A note on travelling-wave solutions to Lax's seventh-order KdV equation

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Abstract

Ganji and Abdollahzadeh [D.D. Ganji, M. Abdollahzadeh, Appl. Math. Comput. 206 (2008) 438–444] derived three supposedly new travelling-wave solutions to Lax's seventh-order KdV equation. Each solution was obtained by a different method. It is shown that any two of the solutions may be obtained trivially from the remaining solution. Furthermore it is noted that one of the solutions has been known for many years.

Key words: Travelling-wave solutions. Lax's seventh-order KdV equation.
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Over the past two decades several expansion methods for finding travelling-wave solutions to nonlinear evolution equations have been proposed, developed and extended. The solutions to dozens of equations have been found by one or other of these methods. Reference [1] and references therein mention some of this activity. Unfortunately, many authors claim that their solutions are 'new', when the truth is that these solutions are merely 'old' solutions in a different guise. Recently, in a series of enlightening papers [2–4], Kudryashov has warned researchers and referees of the danger of not recognizing that apparently different solutions may simply be different forms of the same solution. He has provided numerous examples to illustrate this phenomenon. In the present note we point out one more example.

Recently, Ganji and Abdollahzadeh [1] presented three travelling-wave solutions to Lax's seventh-order KdV equation

$$u_t + (35u^4 + 70(u^2 u_{xx} + u u_x^2) + 7(2u u_{xxx} + 3u_{xx}^2 + 4u_x u_{xxx}) + u_{xxxxxx})_x = 0. \quad (1)$$

Each solution was derived by a different method. We wish to point out that the second and third of these solutions may be derived trivially from the first solution, and that the latter has been known for at least a dozen years.

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The first solution to (1) is given by Eq. (4.5) in [1], namely

$$u = a_0 + 2\alpha^2 \operatorname{sech}^2 \xi, \quad \xi = \alpha(x - \beta t), \quad \beta = 64\alpha^6 + 224\alpha^4 a_0 + 280\alpha^2 a_0^2 + 140a_0^3, \tag{2}$$

where a_0 and α are arbitrary. It was derived by using the sech-function expansion method.

The authors formulated the sec-function expansion method, applied it to (1), and obtained another solution given by (4.8) in [1], namely

$$u = a_0 - 2\alpha^2 \sec^2 \xi, \quad \xi = \alpha(x - \beta t), \quad \beta = -64\alpha^6 + 224\alpha^4 a_0 - 280\alpha^2 a_0^2 + 140a_0^3, \tag{3}$$

where a_0 and α are arbitrary.

Finally, they used the 'rational function in exp' method to get (4.12) in [1], namely

$$u = a_0 + \frac{2\alpha^2}{(1+e^\xi)} - \frac{2\alpha^2}{(1+e^\xi)^2}, \quad \xi = \alpha(x - \beta t), \quad \beta = \alpha^6 + 14\alpha^4 a_0 + 70\alpha^2 a_0^2 + 140a_0^3. \tag{4}$$

Firstly, we note that (3) may be obtained from (2) by using the transformation $\alpha \to i\alpha$ and the identity $\operatorname{sech}(iX) = \sec(X)$. Thus the effort involved in applying the sec-function method, as described in [1], is superfluous.

Secondly, we observe that the identity

$$\tanh X = 1 - \frac{2}{(1+e^{2X})} \tag{5}$$

implies that any solution expressed as a series in $\tanh X$ may be written as a series in inverse powers of $1 + e^{2X}$. In particular, (4) may be obtained from (2) by using the transformation $\alpha \to \alpha/2$ and the identity (5).

Written in terms of the tanh function, (2) becomes

$$u = b_0 - 2\alpha^2 \tanh^2 \xi, \quad \xi = \alpha(x - \beta t), \quad \beta = -4(96\alpha^6 - 196\alpha^4 b_0 + 140\alpha^2 b_0^2 - 35b_0^3), \tag{6}$$

where b_0 and α are arbitrary. This solution has been known for at least a dozen years. For example it is given by Eq. (6.4) in [5] where it was derived by using the Automated Tanh-Function Method (ATFM), a Mathematica package designed to take the drudgery out of applying the tanh-function expansion method by hand. The particular case given by putting $a_0 = 0$ in (2) or, equivalently, $b_0 = 2\alpha^2$ in (6), namely

$$u = 2\alpha^2 \operatorname{sech}^2[\alpha(x - 64\alpha^6 t)] \tag{7}$$

is mentioned in [1] and [5]. In the latter, it is noted that (7) agrees with the solution given over twenty years ago in Table 3 in [6] with appropriate choices of parameter values.

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