Analysis of a cyclotron maser instability with application to space and laboratory plasmas

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When a beam of electrons moves into an increasing magnetic field, conservation of the magnetic moment results in the formation of a crescent, or horseshoe shaped velocity distribution. The resultant horseshoe shaped velocity distribution has been shown to be unstable with respect to a cyclotron-maser type instability. This instability has been postulated as the mechanism responsible for auroral kilometric radiation and also non-thermal radiation from other astrophysical bodies. In this paper the previous theory, that assumed an infinite uniform plasma, is extended to apply to a bounded cylindrical geometry. This more exact theory in bounded cylindrical geometry is also directly relevant to a laboratory experiment currently being carried out.

1. Introduction

When a beam of electrons moves into a converging magnetic field, the velocity distribution function takes on a horseshoe shape as a result of conservation of magnetic moment. A few years ago it was pointed out that such a distribution is unstable to a cyclotron maser type of instability and it was suggested that this instability might be the source of auroral kilometric radiation [1] and also of emission from certain types of star [2].

The theoretical analysis presented in the earlier papers is somewhat approximate and assumes an infinite uniform plasma. We discuss in this paper a more exact theory in cylindrical geometry, obtaining a dispersion relation for the modes in a cylindrical cavity in which the driving electron beam occupies a central cylindrical region or an annular region.

This cylindrical geometry configuration is directly relevant to a laboratory experiment which is currently being carried out and which is described in an accompanying paper [3]. This experiment exploits the fact that the mechanism only depends on dimensionless ratios like the ratio of the wave frequency to the cyclotron and plasma frequencies and the factor by which the magnetic field varies along the path of the electron beam. Thus it is possible to scale the effect to laboratory size and to frequencies of several GHz and above.

2. Theory

Adiabatic invariance of the magnetic moment and particle energy applied to electrons in an increasing magnetic field results in the transformed distribution with a shape of a horseshoe in velocity space [1], having the following form for an initial drifting Maxwellian distribution:

$$f(v_{\|},v_{\perp})=Ae^{-\frac{m}{2T}\left[\left(\sqrt{v_{\|}^{2}+(1-B/B_0)v_{\perp}^{2}}-v_0\right)^2+B/B_0\,v_{\perp}^{2}\right]} \qquad (1)$$

We consider radiation near the fundamental of the electron cyclotron frequency. The relevant elements of the dielectric tensor are calculated for the distribution (1) using standard formulae as, given for example, in [4].

To investigate the properties of the horseshoe distribution instability we look for the growth rate of different modes, determined as the imaginary part of a complex-value k_z in the field propagator $e^{-iwt+ik_z z}$. The frequency is chosen to be just below the cyclotron frequency, in the range where the instability is expected to occur.

To obtain the dispersion equations for k_z we look for a solution of Maxwell's equations with the dielectric tensor described above, in a form of sums of anisotropic TE and TM modes. Each TE or TM mode has transverse components in the form of an "anisotropic sum" of the corresponding TE and TM components for the isotropic case, which means that different components are additive with different weights,

$$\begin{cases} E_r^{TE,TM} = aE_{r\_isotropic}^{TE} + bE_{\varphi\_isotropic}^{TM} \\ E_\varphi^{TE,TM} = cE_{r\_isotropic}^{TE} + dE_{\varphi\_isotropic}^{TM} \end{cases}. \qquad (2)$$

In such a representation the field components are sought in the following form:

$$E_r^{TE,TM}(r,\varphi) = \frac{J_n(\beta_{pl}^{TE,TM} \cdot r/R_1)}{r} \cdot$$

$$\cdot \left(A_1^{TE,TM}\cos(n\varphi) + A_2^{TE,TM}\sin(n\varphi)\right) +$$

$$+ J_n'(\beta_{pl}^{TE,TM} \cdot r/R_1)\left(B_1^{TE,TM}\cos(n\varphi) + B_2^{TE,TM}\sin(n\varphi)\right)$$

and (3)

$$E_\varphi^{TE,TM}(r,\varphi) = \frac{J_n(\beta_{pl}^{TE,TM} \cdot r/R_1)}{r} \cdot$$

$$\cdot \left(C_1^{TE,TM}\cos(n\varphi) + C_2^{TE,TM}\sin(n\varphi)\right)$$

$$+ J_n'(\beta_{pl}^{TE,TM} \cdot r/R_1)\left(D_1^{TE,TM}\cos(n\varphi) + D_2^{TE,TM}\sin(n\varphi)\right)$$

while the z-components have a form similar to that of the isotropic case, because we have only a transverse anisotropy,

$$E_z^{TE}(r,\varphi) = 0,$$

$$E_z^{TM}(r,\varphi) = \frac{J_n(\beta_{pl}^{TM} \cdot r/R_1)}{r} \cdot \left(F_1^{TM}\cos(n\varphi) + F_2^{TM}\sin(n\varphi)\right) \qquad (4)$$

where J_n and J_n' are the Bessel functions of the first kind and their derivatives, respectively.

TE and TM modes inside the plasma region have different β_{pl} determined by the following dispersion equations:

$$\left(k_{11} - k_z^2\right)\left(\frac{\beta_{pl}^{TE^2}}{R_1^2} - k_{11} + k_z^2\right) = k_{12}^2 \qquad (5)$$

for TE modes and

$$\left(k_{11}\left(\frac{\beta_{pl}^{TM^2}}{R_1^2} - k_{33}\right) + k_z^2\right)\left(\frac{\beta_{pl}^{TM^2}}{R_1^2} - k_{11} + k_z^2\right) +$$

$$+ k_{12}^2\left(\frac{\beta_{pl}^{TM^2}}{R_1^2} - k_{33}\right) = 0 \qquad (6)$$

for TM modes, where k_{11} and k_{12} are the elements of the dielectric tensor and depend on the radiation frequency ω.

TE and TM modes in plasma in the proposed representation can exist separately, but when we consider a configuration with either a cylindrical core, or an annulus filled with plasma, then the anisotropic nature of the plasma response requires that both modes be present. Otherwise it is not possible to satisfy the boundary conditions. These modes have different spatial structures due to different β_{pl}. The coefficients for the fields and additional dispersion equation for finding β_{pl} can be obtained by applying boundary conditions at the plasma interface, r=R1, with the field in the vacuum region being expressed in a conventional way as a sum of TE and TM modes.

3. Results

We consider how the growth rate depends on the mode structure and look at the effect of different radial distributions of the driving electron beam. The analytic results confirm that the growth rate is sufficient for us to expect the instability to reach saturation within the dimensions of the experiment, a typical length scale for exponential growth being of the order of 10 cm.

The analytic growth rates are also compatible with the rate of growth seen in computer simulations. The dispersion equations we have obtained allow us to look separately at each spatial mode generated by the horseshoe distribution instability. Calculating an accurate solution is very fast, which is an advantage in comparison with fully numerical simulations like PIC codes.

Typical results for the growth rate per metre of the coupled TE and TM mode for different mode numbers are given in Fig.1. One can see modes growing (white) with a high rate and also modes damping (black).

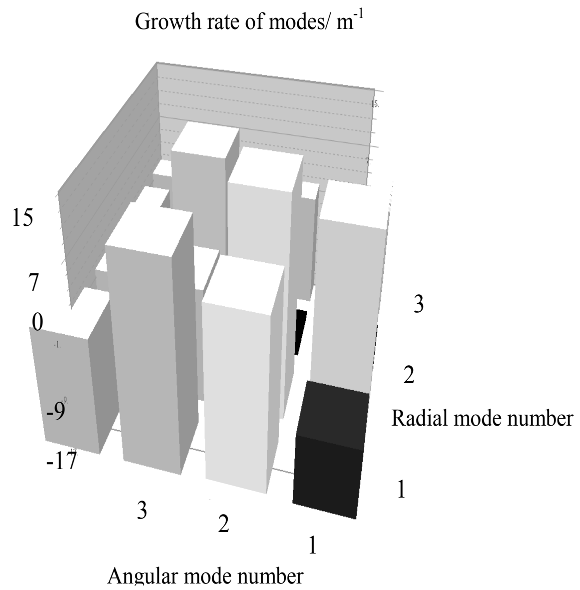

Fig.1. Growth rate of different TE and TM coupled modes

4. Acknowledgements

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5. References

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