$\begin{aligned} u_r &= r\omega^2 - \frac{\mu}{r^2} + \beta\frac{\mu}{r^2}\cos^3\alpha + \frac{T}{m}\cos\alpha \\ &= r\omega^2 - \frac{\mu}{r^2} + (a_S + a_E)\cos\alpha \end{aligned} \tag{11}$$

$$\begin{aligned} u_\theta &= (-2\upsilon\omega + \beta\frac{\mu}{r^2}\cos^2\alpha\sin\alpha + \frac{T}{m}\sin\alpha)/r \\ &= \frac{-2\upsilon\omega + (a_S + a_E)\sin\alpha}{r} \end{aligned} \tag{12}$$

The accelerations generated by a solar sail and a solar electric propulsion system are denoted as a_S and a_E, respectively.

The equations of motion are expressed as

$$\frac{d}{dt}\begin{bmatrix} r \\ \upsilon \\ \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \upsilon \\ \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_r \\ u_\theta \end{bmatrix} \tag{13}$$

It is obvious from Eq. (13) that there is no interference between the motion in the radial and angular directions; thus, Eq. (1) can be transformed to a state equation.

To consider input constraints in the analysis, it is assumed that the sail distance does not exceed the radius of the Mars' orbit and the circumferential velocity of the sailcraft is less than that of Mars. The sail pitch angle and power of the SEP P are subject to $-\pi/2 \le \alpha \le \pi/2$ and $0 \le P \le P_{\max}$, respectively. Therefore, the control inputs u_r and u_θ are subject to the following inequalities.

$$|u_r| < u_{r\max}, \quad |u_\theta| < u_{\theta\max} \tag{14}$$

The proposed idea can be readily applied to this problem because Eq. (13) is expressed in a Brunovsky canonical form.

The performance index is defined and used together with Eq. (6) to generate barriers corresponding to the input constraints in Eq. (14). The barrier function is defined as $B = \ln[\sec(u(t))]$ [11].

The boundary conditions for the transfer orbit problem are set as follows:

$$r(t_0) = r_{\oplus},\ \upsilon(t_0) = 0,\ \theta(t_0) = 0,\ \omega(t_0) = (\mu / r_{\oplus})^{1/2} \tag{15a}$$

$$r(t_f) = r_{♂},\ \upsilon(t_f) = 0,\ \theta(t_f) = \theta_f,\ \omega(t_f) = (\mu / r_{♂})^{1/2} \tag{15b}$$

The initial and terminal conditions of the other parameters are arbitrarily set in the numerical simulation. The control laws for radial and circumferential direction are obtained from Eq. (9) and the transversality condition.

$$u_r^* = -k_r \tan^{-1}\lambda_\upsilon = -\frac{2u_{r\max}}{\pi}\tan^{-1}\lambda_\upsilon \tag{16a}$$

$$u_\theta^* = -k_\theta \tan^{-1}\lambda_\omega = -\frac{2u_{\theta\max}}{\pi}\tan^{-1}\lambda_\omega \tag{16b}$$

When a two-point boundary-value problem is solved, the relationships between the adjoint variables **λ** and the Lagrange multipliers **ν** are given by the following equations.

$$\lambda_r = \nu_r,\ \lambda_\upsilon = \nu_r(t_f - t) + \nu_\upsilon \tag{17a}$$

$$\lambda_\theta = \nu_\theta,\ \lambda_\omega = \nu_\theta(t_f - t) + \nu_\omega \tag{17b}$$

It is difficult to provide the control force in Eqs.(16a) and (16b) separately along the radial and angular directions in an actual system. Therefore, it is necessary to derive control inputs, the sail pitch angle and the electric thruster input power, which realize the control laws. Using Eqs.(11), (12), (16a) and (16b), an optimal sail pitch angle is obtained.

$$\alpha^* = \tan^{-1}\left(\frac{u_\theta^{\ *} r + 2\upsilon\omega}{u_r^* - r\omega^2 + \mu / r^2}\right) \tag{18}$$

The electric thrust input power is similarly derived from Eqs.(2), (3), (16a), and (16b). As a result, the calculation for these control inputs does not require solving any differential equations at every sampling time. Thus, the control method is capable of providing the control inputs in real-time despite the proposed control algorithm developed being explicitly dependent of time.

The equations of motion of the sailcraft treated in this paper consist of two systems in Brunovsky canonical systems. The analytical solutions in terms of state variables in Eq.(13) can now be obtained. For instance, the derivation of the control input in the radial direction is described. The state variables of this system, r and υ, can be derived analytically from Eqs.(16a) and (17a) as follows:

$$r = \frac{k_r}{2\nu_r^2}\Big[-\lambda_\upsilon^{\ 2}\tan^{-1}(\lambda_\upsilon) + \lambda_\upsilon \ln(1+\lambda_\upsilon^{\ 2}) + \tan^{-1}\lambda_\upsilon + (\lambda_\upsilon + 2\nu_r t)\Big] + C_\upsilon t + C_r \tag{19a}$$

$$\upsilon = \frac{k_r}{\nu_r}\Big[\lambda_\upsilon \tan^{-1}(\lambda_\upsilon) - \frac{1}{2}\ln\{1+(\lambda_\upsilon)^2\}\Big] + C_\upsilon \tag{19b}$$

where, C_r and C_υ in Eqs.(18) and (19) are constants of integration and are calculated by equations using terminal values of state variables and Lagrange multipliers.

$$\begin{aligned} C_r = \frac{k_r}{2\nu_r^2}\Big[&((\nu_r t_f + \nu_r)^2 - 1)\tan^{-1}(\nu_r t_f + \nu_\upsilon) \\ &- (\nu_r t_f + \nu_\upsilon)\ln(1+(\nu_r t_f + \nu_\upsilon)^2) \\ &- (\nu_r t_f + \nu_\upsilon)\Big] + r_0 \end{aligned} \tag{20a}$$

$$\begin{aligned} C_\upsilon = \frac{k_r}{\nu_r}\Big[&-(\nu_r t_f + \nu_\upsilon)\tan^{-1}(\nu_r t_f + \nu_\upsilon) \\ &- \frac{1}{2}(\nu_r t_f + \nu_\upsilon)\ln\big(1+(\nu_r t_f + \nu_\upsilon)^2\big)\Big] + \upsilon_0 \end{aligned} \tag{20b}$$

Then, the control input u_r of Eq. (16a) can be rewritten as a variable feedback controller using Eqs.(19a) and (19b):

$$u_r = -K_r\, r - K_\upsilon\, \upsilon \ + K \tag{22}$$

where

$$K_r = \frac{2\nu_r^2}{\lambda_\upsilon} \tag{23a}$$

$$K_\upsilon = \nu_r \tag{23b}$$

$$K = \frac{\nu_r}{\lambda_\upsilon}\left[2\nu_r C_r + \left(C_\upsilon + \frac{k_r}{\nu_r}\right)(\lambda_\upsilon + 2\nu_r t)\right] \tag{23c}$$

where K_r and K_υ in Eq. (20) are like feedback gains.

3.2 Maximum principle

Pontryagin's maximum principle has been used to obtain a control law for a system using a solar sail. An optimal control law for the problem both of the time optimal and the minimum fuel consumption control is derived to maximize the performance index J.

$$J = (1-\eta)\frac{m_f}{m_0} - \eta\, t_f \tag{24}$$

$$H = \lambda_r \upsilon + \lambda_\theta \omega + \lambda_\upsilon u_r + \lambda_\omega u_\theta - \lambda_m \dot{m}_p \tag{25}$$

where η is a tradeoff parameter. $\eta = 1$ and $\eta = 0$ correspond to the minimum-time trajectory and to the minimum-fuel consumption trajectory. Note that the Hamiltonian includes the mass variation. The Euler-Lagrange equations of this system are derived from Eq.(8).

$$\dot{\lambda}_r = -\lambda_\upsilon\left(\frac{\partial a_S}{\partial r}\cos\alpha + \frac{2\mu}{r^3} + \omega^2\right) - \lambda_\omega\left(\frac{\partial a_S}{\partial r}\sin\alpha + \frac{2\upsilon\omega}{r^2}\right) \tag{26a}$$

$$\dot{\lambda}_\theta = 0 \tag{26b}$$

$$\dot{\lambda}_\upsilon = -\lambda_r + 2\lambda_\omega \frac{\omega}{r} \tag{26c}$$

$$\dot{\lambda}_\omega = -\lambda_\theta - 2\lambda_\upsilon r\omega + 2\lambda_\omega \frac{\upsilon}{r} \tag{26d}$$

$$\dot{\lambda}_m = -\frac{1}{mr}\left[\lambda_\upsilon r(a_S + a_E)\cos\alpha + \lambda_\omega (a_S + a_E)\sin\alpha\right] \tag{26e}$$

A control vector $\mathbf{u} = [P \quad \alpha]^T$ maximizes the performance index through an indirect approach. When the optimal control vector $\mathbf{u}^*$ is selected in the domain of the feasible reagion Ω_u, the Hamiltonian H' which coincides with that portion of H of Eq.(25) is an absolute maximum.

$$\mathbf{u}^* = \arg\max_{\mathbf{u}\in\Omega_u} H' \tag{27}$$

$$H' = \lambda_\upsilon (a_S + a_E)\cos\alpha + \lambda_\omega \frac{(a_S + a_E)\sin\alpha}{r} - \dot{\lambda}_m \dot{m}_p \tag{28}$$

That is

$$P = \arg\max_{P\in\Omega_u} \sum_{i=0}^{2} k_i P^i \tag{29a}$$

$$k_i = \frac{r\lambda_\upsilon \cos\alpha + \lambda_\omega \sin\alpha}{mr} b_i - \lambda_m a_i \tag{29b}$$

From Eqs.(30a) and (30b) , the optimal electric thruster input power is obtained as follows:

$$P^* = \begin{cases} 0 & \text{if } -\dfrac{k_1}{2k_2} < P_{\min} \\ -\dfrac{k_1}{2k_2} & \text{if } -\dfrac{k_1}{2k_2} \in \Omega_P \\ P_{\max} & \text{if } -\dfrac{k_1}{2k_2} > P_{\max} \end{cases} \tag{30}$$

The optimal sail pitch angle can be similarly obtained.

$$\alpha^* = \arg\max_{\alpha\in\Omega_\alpha} \frac{1}{r}\left[\lambda_\upsilon r(a_S + a_E)\cos\alpha \quad \lambda_\omega (a_S + a_E)\sin\alpha\right] \tag{31}$$

The results are given by the following nonlinear equation.

$$\left(\lambda_\upsilon + \frac{\lambda_\omega}{r}\tan\alpha\right)\frac{\partial a_S}{\partial \alpha} + (a_S + a_E)\left(-\lambda_\upsilon \tan\alpha + \frac{\lambda_\omega}{r}\right) = 0 \tag{32}$$

The control inputs of the sailcraft can be derived from Eqs.(30) and (31). However, it is difficult to obtain the control inputs in real-time since the differential equations relating to the adjoint variables in Eqs.(26a) to (26e) and the coefficient k_i in Eq.(29) must be solved at every sampling time.

4. NUMERICAL RESULTS

A sailcraft whose thrust is generated by a solar sail and SEP system is controlled to achieve a rendezvous mission toward Mars. The sailcraft has a launch mass m_0 = 400 kg. The initial value of the lightness number is 0.1686. The terminal distance r_f is equal to $r_♂$ = 1.5237 AU. The coefficients of the SEP system a_i and b_i in Eqs.(2) and (3) are referred to values for a plasma thruster (see Table 1). The power of the thruster P is assumed to be less than P_{max} = 1.5 kW. These numerical conditions were selected to match with reference [8] as much as possible.

Table 1 Coefficients of the SEP system.

coefficient	Value
a_1 [mg]	1.953
a_2 [mg/kW]	2.545
a_3 [mg/kW^2]	-0.3716
b_1 [mN]	4.68
b_2 [mN/kW]	60.94
b_3 [mN/kW^2]	-5.1

A transfer trajectory with mission time t_f = 264.7 days under the proposed control law is shown in Fig.2. Figures 3 to 6 show time histories of the Sun-sail distance, radial velocity, polar angle, and angular velocity of the sailcraft, respectively. From these figures, it is apparent that the final values with respect to the state variables satisfy the terminal conditions.

The sail pitch angle varies within its constraint as shown in Fig.7. The sailcraft initially exploits SEP more than the solar radiation pressure shown in Fig.8. It can be seen that the SEP acceleration does not exceed its maximum value.

Figure 9 shows the time history of the sailcraft mass. The SEP consumed about 40 kg propellant. The ratio of the used propellant mass to initial sailcraft mass was 0.099. The ratio can not be simply compared with other results in Ref. [8] because Mengali et al. considered the thrust gimbaling as a control variable. However, it is noted that the necessary propellant mass for the mission is less than the result [8] in this numerical simulation even though there is no thrust gimbaling.

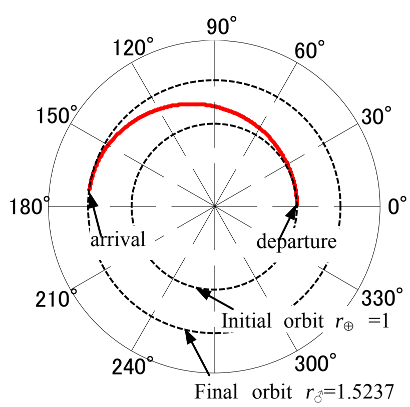

Fig.2 Earth-Mars trajectory with proposed control

5. CONCLUSIONS

The design of a control law based on the barrier function method was introduced for a solar sailcraft with an input constraint on the pitch angle. An SEP system which has a constraint condition in regard to its input power was used to assist the thrust generated by the solar sail. The sail pitch angle and the electric thrust input power were treated as control variables in this paper. A control law was derived analytically for the system with input constraints by applying the proposed control method.

A mission toward Mars was considered to assess the validity of the proposed control law. Numerical results show the possibility to control by an onboard computer because recursive calculation is not required to obtain the control law. Analytical solutions of state variables are also derived under the proposed control law. Therefore, a tracking control around a reference trajectory which is calculated by the analytical solutions can be applied to reduce the effect of disturbances on the sailcraft.

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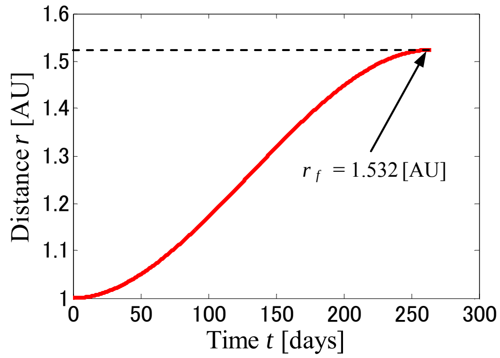

Fig.3 Time history of sail distance.

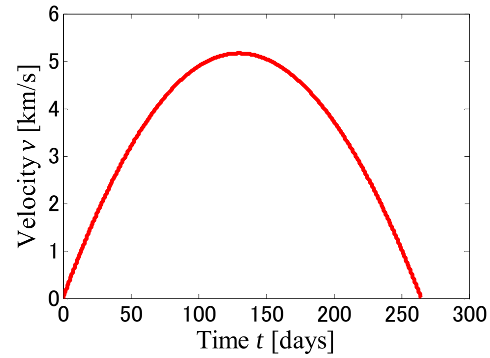

Fig.4 Time history of radial velocity.

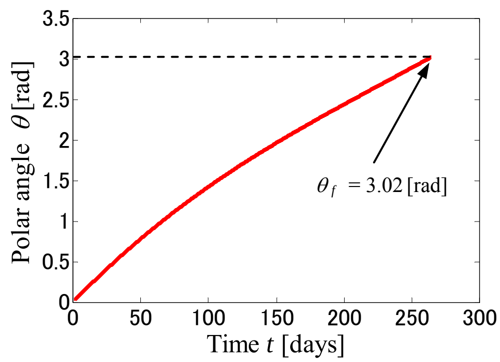

Fig.5 Time history of polar angle.

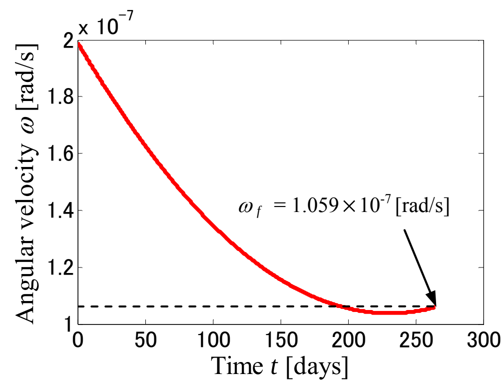

Fig.6 Time history of angular velocity.

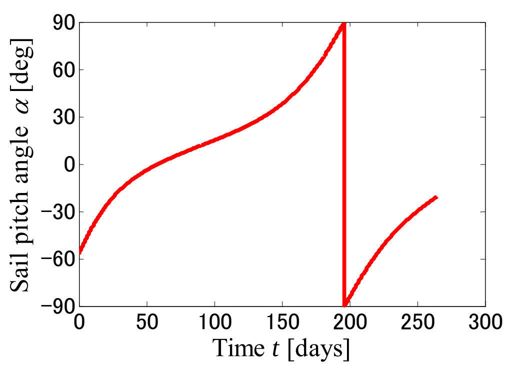

Fig.7 Time history of sail pitch angle.

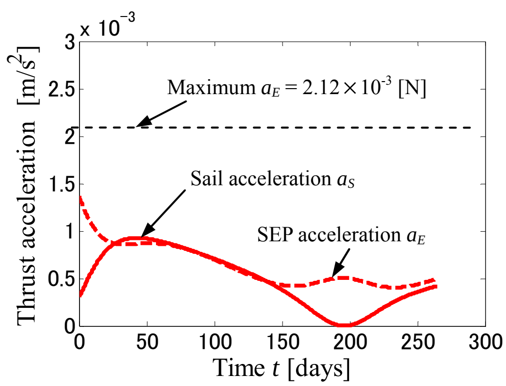

Fig.8 Time history of thrust acceleration of SEP.

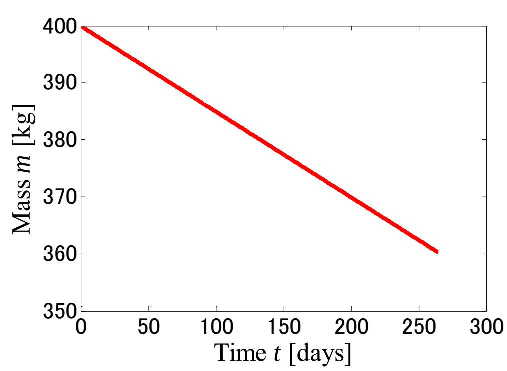

Fig.9 Time history of mass of sailcraft.