The Application of Reliability Methods in the Design of Stiffened FRP Composite Panels for Marine Vessels

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Abstract:
The use of composite laminate materials has increased rapidly in recent years due to their excellent strength to weight ratio and resistance to corrosion. In the construction of marine vessels, stiffened plates are the most commonly used structural elements, forming the deck, bottom hull, side shells and bulkheads. This paper presents the use of a stochastic approach to the design of stiffened marine composite panels as part of a current research programme into developing stochastic methods for composite ship structures, accounting for variations in material properties, geometric indices and processing techniques, from the component level to the full system level. An analytical model for the solution of a stiffened isotropic plate using a grillage analogy is extended by the use of equivalent elastic properties for composite modelling. This methodology is applied in a reliability analysis of an isotropic (steel) stiffened plate before the final application for a reliability analysis for a FRP composite stiffened plate.

Keywords: composites, FRP, reliability, SORM, tophat stiffeners, grillage

Introduction

There is increasing interest in the use of lightweight, polymer composite structures for a variety of applications in underwater structures. These applications are in the form of single skin stiffened structures as well as monocoque single skin and sandwich configurations. The structures could potentially be made up using different fibre types, fibre architectures and weaves, resins, core materials; there could be further variations owing to volume fractions and geometric/topological layouts. Also, there is further choice in processing routes as well. One could consider the use of low temperature cure prepregs or alternatively consider vacuum assisted resin infusion moulding. These processes too have many in-built variabilities. Current trends in marine (ship/boat) design use conservative safety indices based mainly on some limiting strain value.

This approach however does have drawbacks. The limiting strain value (usually an in-plane strain) may not pick up the load transfer mechanism adequately and hence may not model dominant failure modes adequately. Currently little or no allowance is made for variabilities in design parameters, processing parameters and topological indices.

One solution to this problem is to integrate well established reliability techniques with composite structure design. There are various established techniques to carry out reliability analysis such as first or second order reliability methods (FORM or SORM) or simulation (e.g. Monte Carlo) and depending on the type of problem, one or the other methods can be applied. One of the difficult problems in composites will be to define the failure surface for various limit states and also the uncertainties of different design variables involved in the definition of the limit states. Surmounting these issues will reduce the level of uncertainty in adopting composites as a construction material and widen the engineer's choice of design solutions.

There is a significant use of stiffened plating or panels in ship structures. The accurate solution for the mechanical response of these stiffened panels subject to loading is not trivial. However, to avoid conservativeness in design and easing the introduction of new construction materials, the use of probabilistic methods requires a structural model with high levels of confidence. The challenge is to identify an analytical or numerical technique that can meet the compromise between accuracy and speed required in reliability analyses.

In a ship structure, beams and girders are the stiffening members for the plating: girders and beams are usually placed longitudinally and transversely respectively, forming a mesh which intersects orthogonally (Figure 1). The network of these girders and beams is called a "grillage", defined by Clarkson (1965), whereby the plating between the stiffeners is considered as an effective flange (stiffener base plate) between the girders and beams. In this way, the analysis is reduced to that of an unplated grillage where the mechanical response can be obtained using Euler-Bernoulli beam theory, either through the use of a Displacement Method (DM) or a Force Method (FM) (Wunderlich & Pilkey 2003) or approximate methods such as the Orthotropic Plate (OPM) or the Energy method (EM) (Vedeler 1945). Folded Plate methods (FPM) can also be used to solve the mechanical response a stiffened panel or the use of a numerical Finite Element Analysis (FEA).

The displacement method (Clarkson 1965) is the most common method used for grillage analysis and it relies on the analysis of the straight segments of the girders and beams between the intersection points and defines the deflection and slopes at the intersection points. Research applications have considered both static and dynamic problems (Balendra & Shanmugam 1985;Cheung, Bakht, & Jaeger 1982;Evan H.R. et al. 1983;Tan & Montage 1991).

For the force method, at every intersection point of the grillage the condition of equilibrium is satisfied for applied load. The deflection is calculated by using beam theory where reaction force is determined. Lazarides (1952) introduced the calculation procedure of the FM to a square grillage by ignoring the torsion of beams. This calculation procedure used by Clarkson (1963) provided solutions which agreed well with experimental data.

Since the number of equations increases when the number of intersections increases, finding the mechanical solutions of a grillage having a large number of beams requires solution by computer when either the FM or DM is being utilised. Smith (1964) developed a computer program to analyse grillages with up to 100 intersections where two axes of symmetry were present. More recently, Jang *et al*. (1996) employed the FM by ignoring torsional rigidity of girders and beams within their optimisation of a surface effect ship built from aluminium. The complex structures due to longitudinal girder and transverse web frames were represented as a number of grillages.

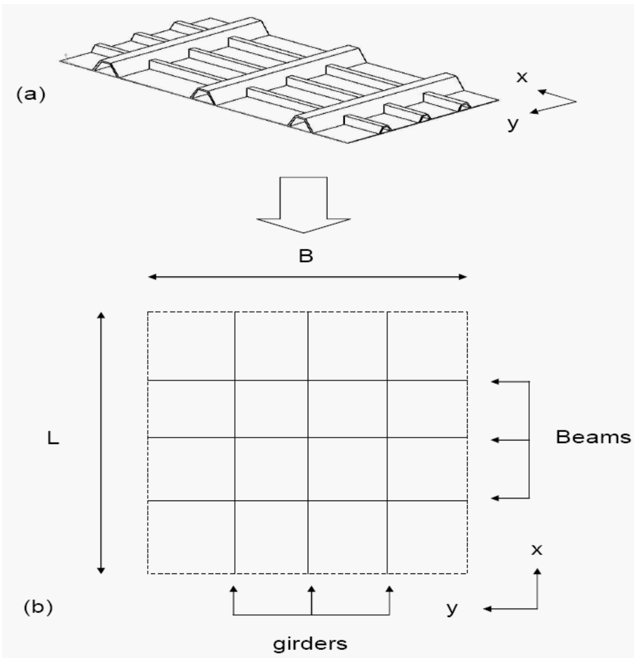


Figure 1.(a)Tophat cross stiffened plate (b)unplated grillage representation for the stiffened plate (Maneepan 2007)

In the context of a probabilistic analysis of stiffened panels, a large number of simulations may be required where each simulation represents an alternative combination of stiffened panel variables. To avoid solving a large number of equations therefore, approximate methods for the mechanical response of a stiffened panel is advantageous.

A number of researchers have utilised one approximating analytical technique, the orthotropic plate method, for a number of applications (Hosseini-Toudeshky, Ovesy, & Kharazi 2005;Krisek, Evan, & Ahmad 1990;Mikami & Niwa 1996;Mikami & Yonezawa 1983). OPM converts the stiffened plate into an equivalent plate with constant thickness by smearing out the stiffeners but is limited in accuracy by the spacing and number of stiffeners considered (Bedair 1997).

An alternative approximating technique was developed by Vedeler (1945) in the 1940's who simplified the solution to the grillage problem by using Navier's energy method (EM) in which the deflection of the grillage is determined by equating the total strain energy of all the beams to work done by the normal load so that only one equation needed to be solved for deflection at every intersection.

Other alternative techniques for the analysis of stiffened panels can be found in the folded plate method (FPM) and finite element analysis (FEA), which is likely to be the most effective means of getting accurate results. Both methods are based on discrete models of an array of beams and plate elements. The continuity conditions are defined along the interconnecting boundary between the plates and beams. The accuracy of the FPM is limited to structures consisting of flat rectangular panels simply supported at one pair of opposite sides and stiffened in one direction only (for orthogonally stiffened plates, the transverse stiffeners are smeared out by adding the stiffness properties into the plate element). The practical application of FPM can be seen in the Canadian bridge design code (CHBDC 2000) which restricts the use of this method to bridges with support conditions closely equivalent to line supports at both ends of the bridge.

The time consumed in the solution for both the FPM and FEA is too high to allow the practical use of these approaches in probabilistic analysis due to the number of equilibrium equations that require solution.

Subsequently the analysis of a stiffened plate will be performed based on the grillage model assumption over the OPM or FPM. The energy method (EM) is considered in this analysis for the grillage solution. This can be rapidly employed to evaluate the reliability without building the FE models.

For the application of anisotropic FRP composite materials, Smith (1990) showed that the grillage analysis for stiffened panels made from isotropic materials could be utilised by consideration of composite beam theory under the assumption that a plane section on the panel was to remain plane when subjected to bending moments and Poisson's ratio effect was considered negligible.

In summary, the probabilistic analysis of a composite stiffened panel will be undertaken on the basis of a structural model that reduces the problem to that of an energy method solution of an analogous grillage.

Grillage analysis for a Composite Stiffened Plate

The analysis of a grillage based on Navier's Energy Method found in Vedeler (1945), originally developed for a structure built of isotropic material, is adapted for composite plated grillages by substituting equivalent elastic properties of a symmetric laminate into the grillage analysis. Consider the grillage (see Figure 1) consisting of b equally spaced beams in the length (L) direction and g equally spaced girders in the width (B) direction.

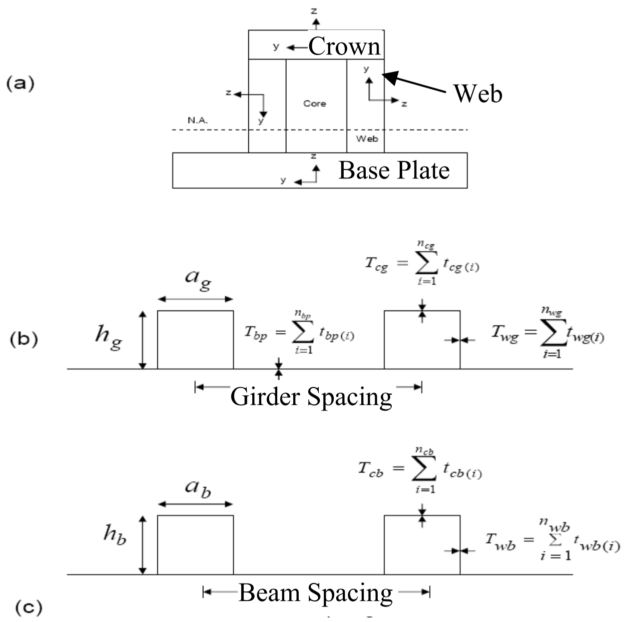


Figure 2(a)Tophat cross-section of girders & beams, describing i elements, with local coordinate system for fibre layup (b)Geometric parameters of girders and (c)Geometric parameters of beams (Maneepan 2007)

To represent the tophat cross stiffened plates, the girders and beams of the grillage have a tophat shape including the base plate, or effective flange, (see Figure 2). Since the structures are made of laminated composite, the tophat, or box, cross section could be comprised of many elements, for example the base plate, vertical webs and the horizontal top crown, having different elastic properties (the core is neglected as it is usually non-structural).

To avoid the section coupling problem between membrane and bending action, the geometry of the cross section must be symmetric. Each laminated element is assumed to be symmetric about its own plane and specially orthotropic in the membrane mode to eliminate the effect of the coupling terms. From Datoo (1991), the membrane equivalent Young's modulus value of a laminate in the axial direction of the i^{th} element (E_i) can be found by,

$$E_i = \frac{\left(A_{11}A_{22} - A_{12}^2\right)}{A_{22}t} \qquad (1)$$

The extension stiffness [A] of the element is expressed as:

$$A_{ij} = \sum_{k=1}^{N} t_k \left(\bar{Q}_{ij}\right)_k \qquad (2)$$

For ij = 11, 12 and 22, the expression of $\bar{Q}_{ij}$, the transformed reduced stiffness of the kth layer, are as follows:

$$\begin{aligned} \bar{Q}_{11} &= c^4 Q_{11} + s^4 Q_{22} + 2c^2 s^2 Q_{12} + 4c^4 s^2 Q_{66} \\ \bar{Q}_{12} &= c^2 s^2 Q_{11} + c^2 s^2 Q_{22} + \left(c^4 + s^4\right) Q_{12} - 4c^2 s^2 Q_{66} \\ \bar{Q}_{22} &= s^4 Q_{11} + c^4 Q_{22} + 2c^2 s^2 Q_{12} + 4c^4 s^2 Q_{66} \end{aligned} \qquad (3)$$

c and *s* are abbreviations for $\cos\theta$ and $\sin\theta$ and θ is the fibre angle in each ply. The reduced stiffness terms Q_{ij} where ij = 1, 2 and 6 are expressed as:

$$\begin{aligned} Q_{11} &= \frac{E_1}{\left(1-\nu_{12}\nu_{21}\right)}, \quad Q_{22} = \frac{E_2}{\left(1-\nu_{12}\nu_{21}\right)} \\ Q_{12} &= \frac{\nu_{21}E_1}{\left(1-\nu_{12}\nu_{21}\right)}, \quad Q_{66} = G_{12} \end{aligned} \qquad (4)$$

If the girders and beams consist of N_g and N_b elements respectively, the flexural rigidity of the girder (D_g) and beam (D_b) can be written as:

$$D_g = \sum_{i=1}^{N_g} E_{g(i)} I_{g(i)}, \quad D_b = \sum_{i=1}^{N_b} E_{b(i)} I_{b(i)} \qquad (5)$$

$E_{g(i)}$ and $E_{b(i)}$ are membrane equivalent Young's moduli in the axial direction of ith element of the girders and beams respectively. $I_{g(i)}$ and $I_{b(i)}$ are the second moment of area of the ith element relative to the Neutral Axis (NA) of the girder's and beam's cross sections respectively. The general form of $I_{g(i)}$ and $I_{b(i)}$ can be presented by $I_{(i)}$ as follows using the standard parallel axis theorem, where $a_{(i)}$ is the area of the ith element and $d_{NA(i)}$ is the distance from the NA of the ith element:

$$I_{(i)} = I_{cx(i)} + a_{(i)} \left(d_{NA(i)}\right)^2 \qquad (6)$$

The deflection, *w(x,y)*, at any point of the grillage is expressed by the following double summation of trigonometric series according to Navier's energy method (Bedair 1997):

$$w\left(x,y\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{B} \qquad (7)$$

m and n are wave numbers and a_{mn} are coefficients which can be determined by the condition that the change in potential energy due to the assumed deflections is a minimum. The potential energy, or strain energy, V, in a deflected grillage can be written as:

$$V = V_g + V_b - W \qquad (8)$$

V_g and V_b are the strain energies in the girders and beams respectively and W is the work done by an external load, P. For minimum potential energy,

$$\frac{\partial V}{\partial a_{mn}} = \frac{\partial V_g}{\partial a_{mn}} + \frac{\partial V_b}{\partial a_{mn}} - \frac{\partial W}{\partial a_{mn}} = 0 \qquad (9)$$

The deflection curve of the qth beam is obtained by giving x the constant value,

$$x_q = \frac{qL}{(b+1)}, \qquad (10)$$

such that,

$$w(y)_{x=x_q} = \sum_{n=1}^{\infty} b_{qn} \sin\frac{n\pi y}{B} \quad \text{where } b_{qn} = \sum_{m=1}^{\infty} a_{mn} \sin\frac{m\pi q}{(b+1)} \qquad (11)$$

Similarly, the deflection curve of the pth girder is obtained by giving y the constant value,

$$y_p = \frac{pB}{(g+1)}, \qquad (12)$$

such that,

$$w(x)_{y=y_p} = \sum_{m=1}^{\infty} c_{pn} \sin\frac{m\pi x}{L} \quad \text{where } c_{pn} = \sum_{n=1}^{\infty} a_{mn} \sin\frac{n\pi p}{(g+1)} \qquad (13)$$

The strain energy for all the girders and beams can now be represented as:

$$V_g + V_b = \int_0^L \frac{D_g}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2_{y=y_p} dx \; + \; \int_0^B \frac{D_b}{2}\left(\frac{\partial^2 w}{\partial y^2}\right)^2_{x=x_q} dy \qquad (14)$$

Meanwhile, the work done, W, by the application of a uniform pressure load, P, is:

$$W = \int_0^L\int_0^B P \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{B}\, dy dx \qquad (15)$$

Therefore,

$$\frac{\partial W}{\partial a_{mn}} = \int_0^L \int_0^B P \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{B} dydx \qquad (16)$$

For the minimum potential energy, equating $\left(\frac{\partial V_g}{\partial a_{mn}} + \frac{\partial V_b}{\partial a_{mn}}\right)$ to Equation (16), we obtain,

$$\frac{\pi^4 D_b}{2B^3}\sum_{q=1}^{b} n^4 b_{qn} \sin\frac{m\pi q}{(b+1)} + \frac{\pi^4 D_g}{2L^3}\sum_{p=1}^{g} m^4 c_{pn} \sin\frac{n\pi p}{(g+1)} = \frac{4PLB}{\pi^2 mn} \qquad (17)$$

where *m* and *n* are odd numbers (integration of even numbered sine functions equals zero). Now the coefficient a_{mn} can be obtained as,

$$a_{mn} = \frac{16PLB}{\pi^6 mn\left\{m^4 (g+1)\frac{D_g}{L^3} + n^4 (b+1)\frac{D_b}{B^3}\right\}} \qquad (18)$$

Hence, the complete expression for the deflection of the tophat stiffened plate can be found by substituting Equation (18) into a double sine series in Equation (7). The bending moment and shear force, respectively, of the pth girder can be obtained by,

$$M_g = -D_g \frac{\partial^2 w}{\partial x^2}, \ Q_g = \frac{\partial M_g}{\partial x} \qquad (19)$$

The direct stress in the axial direction and shear stress at each element on the girder cross section are given by the following expressions,

$$\sigma_g = \frac{E_{g(i)} M_g Z_g}{D_g}, \ \tau_g = -\frac{E_{g(i)} Q_g}{D_g}\int_0^s Z_g \, ds \ (20)$$

where Z_g is the distance from the neutral axis of the girder to the ith element and *s* is the distance around the cross-section from the middle of the crown element to a point where the shear value is of interest. Similar to Equations (19) and (20), the direct stress (σ_b) and shear stress (τ_b) of the beam can be obtained by relevant substitutions.

Validation

The grillage analysis procedure for the response of a composite stiffened panel to uniform pressure loading with simple supports has been detailed in the previous section "Grillage analysis for a Composite Stiffened Plate". Using this same procedure but with the simplification of isotropic material properties for which known solutions exist a validation (Clarkson 1965;Maneepan 2007;Nayak et al. 2006) can be made, likewise a comparison with results obtained from finite element analysis (Maneepan 2007). Extending the EM approach for FRP composites by the use of equivalent elastic properties (equations (1-6)) is trivial.

From four examples presented in Clarkson (1965), the grillages chosen for validatory purposes represent a rectangular and a square panel stiffened by either I-sections or box sections in the following arrangement:

- four equal & evenly spaced girders & five equal and evenly spaced transverse beams for rectangular panel (4x5)
- four equal & evenly spaced girders & transverse beams for square panel (4x4)

The dimensions of the stiffeners for both panels are given in Table 1 and the panel dimensions in Table 2.

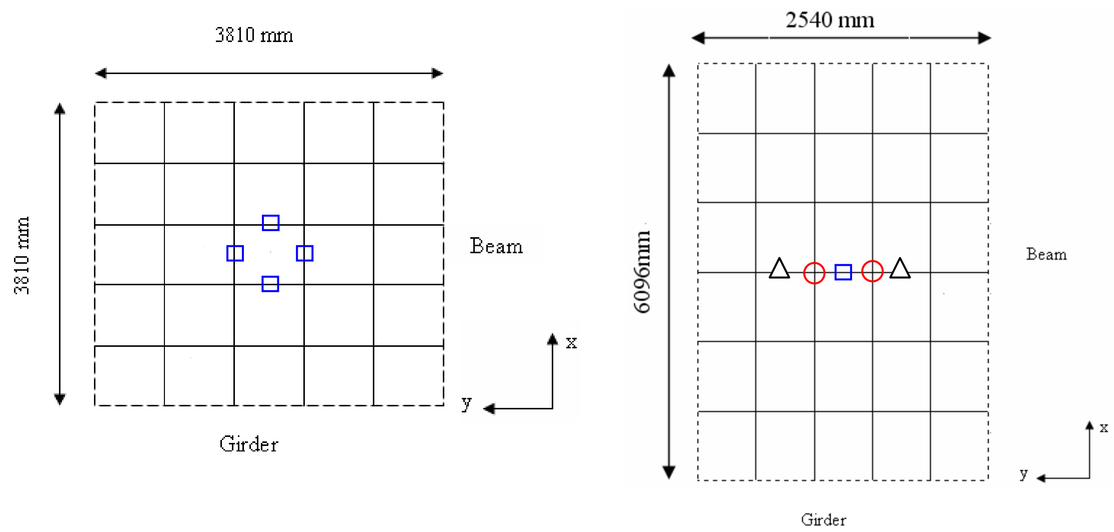
Table 1 Girder & beam dimensions

Dimension	4x5		4x4
	I or Box		I or Box
	Girder	Beam	Girder or Beam
Height (mm)	254	69.85	254
Width (mm)	127	44.45	127
Crown thickness (mm)	18.288	9.525	18.288
Web thicknesses (mm)	9.144	5.08	9.144
Flange thickness (mm)	18.288	9.525	18.288

The 4x4 panel is subjected to 137.895kPa and the 4x5 panel to 34.474kPa (see Table 2). The apparent accuracy to which the design parameters are specified is due to the nature of converting from Imperial units of measurement to metric.

Table 3 compares the results of DM (Clarkson 1965), EM (Vedeler 1945) and FEA (Maneepan 2007) for maximum deflection and stress. For the grillage constructed using I-beams, the prescribed EM results have good agreement with results obtained from FEA (using 4-node, 6 degree of freedom per node, SHELL63 elements). For the box stiffened plate, the maximum stress in the longitudinal and transverse beams differ by up to 20.3%. The assumption that the effects of shear deflections and torsional rigidity of the beams are sufficiently small to be neglected is not realistic as in this case the longitudinal and transverse members have widely differing stiffness.

The extension to an anisotropic composite material requires the definition of the equivalent Young's modulus in the rigidity definitions, equations (5). Comparing the membrane equivalent Young's modulus with that determined by Datoo (1991) shows that the present work is in exact agreement (Maneepan 2007).



□ Grillage displacement and stress calculation position

□ Grillage displacement calculation position
○ Grillage stress calculation position for girder
△ Grillage stress calculation position for beam

(a) 4x4 grillage (b) 4x5 grillage

Figure 1 Grillage geometry and positions of displacement and stress calculations

Table 2. Statistics of random variables for steel grillages (* value used for stress limit state)

Random Variable	COV (%)	Distribution	Mean Value			
			4x4		4x5	
			I	Box	I	Box
L (length) (mm)	3	Normal	3810	3810	6096	6096
B (breadth) (mm)	3	Normal	3810	3810	2540	2540
I_g (Inertia, girder) (mm^4)	3	Normal	$72.48 x 10^6$	$80.31 x 10^6$	$72.48 x 10^6$	$80.31 x 10^6$
I_b (Inertia, beam) (mm^4)	3	Normal	$72.48 x 10^6$	$80.31 x 10^6$	$0.8323 x 10^6$	$0.8878 x 10^6$
E (Young's modulus) (GPa)	2,3,4,5	Normal	207	207	207	207
σ_Y (Yield stress) (MPa)	8	Lognormal	245	245	245	245
P (load) (kPa)	10,15*,20,25,30	Weibull	137.895	137.895	34.474	34.474

Table 3. Comparison between results for DM, EM and the FEA for maximum deflection, δ_{max} , and stress, σ^g and σ^b for girder and beam respectively.

Grillage structure	Beam Type	Solution	Displacement method (Clarkson 1965)		Energy Method (Vedeler 1945) (Shear and torsion neglected)	FEA (Maneepan 2007)	Error between Energy Method and FEA(%FEA)
			Torsion included	Shear and torsion neglected			
4x4	I	δ_{max}(mm)	10.95	10.95	11.55	11.51	0.35
		σ_{max}(MPa)	183.17	183.32	196.12	189.42	3.54
	Box	δ_{max} (mm)	9.296	9.63	10.42	9.83	6.0
		σ_{max}(MPa)	160.16	165.56	176.91	157.48	12.3
4x5	I	δ_{max}(mm)	20.32	20.40	21.96	20.47	7.28
		σ^g_{max}(MPa)	137.15	137.92	142.85	135.10	5.7
		σ^b_{max}(MPa)	206.64	205.41	208.17	209.81	-0.78
	Box	δ_{max}(mm)	16.84	18.34	19.93	17.306	6.65
		σ^g_{max}(MPa)	108.57	125.41	129.58	107.72	20.3
		σ^b_{max}(MPa)	238.15	184.71	189.17	235.14	-19.6

Reliability Analysis

Following the successful analytical approach for representing the mechanical response of an isotropic stiffened panel and extending the approach for anisotropic material by the use of equivalent elastic properties, a reliability analysis can be undertaken with confidence.

The reliability of a structure is defined as the probability that the structure will perform its intended function without failing. Defining a performance function, or limit state function, g(x), as the difference between structural "capacity" and "demand" then:

- $g(x) > 0$ then the structure is safe.
- $g(x) < 0$ then the structure has failed.
- $g(x) = 0$ defines the failure limit state between survivability and failure

In this paper the reliability is given as the probability that the calculated stiffened panel deformations and stresses are less than the permissible values: a stiffness limit state and a strength limit state.

The reliability index or safety index is effectively a measure of how far inside the "safe" zone the structure is operating – approaching a zero value, the probability that a structure will fail approaches 100%.

The importance that each random variable has on the overall grillage response can be examined by the evaluation of the sensitivity index, α. The larger the sensitivity index, the more influential the particular random variable is on the overall limit state function.

For the following cases, all probabilistic computations are carried out with the computer program CALREL using first order and second order reliability methods (FORM/SORM) (Liu, Lin, & Der Kiureghian 2008).

Reliability of stiffened steel plate

Deflection Limit State

The deflection limit state function is defined as follows:

$$g(x) = k \times w_{\max} - w(L, B, I_g, I_b, E, P) \quad (21)$$

where w_{max} is the maximum displacement at any of the locations represented in Figure 3 using the mean values of the design parameters, Table 3. k is an arbitrary factor - it is taken as 2 in this problem. L and B are the length and width of the grillage structure; I_g and I_b are the second moment of area of the girder and beam respectively; E is Young's modulus for steel; P is a uniformly distributed load.

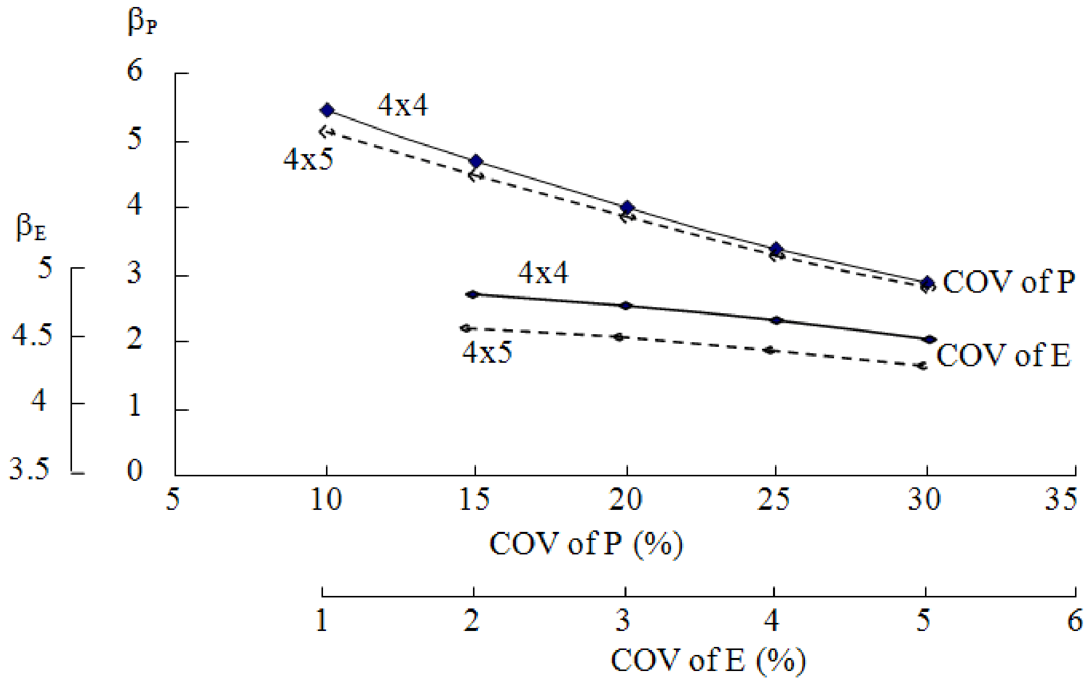


Figure 3. Influence of COV of load, P and Young's Modulus, E on reliability index, β.

Figure 3 shows the effect of increasing uncertainty in the quantity of load uniformly distributed across the stiffened plate has an almost linear reduction in the reliability of the plate's deflection over the specified range. This is not an unsurprising result given the sensitivity of the load quantity on the deflection limit state function (Table 4).

Table 4.Deflection limit state sensitivity indices

Variable	Sensitivity Index, α	
	4x4	4x5
	I or Box	I or Box
L	0.4741	0.6248
B	0.4741	0.2808
I_g	0.1018	0.1631
I_b	0.1018	0.0327
E	0.2063	0.1972
P	0.698	0.6814

From Table 4, the largest contributions to the reliability of either plates is from load and the plate size but for both aspect ratio plates the sensitivity of the Young's modulus on the deflection limit state of the stiffeners cannot be ignored. With increasing uncertainty in the value of Young's modulus, the reliability of stiffened plates with regards limiting deflection can be seen in Figure 3. Increasing the uncertainty in the Young's modulus for the steel from 2% to 5% leads to a reduction in reliability index of 0.25. For both panels this equates to over a threefold increase in the probability of failure.

Stress limit state

A reliability analysis is undertaken assuming the distributions described in Table 3. The COV for load, P, is taken as 15%. The yield stress is defined as being represented by a lognormal distribution.

The stress limit state equation g(x) is defined as,

$$g(x) = \sigma_y - \sigma(L, B, I_g, I_b, P) \quad (22)$$

where σ_g and σ_b are the maximum stresses of the girder and beam respectively, calculated by the grillage analysis. The calculate reliability index and probability of failures are given in Table 5. Stiffening with box beams for either aspect ratio plate leads to a much higher plate reliability in terms of limiting the maximum perceived stress.

Table 5. Stress reliability

Grillage	Stiffener type		Reliability Index	Probability of failure
4×5	I	Beam	0.8359	2.016×10^{-1}
		Girder	3.6051	1.560×10^{-4}
	Box	Beam	1.3785	8.402×10^{-2}
		Girder	4.3604	6.492×10^{-6}
4×4	I	Beam or Girder	1.4753	7.006×10^{-2}
	Box	Beam or Girder	2.1535	1.564×10^{-2}

Sensitivity analyses showed that the relative importance of the variables is almost identical between the two beam types. For brevity therefore, sensitivity factors are given only for I-beam types in Table 6.

Table 6. Stress limit state sensitivity indices

Variable	Sensitivity Index, α		
	4x4	4x5	
	Beam & Girder	Beam	Girder
L	0.4762	0.5993	0.3414
B	0.0922	0.1312	0.3293
I_g	0.1944	0.1501	0.1993
I_b		0.0094	0.0366
σ_Y	0.5133	0.4231	0.6165
P	0.6808	0.6496	0.5949

For the 4x4 grillage sensitivities shown in Table 6, the largest effect on the stress limit state comes from the uncertainty in the applied load but also the effects of the uncertainty in yield stress and panel aspect ratio are significant.

Reliability of a Composite Stiffened Panel

The grillage chosen for investigation is the 4x4 panel with box or tophat stiffening. The structure measures 3810mm square and is simply supported at all edges (cf. Figure 3a). The longitudinal girder and transverse beam dimensions are given in Table 1. A uniform pressure of 137kPa is applied on the grillage structure.

Reliability analyses is performed using the (mean) material properties of the resin and fibre listed in Table 7.

Table 7. Material properties of resin & fibre

	Epoxy	HM Carbon
Young's modulus, E (GPa)	3	826
Poisson's ratio, ν	0.37	-
Shear modulus, G (GPa)	1.09	413
Tensile strength (MPa)	85	2200
Tensile failure strain (%)	5	0.3
Compressive strength (MPa)	130	-

Elastic properties for a unidirectional layer should be established ideally by tests, however, for initial design purposes, it may be obtained by several approximations to the elastic constants with reasonable accuracy (Nayak, Das, Blake, & Shenoi 2006).

Deflection limit state

The deflection limit state function is defined below as a function of the random variables,

$$g(x) = k \times w_{\max} - w(L, B, P, E_f, E_m, G_f, G_m, V_f) \qquad (23)$$

where w_{max} is the maximum displacement using the mean values of the design parameters. k is a safety factor and is equal to 2 in this problem. The reliability analysis is performed with the following statistics of the design variables given in Table 8. The results for the reliability index and probability of failure are listed in Table 10.

Table 8. Statistics for random variable for composite grillage (deflection limit state)

Random Variable	Distribution	Mean Value	C.O.V %
L (Length)	Normal	3810mm	3
B (Width)	Normal	3810mm	3
P (pressure)	Weibull	137kPa	15
E_f (fibre)	Normal	826GPa	3
E_m (resin)	Normal	3.0GPa	3
G_f (fibre)	Normal	413GPa	3
G_m (resin)	Normal	1.09GPa	3
V_f (fibre volume fraction)	Normal	0.6	3
ε_f^* (Tensile failure strain %)	Normal	0.3	3

Table 9. Reliability of composite grillage

Method	Reliability Index, β	Probability of Failure, P_f ($\times 10^{-6}$)
FORM	4.6927	1.348
SORM	4.7446	1.045

From Table 9, the inclusion of second order terms in the linearization of the limit state equation (SORM) has only a marginal consequence on the predicted reliability compared to the consideration of only the first order terms (FORM). The predicted reliability of the composite grillage is 1 in approximately 740000 grillages would be expected to fail the deflection limit state. Comparing with those determined from Figure 3 suggests that the equivalent steel plate is marginally more reliable with 1 in approximately 770000 grillages expected to fail.

The dominant variables in the limit state equation on the reliability of the composite grillage can be seen in Figure 4.

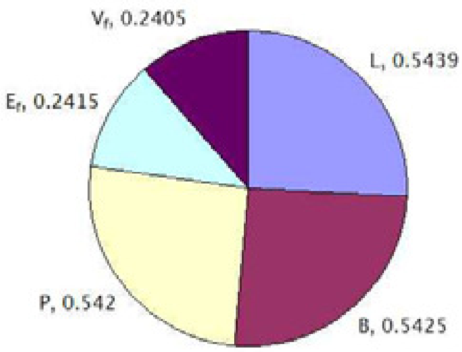


Figure 4. Sensitivity factors 4x4 box stiffened composite grillage

The effect of uncertainty in the stiffened composite plate dimensions, *L* and *B* and load, *P*, have quite sizeable contributions to the probability of the deflection limit state being exceeded which cannot be ignored. It is also noticed that Young's modulus and fibre volume fraction also have an important contribution on the deflection limit state. Unrepresented in this figure are the sensitivities for the Young's modulus of resin E_m and G_f and G_m, the shear modulus of the fibre and the resin, which play such small roles in contributing to the probability of failure that they can be treated as deterministic constants.

It is interesting to compare these results with the results for an equivalent steel grillage, Table 4. The Young's modulus for the fibre and the fibre volume fraction provide a measure of the laminate stiffness which is analogous to the Young's modulus for steel for the isotropic grillage example. Indeed the uncertainty of the Young's modulus for the isotropic example has a significant influence on the deflection limit state. However from Table 4, it is also clear that the moments of inertia of the girders and beams have an influence that should not be neglected statistically, whereas their uncertainty with regards the composite grillage can be ignored and the dimensional accuracy of the stiffeners is assumed to be assured (I_g and I_b are deterministic constants).

Stress limit state

Using maximum stress criteria, the crown of the composite structure is assessed with regards its failure. The stress limit state function is therefore,

$$g(x) = X_t(E_f, E_m, V_f, \varepsilon_f^*) - \sigma_{\max}(L, B, P, E_f, E_m, G_f, G_m, V_f) \qquad (34)$$

in which X_t is the ultimate tensile strength determined by the mean values of its dependent variables and σ_{max} is the maximum stress in the crown. The reliability analysis is performed with the statistics for the design variables described in Table 8. The results for the reliability index are very large, over 20 in value, which is equivalent to a probability of failure equal to zero - the "demand" contribution to the stress limit state function is far removed from the grillage "capacity" and in fact the average maximum stress level in the grillage crown is 175MPa whereas the average maximum tensile strength of the crown is an order higher at 1470MPa. It appears that in comparison to the steel grillage example, the composite grillage is effectively "infinitely" more reliable when the limit state is maximum stress.

The sensitivity of the limit state function on the random variables is shown in Figure 5 where only the dominant variables are shown. Any uncertainty in the true value of the tensile failure strain significantly affects the stress limit state equation. The stress is also dependent on the fibre volume fraction and the Young's modulus for the fibre. Most importantly though is the variation in the length dimension of the panel. Young's modulus of resin E_m and shear modulus of fibre and resin, G_f and G_m, unrepresented in Figure 5, again play a small role in the reliability analysis and as such can be treated as deterministic constants.

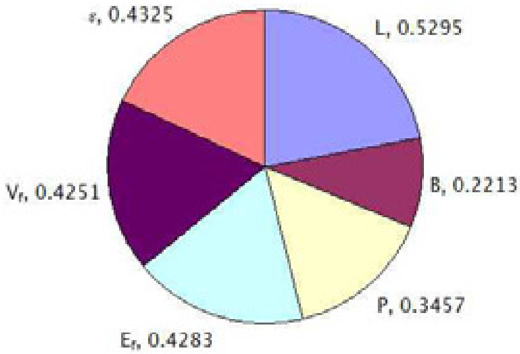


Figure 5. Sensitivity factors 4x4 box stiffened composite grillage

Conclusions

Using a grillage analogy for a stiffened plate, a structural model has been generated, validated against known solutions for steel grillages, and extended using equivalent elastic properties for laminates to consider anisotropy. From limited data for composite stiffened plate analyses, there is confidence in the approach but it is anticipated that future experimental tests will be required for improvements to be made to the structural model.

Reliability analyses have been performed on the isotropic steel grillage and on the anisotropic composite grillage, providing reliability indices and corresponding probabilities of failure can therefore be determined for the two limit states presented in this paper of deflection and stress. The importance of the random variables in the prediction of reliability can be determined through investigation of the sensitivity indices and it is apparent that load is important. Load typically is considered a subjective uncertainty as often the phenomenological behaviour, such as wave loads, is not wholly understood. Accumulating good qualitative data is important to forming target structural reliability and therefore efficient design.

From analyses of the composite grillage, one benefit of reliability methods can be readily seen. Composite design, manufacture and processing have many random variables that can be considered to affect the structural performance of the finished product. The subsequent operation of that product can induce cracking, water ingress, durability issues, material failure and so on, each of which is influenced by the very nature of the composite material itself. Using reliability analyses can identify which random variables are more influential on the resulting performance. In terms of manufacture or repair this may have the more obvious advantage of allowing the engineer to concentrate on these more important areas – for example, geometry, fibre angle as cloths are stacked, process technique to maximise fibre volume fraction and so forth. With the methods presented in this paper, further developments are now possible.

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